Finite Element Simulation of Motional Eddy Currents in Slotted Geometries

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Abstract— **Motional eddy current effects in rotational machines are considered. Expensive transient finite element simulation can be replaced by a stationary motional formulation equipped with geometrical homogenisation or by a time-harmonic approach using slip transformation. The importance of the assumptions involved with both stationary approaches are pointed out and verified for a technical induction machine.**

Keywords— **Finite element methods, motional eddy currents, induction machines.**

I. Introduction

The simulation of electrical machines operated at steady-state, i.e., with a steady DC or AC voltage supply and constant mechanical load, has a large technical importance. It is the most relevant design tool for almost all electrical machines. It is embedded in iterative design procedures and optimisation routines. Finite element (FE) approaches are attractive because complicated geometries and local phenomena such as ferromagnetic saturation and eddy currents are easily incorporated. The accurate FE simulation of motional eddy current effects, however, is still challenging [1]. A common approach is transient FE simulation [2], i.e., solving

$$-\nabla \cdot (\nu \nabla A_z) + \sigma \frac{\partial A_z}{\partial t} = \frac{\sigma}{\ell_z} \Delta V \qquad (1)$$

with A_z the z-component of the magnetic vector potential, ν the reluctivity, σ the conductivity, ℓ_z the length of the 2D model and ΔV the voltage drop applied between the front and rear ends of the device. To incorporate motional effects, the mesh and the solution of the previous time step, are moved together with the rotating and/or translating geometry. Motional eddy current phenomena are introduced by the varying fields at the interfaces between the moving bodies. The relative motion of the FE meshes requires a special treatment, e.g., the moving band approach [3].

Obviously, transient simulation is the most general approach (Table I). It captures all phenomena if sufficiently small time steps are applied. Transient FE simulation is however to expensive if only steady-state results are required. Stationary FE methods should be sufficient to simulate steady-state conditions. If moving parts are involved, and especially when the moving bodies are non-uniform, standard stationary methods do not further apply. In this paper, the stationary Eulerian formulation is accomplished by a homogenisation procedure to consider slotted moving bodies. Also, for particular configurations, motional effects can be treated by the slip transformation technique. Both stationary approaches are discussed and compared to transient simulation for an induction machine example.

II. Motional Formulation with Homogenisation

The Eulerian formulation

$$-\nabla \cdot (\nu \nabla A_z) + \sigma \mathbf{v} \cdot \nabla A_z + \sigma \frac{\partial A_z}{\partial t} = \frac{\sigma}{\ell_z} \Delta V \qquad (2)$$

with $\mathbf{v}$ the velocity, explicitly accounts for motional eddy currents [4]. Hence, if the moving bodies are *uniform*, i.e., have constant cross-sections, material distributions and excitations with respect to the direction of motion, remeshing is avoided. Furthermore, if excited by a DC source, $\sigma \frac{\partial A_z}{\partial t}$ vanishes. If an AC source at frequency f is applied, one can adopt to time-harmonic simulation:

$$-\nabla \cdot (\nu_{\text{eff}} \nabla \underline{A}_z) + \sigma \mathbf{v} \cdot \nabla \underline{A}_z + \jmath\omega\sigma \underline{A}_z = \frac{\sigma}{\ell_z} \Delta \underline{V} \quad . \qquad (3)$$

with $\underline{A}_z$ and $\Delta\underline{V}$ the phasors corresponding to A_z and ΔV, ν_{eff} the effective reluctivity [5] and $\omega = 2\pi f$ the pulsation. To overcome spurious numerical oscillations appearing in FE solutions if large velocities are involved, a combined approach based on upwinding and adaptive mesh refinement is incorporated [6]. In this form, the method applies to e.g. rail braking systems and solid rotor induction machines. The formulation is, however, not directly applicable to slotted geometries because of their lack of uniformity.

Steady-state Eulerian simulation can be made possible for slotted moving bodies by applying *geometrical homogenisation* [7], [8]. The rotor bars and rotor teeth of a rotating machine are replaced by a homogeneous ring which is, by construction, uniform with respect to the rotation. For linear motors, homogenisation is carried out in the direction of translation.

