Finite element modelling of the induction heating of a moving wire

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Abstract— **This paper presents the finite element modelling of the induction heating of a moving ferromagnetic wire. Rather than focusing on the numerical techniques that have been applied, which are rather classical, the paper presents an unusual modelling methodology for strongly coupled and nonlinear problems that are hard, or even impossible to solve by classical approaches. By doing pertinent simplifications, the analysis is applied at an intermediary level, a well-chosen cell, that is representative of the phenomenology of the system but for which the finite element method is well-behaved.**

Keywords— **Computer aided engineering, electromagnetic heating, finite element methods, modelling, skin effect, nonlinearities.**

I. INTRODUCTION

The finite element (FE) modelling of the induction heating of a moving wire is quite a challenging task. All of the following features, which usually lead to serious difficulties in FE modelling, i.e.
— strongly coupled phenomena (magnetic, thermal, exterior circuit),
— very different characteristic times,
— very different characteristic lengths (small skin depth in large conductors),
— strong nonlinearities (magnetic phase change at Curie temperature, thermal radiation, magnetic saturation),
— moving pieces,
affect induction heating applications. Efficient numerical techniques already exist to overcome those difficulties individually but the major hindrance, in this case, comes from the fact that they are all concentrated in the same system.

In order to understand the complex behaviour of a strongly coupled system, it is however essential to analyse it as a whole, what seems at first sight to require the combination of several submodels and several numerical techniques. Moreover, considering the impressive development in the recent years of computer performances and algorithms, one might also feel free to imagine that nothing is able to set any limit to the power of representation of FE models and, consequently, that all the aspects of a system should be taken at once into consideration, i.e. within one single model.

But it rapidly arises from experience that *coupling is more than just putting things together*. Algorithmic and methodological innovations are also necessary. The direct combination of the different aspects of a problem deteriorates dramatically the numerical properties of the model and increases rapidly the

amount of computer resources and time needed. This leads generally to a terrible lack of efficiency, of accuracy and, above all, of reliability. The difficulty when modelling a complex system is not so much to write an appropriate set of equations but rather to be able to solve it suitably at the end.

II. THE "CELL" METHODOLOGY

The discussion of the previous section seems to indicate that the FE modelling of complete induction heating systems is practically out of reach at present. We propose in this paper a different approach based on the following observations:
— At R&D stage, engineers are still mostly busy with qualitative concerns. Numerical models are at this stage less intended to describe accurately an existing system than to estimate the performances of one that is under development.
— Practically, designers must constantly work on basis of incomplete, not yet fixed or inaccurate data sets.
— Conceivers think in term of systems with a very limited amount of degrees of freedom.
— The quality of a numerical model is related to its ability to answer technical questions.

The proposed idea is to analyse a *simplified cell*, extracted from the system by applying well-thought-out simplifications. The cell is defined in such a way that
— it behaves *similarly* to the complete system under a wide range of sollicitations,
— it share as much degrees of freedom as possible with the complete system,
— it allows to exploit the power of the FE method (flexibility, wide applicability) and to avoid its drawbacks (i.e. by favouring as much as possible sparse, symmetric and positive definite matrix),
— it is small enough to give relevant numerical results with a personal computer within a reasonable time.

Even if it is clear that the analysis of the cell can not stand for a good representation of the complete system by itself, it is the opinion of the authors that it already gives relevant pieces of information concerning the *interrelations between observable quantities* in the complete system. The numerical analysis performed at the level of the cell contributes thereby to answer part of the technical questions that arise at R&D stage. According to this approach, numerical models are not intended to be exact copies of reality (though of a virtual nature) but rather *exploratory tools*.

