Air Gap Flux Splitting for the Time-Harmonic Finite Element Simulation of Single-Phase Induction Machines

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Abstract— **A time-harmonic finite element formulation is strongly coupled to a Fast Fourier discretisation. The approach enables the splitting of the alternating air gap field typical for single-phase induction machines into two counter-rotating parts which are applied to two distinct rotor models. Motional effects are incorporated by slip transformations. The approach is illustrated by the simulation of a capacitor-run motor.**

INTRODUCTION

Single-phase induction motors (IMs) mainly occur in domestic applications such as e.g. fans and lawn movers [1]. Because of the increased interest in efficient and qualitative operation of electrical machines, improved designs and hence also fast and accurate simulation techniques are required. In contrast to the case of three-phase IMs, simple time-harmonic (TH) finite element (FE) solvers do not apply. This paper develops a novel TH FE approach for single-phase IMs.

TIME-HARMONIC FINITE ELEMENT SIMULATION OF INDUCTION MACHINES

A fast and sufficiently accurate simulation of three-phase IMs at steady-state is offered by 2D TH magnetodynamic FE simulation [2]. The external electric connections, the supply and the influence of the end-winding and end-ring resistances and inductances are resolved by a field-circuit coupling approach [3]. The motional effects in the rotor are incorporated by applying the *slip transformation*, i.e. multiplying the rotor conductivities by the slip $s = \frac{\omega-\lambda\omega_m}{\omega}$ with ω the pulsation of the stator field, λ the pole-pair number and ω_m the mechanical velocity of the rotor [2]. This approach assumes a sinusoidally distributed revolving field in the air gap of the IM [4].

The elliptical air gap field excited by a single-phase IM is principally alternating instead of rotating. Hence, the slip transformation is not longer applicable. Single-phase IMs are commonly simulated by a transient FE solver [5] or by a combined FE, equivalent-circuit approach [6]. Both are troublesome, the former because of excessive computation times, the latter because of difficulties in considering local effects such as ferromagnetic saturation and eddy currents. This motivates the development of a TH FE simulation scheme for single-phase IMs.

AIR GAP FLUX SPLITTING

The analytical theory for single-phase IMs decomposes the alternating air gap field into two counter-rotating fields, corresponding to two fictitious three-phase IMs [1]. The electromotive force and the torque follow by the superposition of the three-phase characteristics. The approach, presented here, transfers this idea to the FE method. Unfortunately, the splitting of the air gap field within the FE context, is not obvious.

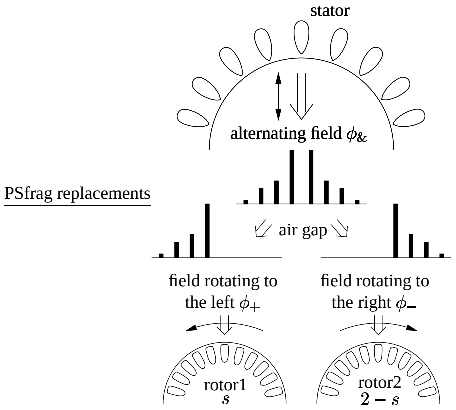

Fig. 1. Scheme of the air gap flux splitting approach.

The alternating field $\phi_\&$ excited by the stator in the air gap, is split into its Fourier components (Fig. 1). The forward and backward rotating components ϕ_+ and ϕ_- respectively, are propagated towards two distinct rotor models rotating at the same mechanical speed. Because the left and right rotor models experience magnetic fields ’rotating in the opposite direction, the slip transformation technique amounts to a multiplicating of the rotor conductivities by s and $2-s$ respectively.

The procedure is for a simple solid-rotor single-phase IM (Fig. 2). The stator excitation is replaced by an alternating field boundary condition at the outer radius of the model. The air gap is modelled by an analytical expression [7]. The flux splitting approach is applied at the outer radius of the rotor models. The magnetic flux lines are plotted for two instants of time, t_0 and t_1, a quarter of a period shifted in time. The superposed solution is shown on the rotor in the middle of the stator. The rotating fields in the left and right rotor models at t_0 and t_1 feature the same flux patterns shifted over 90 degrees mechanically. At standstill, $s = 1 = 2-s$ and both rotor models behave similarly yielding a symmetric alternating field. In the case of rotor movement, $s \neq 2-s$ and different flux patterns are observed.