The homogenisation procedure is schematically represented in Fig. 1, adopting a lumped parameter representation. The rotors slots and teeth are characterised by their permeances Υ_{slot} and Υ_{tooth} or reluctances Λ_{slot} and Λ_{tooth} and conductances G_{slot} and G_{tooth}. Two magnetic paths are considered. The magnetic flux ϕ_r following a

TABLE I
FINITE ELEMENT APPROACHES FOR MOTIONAL EDDY CURRENT PROBLEMS

	transient		time-harmonic + slip transformation		motional + homogenisation
+	arbitrary geometries	+	arbitrary geometries	−	uniform geometries
+	arbitrary air gap fields	−	air gap wave	+	arbitrary air gap fields
−	huge simulation times	+	small simulation times	+	acceptable simulation times
+	accurate description of non-linearities	+	sufficiently accurate description of non-linearities	−	cumbersome for non-linearities
+	external circuit coupling	+	external circuit coupling	−	difficult to consider external circuit coupling

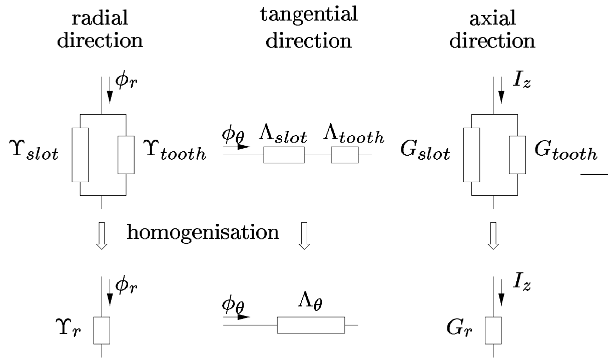

Fig. 1. Homogenisation procedure for slotted moving bodies.

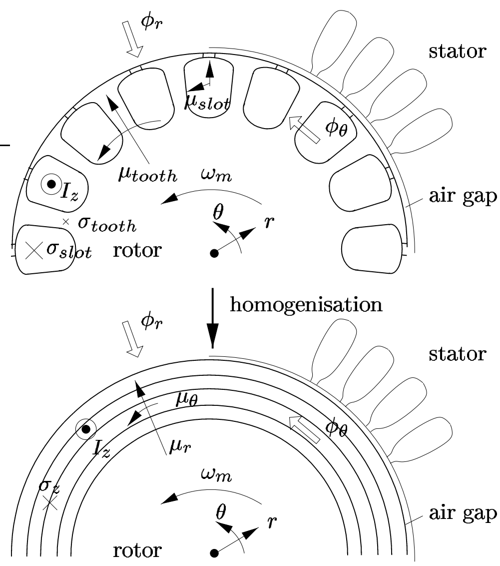

Fig. 2. Homogenisation procedure for slotted moving bodies.

radial path from the inside of the rotor to the air gap, is divided into a large part going through the teeth and a small path through the slots of the rotor. The permeance of the parallel connection of both paths is

$$\Upsilon_r = \sum_{slots} \Upsilon_{slot} + \sum_{teeth} \Upsilon_{tooth} \quad . \tag{4}$$

The magnetic flux ϕ_θ in the tangential direction experiences the series connection of the slot and teeth regions. The full reluctance is

$$\Lambda_\theta = \sum_{slots} \Lambda_{slot} + \sum_{teeth} \Lambda_{tooth} \quad . \tag{5}$$

The slots and teeth form a parallel connection for the electric currents $I_z$in the axial direction with conductance

$$G_z = \sum_{slots} G_{slot} + \sum_{teeth} G_{tooth} \quad . \tag{6}$$

The homogenisation method, proposed here within a FE context, is schematically depicted in Fig. 2 and is the continuous equivalent of the parallel and series connections shown in Fig. 1. The procedure consists of averaging the material parameters in the angular direction. Because of the periodicity of the rotor geometry, it is sufficient to average the material parameters over one rotor pitch τ_p. This is done for different values of the radius r:

