A Preconditioned Krylov Subspace Solver for a Saddle-Point Model of a Single-Phase Induction Machine

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ABSTRACT

To simulate steady-state working conditions of single-phase induction machines, a modified time-harmonic magnetodynamic formulation is applied [1]. The elliptical magnetic field in the air gap is decomposed into its forward and backward rotating components which are applied to two distinct rotor models (Fig. 1). This enables the application of the slip transformation technique to account for

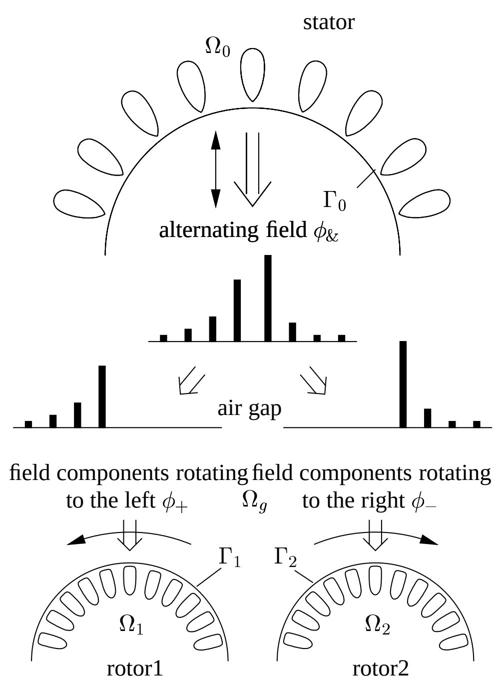

Figure 1: Scheme of the air gap flux splitting approach.

motional eddy current effects and hence avoids transient simulation. A 2D finite element discretisation is applied to the stator, the two rotors and the air gap. The vector u_* contains the degrees of freedom associated with the nodes inside the stator and rotor domains. The vectors u_0, u_1 and u_2 contain those associated with an equidistant, matching grid along the common interface formed by the stator inner boundary Γ_0 and the rotor outer boundaries Γ_1 and Γ_2. The decomposition of the air gap field corresponds to the constraints

$$\underbrace{\begin{bmatrix} 0 & R_+F & F & 0 \\ 0 & R_-F & 0 & F \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} u_* \\ u_0 \\ u_1 \\ u_2 \end{bmatrix}}_{u} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (1)$$

with F the discrete Fourier transform and R_+ and R_- restricting the set of harmonic components to the forward and backward rotating ones. The simulation of a device rotating at steady-state corresponds to solving the saddle-point problem

$$\begin{bmatrix} K & B^H \\ B & 0 \end{bmatrix} \begin{bmatrix} u \\ p \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix} \quad (2)$$

with p a vector of Lagrange multipliers, K a nonlinear complex symmetric system matrix and f a load vector. The application of B and B^H implies fast Fourier transforms, inverse fast Fourier transforms and explicit restrictions. Equation (2) is solved by the Bi-Conjugate Gradient Stabilised method with the block preconditioners proposed in [2], [3] and [4]. As a preconditioner M for K, an exact solution, symmetric successive overrelaxation or an algebraic multigrid step are considered. The Schur complement $S = BK^{\dagger}B^H$ with $K^{\dagger}$ the pseudoinverse of K, is approximated by $\tilde{S} = BM^{-1}B^H$. We will also discuss the performance of other preconditioners for S, such as interface preconditioners proposed in domain decomposition literature [5] and preconditioners based on existing semi-analytical approaches for single-phase induction machine simulation. The mixed formulation and corresponding iterative solution techniques are applied to a model problem and to a technical capacitor motor.

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