A Multi-Conductor Model for Finite Element Eddy Current Simulation

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Abstract— **The stranded conductor finite element model does not account for the skin and proximity effects in a multi-conductor system. The solid conductor model considers the true geometry, all the individual conductors and their connections which results in unmanageably huge models. The multi-conductor model proposed here, avoids meshing the inner geometry, discretises the voltage and enforces the current redistribution typical for multi-conductors by a weak formulation. The magnetic and electric meshes are independently and adaptively refined, yielding accurate results for relatively small models.**

INTRODUCTION

Multi-conductor (MC) systems arise in almost all quasi-static electrotechnical devices [1] (Fig. 1). Dependent on the frequency f, the permeability μ, the conductivity σ and the characteristic diameter d, the individual conductors experience skin and proximity effects related to the skin depth $\delta = \sqrt{\frac{1}{f\pi\mu\sigma}}$. If $\delta \gg d$, the stranded conductor finite element model is appropriate. If $\delta \ll d$, impedance boundary conditions are commonly applied. If δ and d are of the same order of magnitude, the skin effect is resolved by eddy current simulation relying upon the solid conductor model. The unidirectional skin effect in foil conductors is considered in [2].

Devices may feature a large amount of MC systems, each consisting of a considerable number of turns. This may hamper the simulation of the overall device. Several model reduction techniques, such as e.g. analytical macro-elements [3] and inner node elimination techniques [1], exist. They reduce the multi-conductor model parts in advance, and hence, lack adaptive error control for them during the proper finite element simulation. Here, the troublesome geometrical details are approximated by a discretisation and are incorporated as such in the finite element model. An error estimator updates the MC model during the simulation.

MULTI-CONDUCTOR MODEL

The MC has a cross-section Ω_{mc} in the xy-plane and a length ℓ_{mc}. An additional electric mesh is constructed on Ω_{mc} (Fig. 2). To simplify the implementation, a tensor grid is preferred. The voltage $\Delta V(x,y)$ across Ω_{mc} vary with x and y due to the vicinity of other conductors, permeable materials and local heating effects. $\Delta V(x,y)$ is resolved by n_{mc} *voltage shape functions* (VSFs) $M_q(x,y)$, on Ω_{mc}:

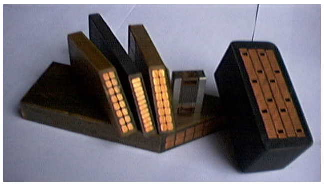

Fig. 1. Parts of technical multi-conductors used in high-voltage induction machines.

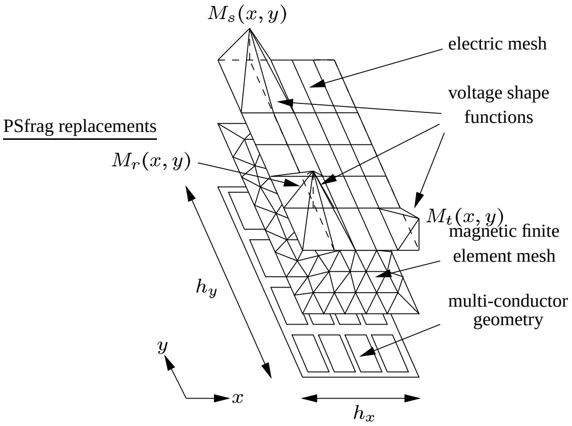

Fig. 2. Cross-section of the multi-conductor, the magnetic mesh, the electric mesh and three voltage shape functions.

$$\Delta V(x,y) = \sum_{q}^{n_{mc}} \Delta V_q M_q(x,y)\,. \tag{1}$$

Insulation material and gaps are accounted for by the fill factor $f_{mc} = \frac{N_{mc}\Delta_b}{\Delta_{mc}}$ with N_{mc} the number of turns and Δ_b and Δ_{mc} the cross-sections of an individual conductor and the entire MC respectively. The electric mesh does not coincide with the magnetic one nor with the true MC geometry. The consistency of the discretisation, however, required the voltage mesh to tend to the MC geometry if refinement is applied. Also, f_{mc} has to convergence to 1. Hence, the gaps and the insulation regions disappear out of the support of the electric mesh causing the latter to become disconnected.

