Solving Non-Linear Magnetic Problems using Newton Trust Region Methods

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Abstract – **In this paper, a Newton trust region method is presented as an improvement, with no extra computational cost, of the underrelaxed Newton-Raphson method. The step is now determined by minimizing the local quadratic approximation of the energy functional within a trust region whose dimension is automatically adapted.**

Introduction

The solution of non-linear magnetic problems can be determined by minimizing the energy functional F:

$$F(\mathbf{A}) = \int_{\Omega} w_{\mathrm{m}}(\mathbf{A})\,\mathrm{d}\Omega \; - \; \mathbf{A}^{\mathrm{T}}\mathbf{T} \quad , \tag{1}$$

with $\mathbf{A}$ the vector of unknown vector potentials (in Vs/m), w_{m} the energy density (in J/m^3) and $\mathbf{T}$ the source vector (in A) [1]. The local quadratic approximation of F around the solution at the k$^{\mathrm{th}}$ iteration equals

$$F_{\mathrm{qm}}(\mathbf{A}_k + \mathbf{s}_k) \approx F(\mathbf{A}_k) + \mathbf{s}_k^T \nabla F(\mathbf{A}_k) + \frac{1}{2}\mathbf{s}_k^T \nabla^2 F(\mathbf{A}_k)\mathbf{s}_k \tag{2}$$

and exhibits ellipsoidal isovalues in a multidimensional parameter space (Fig. 1). Analytical formulas for the gradient ∇F and the Jacobian $\nabla^2 F$ are given in [1]. The Newton step towards the minimum of (2) equals

$$\mathbf{s}_k^{\mathrm{N}} = -\left[\nabla^2 F(\mathbf{A}_k)\right]^{-1} \nabla F(\mathbf{A}_k) \quad . \tag{3}$$

It is the direction along which a line search is performed in order to determine the underrelaxation factor á$_k$ of the enhanced Newton-Raphson method (Fig. 1) [2].

Trust Region Methods

As $\mathbf{s}_k^{\mathrm{N}}$ and á$_k$ are computed separately, the underrelaxed Newton-Raphson method may fail to realize the minimum of (2) in a sphere of radius $\alpha_k \left\| \mathbf{s}_k^{\mathrm{N}} \right\|$. In trust region (TR) methods, both the direction and the step length are computed simultaneously by minimizing (2) within a TR-radius Ä$_k$ around $\mathbf{A}_k$ (Fig. 1) [3]. The trust region can be a hypercube as well. This constrained minimization problem is efficiently solved by a variant of the conjugate gradient method [4].

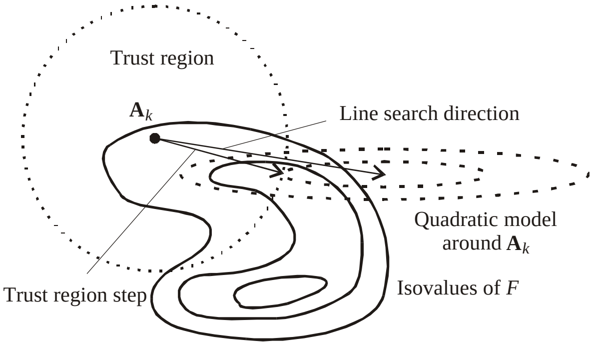

Fig. 1: Graphical interpretation of a trust region method applied to a function F with two variables.

Fig. 1 shows that the decrease of the functional may be larger than possible along the Newton direction. Obviously, the performance of a TR method depends on the size of Ä$_k$. For large Ä$_k$, the step equals the Newton step. For small Ä$_k$, only little progress is made towards the solution. Therefore, the TR radius is adjusted at each new iteration. If the ratio

$$\rho_k = \frac{F(\mathbf{A}_k) - F(\mathbf{A}_k + \mathbf{s}_k)}{F_{\mathrm{qm}}(\mathbf{A}_k) - F_{\mathrm{qm}}(\mathbf{A}_k + \mathbf{s}_k)} \tag{4}$$

is close to 1, the actual reduction of the functional more or less equals the predicted reduction by the local quadratic model in (2). Hence, the latter is a reliable representation of the functional and Ä$_k$ may be increased. If there is less similarity, e.g. if $\rho_k \in [0.25, 0.75]$, the radius is not modified. Otherwise, Ä$_k$ is decreased and the last iterate is rejected [3].

Conclusion

The computational cost for the trust region is approximately equal to the one of a Newton-Raphson step. Per iteration, a trust method requires only one evaluation of the functional (1), which is an advantage compared to the underrelaxed Newton-Raphson method. The convergence is guaranteed by the ratio check in (4) and it becomes quadratic close to the solution. The full paper discusses this method in detail for a practical application in computational magnetics.

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