An algorithm to construct the discrete cohomology basis functions required for magnetic scalar potential formulations without cuts

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Abstract— **Magnetic scalar potential formulation without cuts require the definition of a basis for the cohomology structure of the magnetic field function space. This paper presents an algorithm to construct such a basis in the general case thanks to an adapted spanning tree.**

Introduction

Magnetic potential formulations are very appealing for 3D problems. However, in the classical t-ω formulation, the magnetic field in the non-conductive region is represented by the gradient of the scalar potential ω only. That makes it necessary to define cuts in order to get rid of multivalued potentials, what is not an easy matter in the general case. In order to always avoid cuts, special topological functions have to be build that form a basis for the cohomology structure of the function space containing the magnetic field in the geometry under consideration. This paper demonstrates how to do so.

Topological Structure

Let Ω be a connected (2D or 3D) mesh, Γ_b and Γ_h be the complementary parts of the boundary $\partial\Omega$ of Ω where the fields $b_0.n$ and $h_0 \wedge n$ respectively are known. Let C be the domain occupied by all the conductors of the problem, $C \subset \Omega$, and ∂C be the boundary of C.

While $\operatorname{div} \mathrm{b} = 0$ can be automatically satisfied without any restriction by defining the vector potential a as $b = \operatorname{curl} \mathrm{a}$, the situation is more complicated for $\operatorname{curl} \mathrm{h} = 0$. One has to consider the topological structure of the functional space $F^{(1)}(\Omega - C)$. Let B^1 be the set of the gradients defined on $\Omega - C$ and Z^1 be the set of the curl-free fields defined on $\Omega - C$. Both are vector spaces. The quotient $H^1 = Z^1/B^1$ is the cohomology group of $F^{(1)}(\Omega - C)$. It is also a vector space but of finite dimension. If the conductors of the problem under consideration form an electrical circuit with N_Σ linearly independent loops, and if the sections of the conductors in which the currents $\{I_k, k = 0, \ldots, N_\Sigma\}$ can be imposed independently are called $\{\Sigma_k, k = 0, \ldots, N_\Sigma\}$, it can be shown that the dimension of H^1 is N_Σ.

The magnetic field in the non-conducting region can now be represented in the most general manner by

$$h = \sum_{k=0}^{N_\Sigma} I_p t^p + \operatorname{grad} \omega \tag{1}$$

where the t^p's form a basis for H^1 and ω is a continuous (no cut) scalar potential.

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Construction of the t_p's

The paper gives first an algorithm to build a spanning tree on a given region. A second algorithm is then given to construct one function t such that $\operatorname{curl} \mathrm{t} = 0$. Now, the source field must obey the following constraints:

$$\begin{cases} \operatorname{curl} \mathrm{t}^{\mathrm{p}} = 0 \ \text{ in } \Omega - \mathrm{C} \\ (t^p - h^0) \wedge n = 0 \ \text{ on } \Gamma_h \\ t^p \wedge n = 0 \ \text{ on } S \\ \operatorname{curl} \mathrm{t}^{\mathrm{p}}.\mathrm{n} = 0 \ \text{ on } \partial\mathrm{C} \\ \int_{\partial\Sigma_k} t^p = I_k \delta^{kp}, \ k = 1, \ldots, N_\Sigma \end{cases} \tag{2}$$

In order to ensure that those constraints are fulfilled, the spanning tree itself must be so that the subsets of the spanning tree edges that belong to the surfaces Γ_h, S and ∂C and to the curves $\partial\Sigma_k$ form also spanning trees on those surfaces and curves. However, the union of two trees defined independently on two meshes is not generally a tree on the union of the two meshes. A global approach is therefore needed which can be worked out thanks to the rule: *"The spanning tree build on the union of two meshes $D \cup D'$ includes a spanning tree on D and a spanning tree on D' if it includes a spanning tree on $D \cap D'$."* For that purpose, a more advanced spanning-tree algorithm is described that works by organizing the edges of the mesh into a hierarchy.

Conclusion

The magnetic scalar potential formulations require the definition of special topological fields, in the static case as well as in the dynamic case. The rules governing the construction of the those fields have been reviewed and an algorithm that follow those rules has been presented. The algorithm works without restriction provided the geometry is known. The user has just to define the surfaces whereon the magnetic field is subjected to constraints. The algorithm does not require the solution of any system of equation. Finally, it considers the problem in the most general case and it can be implemented once and for all.

References

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