III. DESCRIPTION OF THE SYSTEM

The system under consideration is represented Fig. 1. The inductor set is an assembly of coils partly connected in series and partly connected in parallel. The coils are hollow water-cooled conductors wound around the wire, forming regular he-
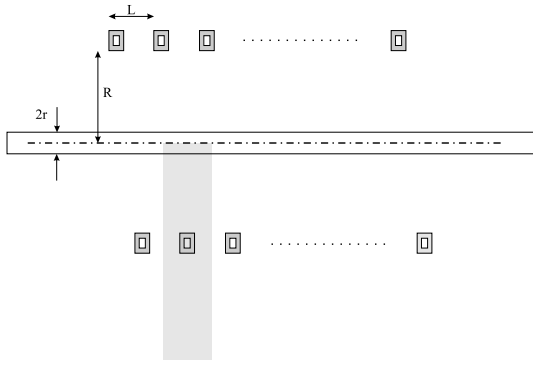
Fig. 1. Geometry of the induction furnace

lices with an axial pitch L. The inductor set is supplied with ac currents via an inverter at a frequency of a few tens kilohertz. The wire of radius r is made of iron and passes along the axis through the coil with a velocity v.

The FE equations describing the magnetic system in terms of the vector magnetic potential a, the magnetic permeability $\nu(b, T)$ and the scalar electric potential V are

$$
\begin{aligned}
(\nu(b,T)\,\mathrm{curl}\,a, \mathrm{curl}\,a') & \\
- (J(a,T), a') = 0 \quad \forall a' \, ;
\end{aligned}
\tag{1}
$$

with

$$
J(a,T) = -\sigma(T)\,(j\omega a + \mathrm{grad}\,V)\,.
\tag{2}
$$

where $\sigma(T)$ is the electric conductivity. The equations describing the thermal system in terms of the temperature T, the material derivative $\dot{h}$ of the enthalpy h and the thermal conductivity λ are

$$
\begin{aligned}
\left(\dot{h}(T), T'\right) + (\lambda(T)\,\mathrm{grad}\,T, \mathrm{grad}\,T') & \\
+ (q(t), T')_\Gamma - (Q(T, a), T') = 0 \quad \forall T'
\end{aligned}
\tag{3}
$$

with the auxiliary quantities

$$
h(T) = \rho c(T) T,
\tag{4}
$$

$$
Q(T, a) = \frac{1}{2}\sigma(T)\,|j\omega a + \mathrm{grad}\,V|^2 \,,
\tag{5}
$$

$$
q(T) = \epsilon_c(T)(T - T_o) + \epsilon_r(T)(T^4 - T_o^4).
\tag{6}
$$

where $\rho c(T)$ is the specific heat. Equation (6) expresses the heat flux, by *fluid convection* (with the surrounding air, parameter ϵ_c) and *radiation* (parameter ϵ_r), through the surface of the wire Γ. T_o is the average temperature in the oven.

The interaction between the magnetic and thermal quantities is mutual. Joule losses are the main heat source in the thermal problem and all the material characteristics depend upon temperature. In particular, the magnetic reluctivity of the wire iron exhibits a strong dependency on both b and T, i.e. $\nu(b, T)$. As the temperature increases, the magnetisation $M(h, T)$ decreases following a square root rule [1]. This writes as

$$
b(T) = \mu_0 \left(h + \sqrt{1 - \frac{T}{T_c}} M(h, 0) \right).
\tag{7}
$$

A correct representation of this *phase change*, is a major concerns in induction heating. At $T = T_c$, the so-called *Curie temperature*, the material has ceased to be magnetic and behaves like air. It should be noted that handling this *double dependency*, remains difficult in most commercial packages.

On the other hand, the magnetic phase change is accompanied by the absorption by the material of a certain amount of latent energy. This phenomenon is described by a strong dependency with temperature of the specific heat ρc of the wire material, as shown Fig. 2, where a sharp peak is observed when approaching T_c.
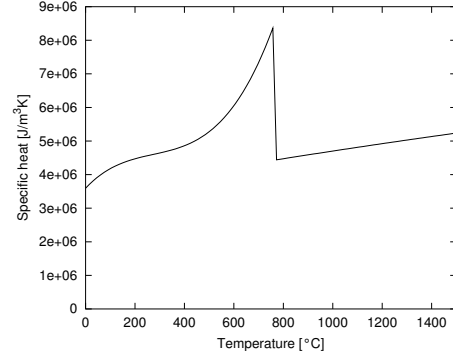

Fig. 2. Specific heat of wire material as a function of temperature.

