The eggshell method for the computation of electromagnetic forces on rigid bodies in 2D and 3D

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Abstract—**This paper presents a technique to compute electromagnetic forces on rigid bodies. It is applicable in 2D and in 3D, and with dual formulations. The method requires only volume integrations although it originates from the Maxwell stress tensor. It clarifies the relationship between the latter and the virtual work principle.**

MAXWELL STRESS TENSOR

A system Ω with a moving rigid piece is considered. An eggshell shaped region S, of thickness $\delta <<$, is placed so to enclose the moving piece. The shell can be in contact or not with the moving piece but it must entirely lie in air. The interior and exterior of the egg are called Y and Z, so that $Y \cup S \cup Z = \Omega$. The value of the magnetic force acting on Y is $F_i = \int_\Gamma \sigma_{ij} n^j \, d\Gamma$ where σ_{ij} is the Maxwell stress tensor [1] and Γ a closed surface that encloses the moving piece.

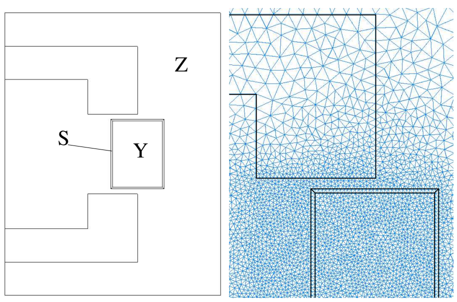

Fig. 1. Geometry of the c-core and detail of the mesh in the airgap.

THE EGGSHELL METHOD

The eggshell method is a generalisation of the Arkkio formula for torque in electrical machines [2]. The idea is to consider the eggshell S as a family of concentric equivalent surfaces for the evaluation of the magnetic force by the Maxwell stress tensor method, and to average them

$$F_i = \frac{1}{\delta} \int_0^\delta \int_{\Gamma(\delta)} \sigma_{ij} n^j(\delta) d\Gamma \; d\delta \qquad (1)$$

which writes more simply $F_i = \int_S \sigma_{ij} \operatorname{grad} \phi \; \mathrm{dS}$ where ϕ is a scalar function that varies smoothly from zero on the inner surface of the eggshell to 1 on the outer surface.

RESULTS

The eggshell method is tested on a c-core problem (Fig. 1), solved with dual finite element formulations, in 2D and 3D. A block of ferromagnetic material is inserted in the airgap of the core. The force computed with the eggshell method is compared with the force computed by direct differentiation of (co)energy, using a second order finite difference approximation (Fig. 2). A perfect match of the curves is observed, which shows that the methods are perfectly equivalent. The eggshell method however, requires only one resolution of the system. The difference between the values computed with the $b-$formulations and those computed with the $h-$formulation are due to discretisation errors.

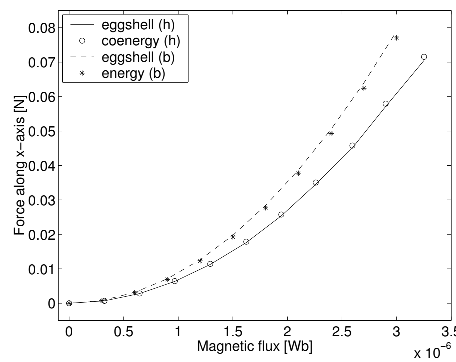

Fig. 2. Comparison of the forces computed with the eggshell method and the derivation of energy (b formulation) or of coenergy (h formulation).

The method leaves quite a freedom in the definition of the eggshell : its thickness, position and discretisation are free parameters of the method. The effect of those parameters will be shown in the full paper.

CONCLUSION

As will be shown in the full paper, the eggshell method helps making the link between the virtual work principle and the Maxwell stress tensor, of which it contributes therefore to clarify the origin and the applicability conditions. The accuracy of the method is identical to the one of equivalent classical techniques. It behaves well in case of flat elements. It has also a practical interest because it is very general, flexible, efficient and easy to implement (only volume integrations are involved). The eggshell region has however to be described explicitely in the geometry of the problem.

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REFERENCES

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[2] Arkkio. Analysis of induction motors based on the numerical solution of the magnetic field and circuit equations. *Acta Polytechnica Scandinavica*, page 56, 1987.