$$\mu_r^{(hom)}(r) = \frac{1}{\nu_r^{(hom)}(r)} = \frac{1}{\tau_p}\int_0^{\tau_p} \mu(r,\theta)\mathrm{d}\theta \quad ; \tag{7}$$

$$\frac{1}{\mu_\theta^{(hom)}(r)} = \nu_\theta^{(hom)}(r) = \frac{1}{\tau_p}\int_0^{\tau_p} \nu(r,\theta)\mathrm{d}\theta \quad ; \tag{8}$$

$$\sigma_z^{(hom)}(r) = \frac{1}{\tau_p}\int_0^{\tau_p} \sigma(r,\theta)\mathrm{d}\theta \quad . \tag{9}$$

The permeability and conductivities of the slot and tooth materials are represented in Fig. 2 by arrows and crosses respectively.

The homogenisation procedure in general yields anisotropic permeabilities, even if the true permeabilities are isotropic. The rotor ring is considerably more permeable in the radial direction than in the tangential direction. The first term in (2) becomes

$$-\frac{1}{r}\frac{\partial}{\partial\theta}\left(\nu_r^{(hom)}\frac{1}{r}\frac{\partial \underline{A}_z}{\partial\theta}\right) - \frac{1}{r}\frac{\partial}{\partial r}\left(\nu_\theta^{(hom)} r\frac{\partial \underline{A}_z}{\partial r}\right) \quad . \tag{10}$$

When applied in a cartesian FE formulation, the reluctivity tensor

$$\begin{bmatrix} \nu_{xx} & \nu_{xy} \\ \nu_{yx} & \nu_{yy} \end{bmatrix} = P^T \begin{bmatrix} \nu_r^{(hom)} & \\ & \nu_\theta^{(hom)} \end{bmatrix} P \tag{11}$$

with

$$P = \begin{bmatrix} sin\theta & -cos\theta \\ cos\theta & sin\theta \end{bmatrix} \tag{12}$$

is inserted in (2).

The homogenisation approach allows for the use of stationary Eulerian formulations for slotted moving bodies but has also some severe drawbacks:

1. Anisotropy: The homogenised material is anisotropic with respect to the radial and tangential axes. The FE solver has to cope with the reluctivity tensor (11). Moreover, the relative difference between the permeabilities of both axes is substantial, e.g. a factor 870 for the induction machine considered below. A fast and accurate algorithm may require graded meshes and specialised iterative solvers.
2. Non-linearities: The rotor iron is ferromagnetic. The non-linear material properties are propagated to the homogenised parameters $\nu_\theta^{(hom)}$ and $\mu_r^{(hom)}$. This does, however, not reflect the true saturation level. The true field solution is artificially smeared out over the homogenised region. As a consequence, local high flux densities are less likely to occur. Because of the slotting and in particularly the slot wedges, significant saturation arises locally, having a considerable influence on the leakage flux and therefore on the overall device behaviour. The homogenised field does not account for local saturation. In general, the homogenisation approach yields lower saturation levels when compared to transient simulation.
3. External circuit coupling: Besides the material distribution in the 2D cross-section of the model, also the external circuit model which may be coupled to the FE model, has to be homogenised. It is, however, not obvious to extract continuous, homogenised quantities for the lumped parameters which model the rotor end-rings. The resistances and inductances representing the end-rings in the coupled model have to be replaced by a continuous end-ring model. This model has to account for external resistances and impedances for all possible closing paths between the points of the homogenised region. A coupling to a 3D model of the end-region would be appropriate but would destroy the efficiency of the simulation.