The FE discretisation error is controlled adaptive mesh refinement. If $\omega_m \neq 0$, both rotor models respond differently. As a consequence, the error estimation marks different elements for refinement, yielding non-coinciding rotor meshes.

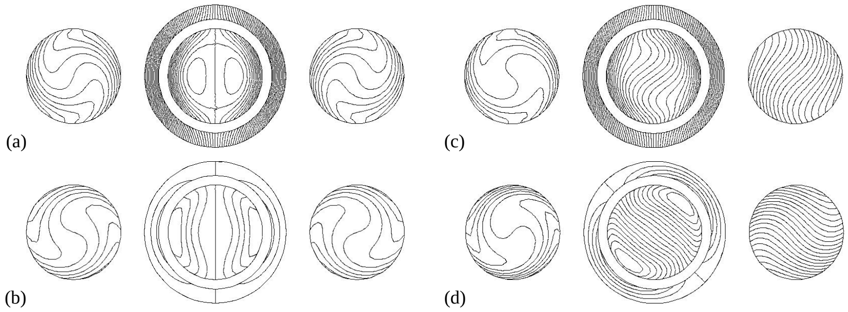

Fig. 2. Solid-rotor single-phase IM model: magnetic flux line plots at t_0 (above) and t_1 (below) for a standstill (left) and a moving (right) rotor excited by an alternating field.

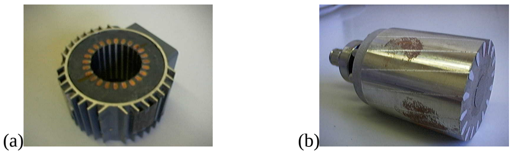

Fig. 3. (a) Stator and (b) rotor of a capacitor-run motor.

The effective magnetisation characteristics of the ferromagnetic materials are evaluated for the superposed solution at the middle rotor model (Fig. 2) [8]. The obtained permeabilities are projected onto the left and right rotor models before invoking the next step in the non-linear loop.

APPLICATION

The FE flux splitting approach is applied to simulate a loaded capacitor-run single-phase IM (Fig. 3). Both the stator and the rotor are slotted. The alternating field excited by the main winding is augmented by a field generated by the auxiliary winding, put in the quadrature axis of the main one. The connections of both windings to the supply, the running capacitor and additional impedances accounting for the end-windings and the end-rings, are represented by a circuit. The middle rotor model serves for superposing the solutions, evaluating the non-linear characteristic of the ferromagnetic iron and visualising the final solution. The magnetic flux lines for a slip of 0.5 are plotted in Fig. 4. The method enables the computation of the torque, the eddy currents and the local losses at a certain speed, up to the technically required accuracy, within a single computational step. The same method applies without modifications to shaded-pole IMs and reluctance-start IMs [1].

CONCLUSIONS

The air gap flux splitting approach is successfully applied within TH FE simulation and provides a fast and reliable design tool for single-phase IMs.

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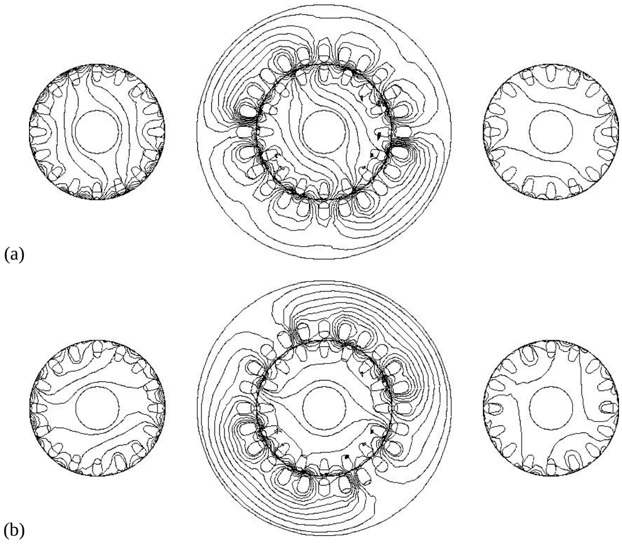

Fig. 4. Plot of the magnetic flux lines at t_0 and t_1 of the capacitor-run motor model operating at slip 0.5.

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