IV. THE "CELL" SIMPLIFICATIONS

The simplifications that lead to the definition of the cell are now detailed.

A. Geometrical simplifications

An infinitely long coil is assumed, i.e. end-effects are disregarded. The turns are assumed to be connected in series and carry the same current. One may for these reasons restrict the analysis to a domain limited by two planes perpendicular to the wire and separated by a distance L, i.e. such that exactly one turn fits into the domain. Because of the absence of any shielding, the problem is open. This is taken into account by using a shell transformation [2]. The problem is finally assumed to be axisymmetrical, i.e. helix effects are neglected. Motion induced currents are negligible.

B. Lagrangian approach, adiabatic boundary condition

The *Peclet number*, $Pe = \frac{\rho c v L}{\lambda}$, represents the ratio of the heat transported by *material convection* (due to the movement of the wire) and the heat transported by conduction. In this application, $Pe > 200$ at any temperature. That means that conduction is negligible in comparison with convection *in the direction of the movement*. This situation is likely to cause severe numerical instabilities if an adequate scheme (up-winding) is not chosen for the discretisation of the material derivative in (3).

In this case however, due to the assumed axial symmetry of the simplified system, a better approach exists that preserves the numerical properties of the system to solve (auto-adjointness). The cell domain (grey zone on Fig. 1) is defined as enclosing a fixed section of the wire, i.e. the cell drifts along

at the same speed as the wire. As the motion goes, the cell domain crosses successively the turns of the coils but contains exactly one turn at each instant of time, as long as one remains inside the oven. This *Lagrangian approach* allows to assume an *adiabatic thermal boundary condition* at the external boundaries of the cell, i.e. conduction is neglected in comparison with convection *in the direction of the movement*, what is now *justified* by the high Pe number. Only the boundary Γ, which is internal, is crossed by the heat flux $q(T)$ defined by (6).

C. Constitutive law

In a nonlinear problem, a substantial part of the assembly time is devoted to the evaluation of the constitutive laws. Costly expressions obtained by interpolation have therefore to be avoided. For the material under consideration, the depen-
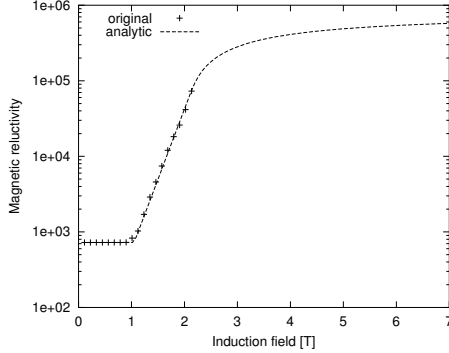

Fig. 3. Concise analytical expression of the magnetic reluctivity compared with the original curve.

dency with induction b of the magnetic reluctivity $\nu(b)$ can be represented with enough accuracy (See Fig. 3) by

$$\nu(b) = \begin{cases} \frac{\nu_0}{\mu_r} & \text{if } b \leq b1 \\ \frac{\nu_0}{\mu_r} 10^{\alpha(b-b_1)} & \text{if } b \leq b2 \\ \frac{\nu(b_2)b_2 + \nu_0(b-b2)}{b} & \text{if } b > b2 \end{cases} \quad (8)$$

whence, after a few algebraic manipulations, the concise expression

$$\frac{1}{\nu(b,T)} = \frac{1}{\nu_0} + \sqrt{\frac{T_c - T}{T_c - T_0}} \left(\frac{1}{\nu(b)} - \frac{1}{\nu_0} \right) \quad (9)$$

where T_0 is the temperature at which $\nu(b)$ has been measured.