III. Time-Harmonic Formulation with Slip Transformation

The second alternative for time-stepping, which is in theory only applicable if the field at the interface between the standstill and the moving body has a particular nature, is commonly applied to simulate induction machines rotating atconstant velocity ω_m [9]. Suppose that the magnetic field in the air gap is a rotating wave, i.e.,

$$A_z(\theta) = \mathrm{Re}\left\{\underline{\alpha} e^{\jmath(\omega t - \lambda\theta)}\right\} \qquad (13)$$

with α a phasor, λ the pole pair number and θ the tangential coordinate. Then, all eddy current effects are correctly modelled by

$$-\nabla \cdot (\nu_{\mathrm{eff}} \nabla \underline{A}_z) + \jmath\omega_s \sigma \underline{A}_z = \frac{\sigma}{\ell_z} \Delta \underline{V} \quad . \qquad (14)$$

with $\omega_s = \omega - \lambda\omega_m$ is the *slip pulsation.* This technique, called *slip transformation*, also approximately applies to models with nearly sinusoidal air gap waves and can be seen as homogenising the air gap field instead of the rotor geometry. This approach does not account for slot harmonics or higher harmonic air gap fields.

TABLE II

Comparison of the torque of the three-phase induction machine simulated by different FE formulations and motional eddy current treatment.

FE formulation	treatment of motional effects	torque
transient	moving band technique	293.0 Nm
Eulerian	geometrical homogenisation	225.7 Nm
time-harmonic	slip transformation	292.6 Nm

TABLE III

True and homogenisation material coefficients of the outer ring of the rotor (for ferromagnetic materials, the slope of the BH-characteristic is given, the homogenised coefficients are evaluated at a contour $r_a = 94$ mm).

coefficient	rotor slots	rotor teeth	homogenised material
$\mu_\theta^{(hom)}(r_a)/\mu_0$	1	4.15e+03	3.33e+00
$\mu_r^{(hom)}(r_a)/\mu_0$	1	4.15e+03	2.90e+03
$\sigma_z^{(hom)}(r_a)$	20.8e+06 S/m	0 S/m	6.25e+06 S/m

IV. Application

Both stationary FE approaches are compared to the reference transient method. The torque of a loaded three-phase induction machine with squirrel-cage rotor operated at steady-state is computed (Table II). The 2D cross-section of the device is discretised by 2D linear FEs. In the transient case, a linear single-step method is applied. The voltage supply, end-winding and end-ring effects are taken into account by an external circuit coupling [10].

The application of the motional formulation requires the homogenisation of the slotted rotor. The material coefficients of the aluminum bar material and those of the non-conductive and ferromagnetic rotor iron, are combined into one partially conductive and modest permeable material. The true and homogenised material parameters are collected in Table III. This homogenisation procedure is repeated for each step of the non-linear loop in order to take ferromagnetic saturation into account. The magnetic flux line plot shown in Fig. 3a, exhibits the strong anisotropy of the homogenised material of the disk between r_a and r_b. Comparing the torque to the transient reference solution, it is concluded that the motional approach with homogenisation is not reliable for induction machine simulation. The homogenisation procedure does not account for local spots of saturation, e.g. at the rotor brides, and only incorporates the external circuit coupling modelling the rotor end ring, in a very approximate way. Homogenisation approaches may yield reliable models if one of the contributing materials is substantially more important than the others [8]. For common induction machines, however, the fractions of slot and tooth material are of the same order of magnitude.

The time-harmonic solution dealing with motional effect by slip transformation is shown in Fig. 3b. The torque computation based on this solution reaches the technical

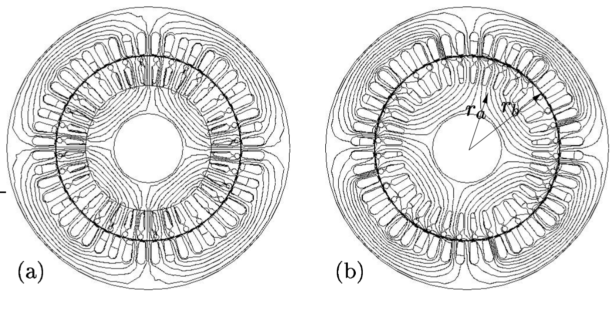

Fig. 3. Magnetic flux lines of (a) the motional FE solution with homogenisation of the slotted rotor and (b) the time-harmonic FE solution with slip transformation of a four-pole, three-phase induction machine with squirrel-cage rotor. The material properties of the disk between r_a and r_b in (a) are highly anisotropic, they are homogeneous with respect to θ and varying with respect to r. The drawing of the rotor slots is indicative. The flux lines reflect a larger permeability in the radial direction than in the tangential direction.