Consider e.g. the 2D time-harmonic, magnetodynamic formulation in terms of A_z, the z-components of the magnetic vector potentials. The current density is

$$J_{mc}(x,y) = \frac{\sigma f_{mc}}{\ell_{mc}} \Delta V(x,y) - \jmath\omega\sigma A_z(x,y) \tag{2}$$

with $\omega = 2\pi f$ the pulsation. The average current density Z_{mc} in the MC is related to the MC current I_{mc} by $Z_{mc} = \frac{N_{mc}}{\Delta_{mc}} I_{mc}$. The restrictions to the current distribution due to insulation,

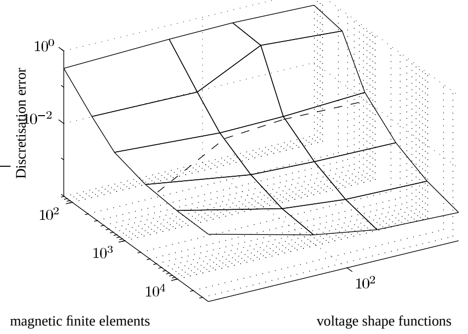

Fig. 3. Convergence of the discretisation error of the multi-conductor model.

are fulfilled in a weak sense. The VSFs serve as weighting functions in a Galerkin formulation forcing the average current in each rectangular electric element to equal Z_{mc}:

$$\int_{\Omega_{mc}} \left(J_{mc}(x,y) - Z_{mc}\right) M_p(x,y)\mathrm{d}\Omega = 0\,. \tag{3}$$

The voltage across the entire MC is attained by homogenising $\Delta V(x,y)$ over Ω_{mc}:

$$\Delta V_{mc} = \frac{1}{\Delta_{mc}} \int_{\Omega_{mc}} \Delta V(x,y)\mathrm{d}\Omega\,. \tag{4}$$

The weak formulation of the magnetodynamic problem, (3) and (4) are assembled into the coupled system of equations

$$\begin{bmatrix} k_{ij} & z_{iq} & 0 \\ z_{pj} & \xi g_{pq} & \xi s_p \\ 0 & \xi s_q & 0 \end{bmatrix} \begin{bmatrix} A_j \\ \Delta V_q \\ I_{mc} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \xi \Delta V_{mc} \end{bmatrix}. \tag{5}$$

with

$$k_{ij} = \int_\Omega \frac{1}{\mu} \nabla N_i \cdot \nabla N_j \mathrm{d}\Omega\,; \tag{6}$$

$$z_{iq} = -\int_\Omega \frac{\sigma}{\ell_{mc}} N_i M_q \mathrm{d}\Omega\,; \tag{7}$$

$$g_{pq} = \int_\Omega \frac{\sigma f_{mc}}{\ell_{mc}} M_p M_q \mathrm{d}\Omega\,; \tag{8}$$

$$s_p = -\frac{N_{mc}}{\Delta_{mc}} \int_\Omega M_p d\Omega\,. \tag{9}$$

Ω is the computational domain. A_j are the coefficients of $A_z(x,y)$ with respect to the magnetic finite elements $N_j(x,y)$. $\xi = 1/\jmath\omega\ell_{mc}$ is a symmetrisation factor. The MC model fits within the field-circuit coupling approach developed in [4].

CONVERGENCE

The convergence of the mixed discretisation technique is studied for an analytical example. The discretisation error is plotted in Fig. 3. The error decays both, when the magnetic mesh is refined and when the number of VSFs is increased. The dashed line denotes loci for which the error is identical. The experiment indicates that it is sometimes more advantageous to apply a finer magnetic mesh than to consider all geometrical details due to the electrical insulation in the MC system.

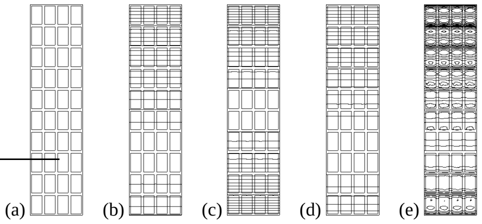

Fig. 4. (a) Geometry, (b) real and (c) imaginary components of the magnetic flux in the multi-conductor model at 50 Hz and (d) real and (e) imaginary components of the magnetic flux in the multi-conductor model at 500 Hz.

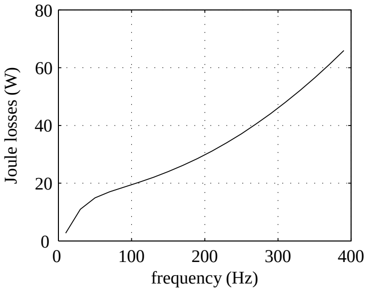

Fig. 5. Harmonic losses in an armature winding of an induction machine.

APPLICATION

The MC model is applied to simulate the harmonic losses in induction machine windings [5] (Fig. 4). At 50 Hz, no significant skin effect is observed. At 500 Hz, substantial losses are introduced. The MC model enables the simulations of all situations in Fig. 5 by the same conductor model. The automated mesh refinement techique only inserts elements where required, e.g. where the error estimator indicates high losses.

CONCLUSIONS

The multi-conductor model developed here, enables the simulation of complicated coil geometries with relatively small models. It offers more modelling flexibility when compares to the solid and stranded conductor models.

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