D. Time stepping and non linear iteration

It is classical in induction heating problems to assume a harmonic induction field, represented by phasors, coupled with a transient temperature field. The time step is chosen according to the time scale of the thermal phenomena. In order to preserve the numerical properties of the system to solve (symmetry and positive definiteness of the matrix), the coupled system is solved by *weak coupling*, i.e. one nonlinear iteration is done alternately for each subsystem (magnetic, thermal), the field values being each time updated for the evaluation of the material characteristics. This extra iteration level is not disadvantageous because of the importance of the nonlinearities of the subsystems.

The relaxed Picard technique (with relaxation factor 0.2) is preferred to the costly computation of the tangent matrix. It has been noticed that adding the tangent matrix for the radiation term only (which involves the fourth power of the surface temperature) leads to a significant acceleration of the convergence.
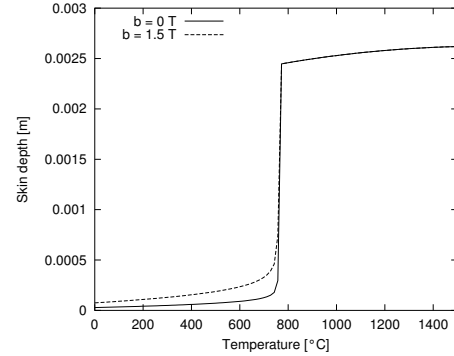
V. ADAPTED MESHING

Fig. 4. Skin depth, $\delta = \sqrt{\frac{2}{\omega\sigma\mu}}$, in the wire, as a function of temperature.

The computation of eddy currents does not cause major difficulties provided a sufficiently fine mesh is used in the skin of the conductors. The skin depth, which varies with temperature (Fig. 4), is $70\mu m$ in the worst case, i.e. at the beginning of the heating process. If that value is very small with respect to the overall characteristic length of the oven, it is already more reasonable if compared to the characteristic length of the cell.
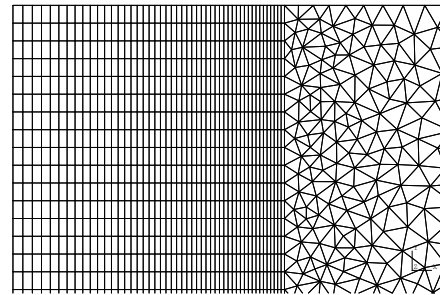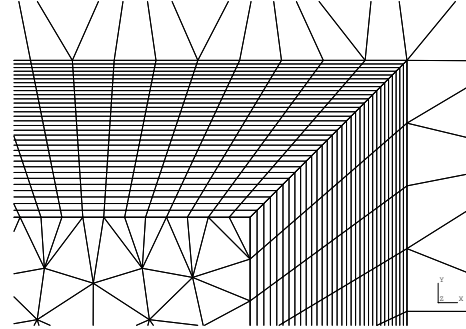

Fig. 5. A mesh adapted for skin effect in the inductor (above) and in the wire (below).

No theoretical rule exists to optimise a mesh for a good representation of dynamical effects such as eddy currents. Moreover, because of the existence of a moving demagnetisation

front (See Fig. 7 below), the skin effect is not always located at the same place. A non uniform regular mesh with rectangle elements, manually refined according to experience and convergence observations, has been used in the conductors (See Fig. 5).
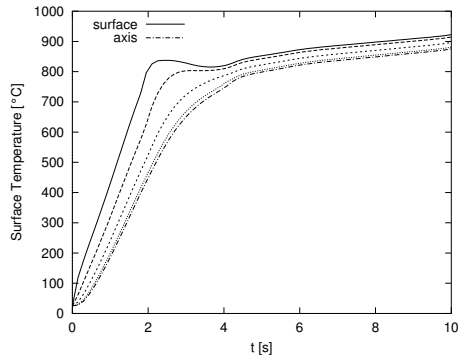
VI. RESULTS

Fig. 6. Time evolution of the temperature at different positions inside the wire $(x = 0, 0.25 \, r, 0.5 \, r, 0.75 \, r, r)$.