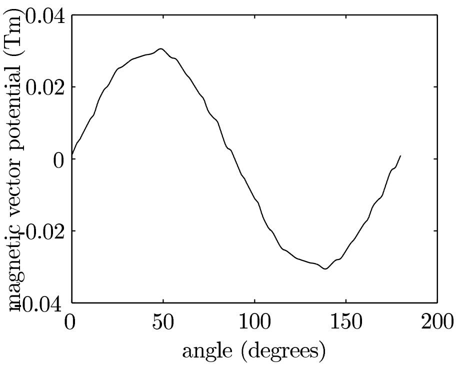

Fig. 4. Magnetic vector potential along a contour in the middle of the air gap of the induction machine.

accuracy of 0.5%. It, however, requires only a factor of 100 less of computation time compared to the transient approach. The chorded multi-layer windings of contemporary induction machines yield an almost sinusoidal rotating air gap field (Fig. 4) which explains the high accuracy of this approach. Hence, for the steady-state simulation of slotted moving bodies, the slip transformation technique is favoured over geometrical homogenisation.

V. Conclusions

Two stationary FE approaches for motional eddy current simulation are presented. The Eulerian formulation is accomplished by a geometrical homogenisation procedure. However, a time-harmonic approach with slip transformation, if applicable, provides the most accurate solution, as indicated by the three-phase induction machine example.

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References

[1] C.R.I. Emson, C.P. Riley, D.A. Walsh, K. Ueda, and T. Kumano, "Modelling eddy currents induced by rotating systems," *IEEE Transactions on Magnetics*, vol. 34, no. 5, pp. 2593–2596, Sept. 1998.

[2] A. Arkkio, "Time-stepping finite element analysis of induction motors," in *Proceedings of the International Conference on Electrical Machines (ICEM 88)*, Pisa, Italy, Sept. 1988, vol. 1, pp. 275–280.

[3] N. Sadowski, Y. Lefèvre, M. Lajoie-Mazenc, and J. Cros, "Finite element torque calculation in electrical machines while considering the movement," *IEEE Transactions on Magnetics*, vol. 28, no. 2, pp. 1410–1413, Mar. 1992.

[4] D. Rodger, P.J. Leonard, and T. Karaguler, "An optimal formulation for 3D moving conductor eddy current problems with smooth rotors," *IEEE Transactions on Magnetics*, vol. 26, no. 5, pp. 2359–2363, Sept. 1990.

[5] D. Lederer, H. Igarashi, and A. Kost, "Modelling of nonlinear magnetic material using an effective permeability - comparison between different formulations," *ACES Journal*, vol. 12, pp. 113–116, July 1997.

[6] H. De Gersem, H. Vande Sande, and K. Hameyer, "Motional magnetic finite element method applied to high speed rotating devices," in *Proceedings of the 9th International Symposium on Electromagnetic Fields-ISEF"99*, Pavia, Italy, Sept. 1999, pp. 228–231.

[7] M. Andriollo, G. Martinelli, A. Morini, and A. Tortella, "Finite-element modelization of slotted configurations by means of magnetically anisotropic smooth cores," in *Proceedings of the International Workshop on Electric and Magnetic Fields (EMF00)*, Ghent, Belgium, May 2000, p. 2 pages, abstract of 2 pages.

[8] G. Vinsard, S. Dufour, and B. Laporte, "An approximation for the air-slots of complex shaped motors," *IEEE Transactions on Magnetics*, vol. 34, no. 5, pp. 2449–2452, Sept. 1998.

[9] E. Vassent, G. Meunier, and J.C. Sabonnadière, "Simulation of induction machine operation using complex magnetodynamic finite elements," *IEEE Transactions on Magnetics*, vol. 25, no. 4, pp. 3064–3066, July 1989.

[10] H. De Gersem, R. Mertens, U. Pahner, R. Belmans, and K. Hameyer, "A topological method used for field-circuit coupling," *IEEE Transactions on Magnetics*, vol. 34, no. 5, pp. 3190–3193, Sept. 1998.