Fig. 6 shows the heating up of the surface of the wire. A steep slope is observed as long as the material is completely ferromagnetic. The temperature is then levelled off during the phase change and starts to increase again, with a less steep slope, when the material is completely demagnetised.
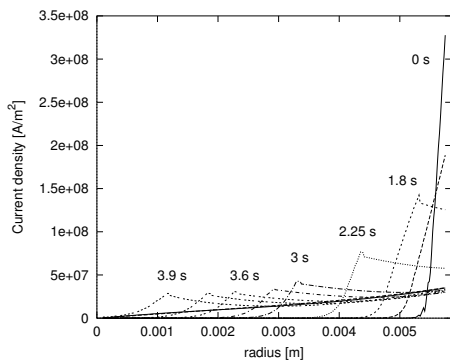

Fig. 7. Current density in the wires at different instants of time.

Fig. 7 shows radial plots of the current density in the wire at different instants of time. A front of demagnetisation is clearly seen to advance from the surface of the wire inwards, pushing along a zone with skin effect. The interval of time during which the front progresses corresponds with the constant temperature observed on Fig.6.

The two previous graphs are typical outcomes of the analysis of a strongly magneto-thermal coupled problem. The cell has been designed in such a way that those results are faithful to the phenomenology in play in the global system.

However, the cell methodology is more than a *local* analysis of the coupling. The cell retains as much as possible relevant parameters of the complete system: materials, radius of the coil, supply, . . . As an example, the time evolution of the power supplied to the cell and of Joule losses in the wire (Fig. 8), placed in perspective with the timing of the heating-up of the

wire (Fig. 6), gives a pertinent insight into the interrelations between the observable quantities that describe the system. That knowledge is likely to be formalised at the cell level in order to be reusable for an analysis at the level of the complete oven.
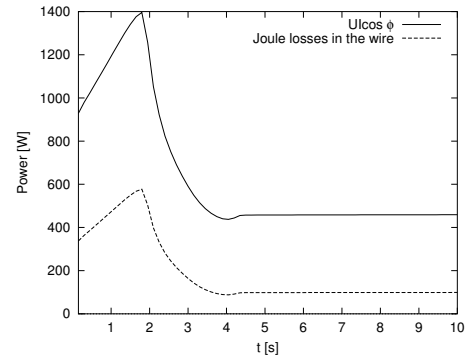

Fig. 8. Active power supplied to the cell and Joule losses in the wire as a function of time.

The study of the cell already represents a few hours of computation on a personal computer. The mesh has about 8000 nodes. That represents 2*8000 (a) + 5000 (T) unknowns. The transient calculations were done with 200 time steps, comprising each 20 nonlinear iterations in average, i.e. about 4000 (a) + 4000 (T) linear systems solved with GMRES+ILUT. All computations were done with the packages [3], [4].

VII. CONCLUSION

The numerous simplifications that have been granted to delimit the cell model (2D, axisymmetry, Lagrangian, adiabatic) make so that the numerical values computed on basis of the cell model are certainly different from their value in the global system. The cell model is not intended to be a good model of the complete system. The cell methodology is rather a way to exploit the power of the FE method at an *intermediary level* whenever the modelling of the complete system is out of reach. The cell has been designed in such a way that the intricate interrelations between observable quantities of the complete system (e.g. the dynamics of the demagnetisation front and how it affects the distribution of the eddy currents and losses) can be analysed at the cell level by in-depth numerical analysis of its behaviour under a wide range of sollicitations. Furthermore, the knowledge gained at the cell scale can, in a further step, be exploited to work on a description of the complete system, what would certainly not have been possible in this case without the proposed intermediary step.

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