

A Finite Element Model for Foil Winding Simulation

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Abstract—Foil windings, inductors consisting of a considerable number of foils with a long rectangular cross-section, give raise to severe meshing problems if one tries to consider all geometrical details. On the other hand, treating the winding as a whole by a solid or stranded conductor model, passes over the particular skin effects occurring in this type of winding. Here, a foil conductor model is developed. An additional discretization of finite elements in the foiled region deals with the voltage variation perpendicular to the foils. The constant current condition is weakly imposed. The foil mesh does not coincide with the geometry nor the magnetic mesh, enabling small meshes and independent mesh refinement. The application to a three-phase foil-winding transformer shows a significant saving of computational time, up to 99% for the test problem.

Index Terms—Coupling circuits, finite element methods, foil windings, transformer windings.

I. INTRODUCTION

A LARGE range of electrotechnical devices is commonly simulated by transient or time-harmonic magnetodynamic finite element (FE) models. The conductors may exceed the FE model and are connected to an external electric circuit. External loads, external current and voltage sources and also parts of the device itself are represented by additional electric lumped parameters. Field-circuit coupled simulation techniques are developed and commonly applied [1], [2]. Two conductor models exist: the solid conductor and the stranded conductor model, the former for massive bars, the latter for coil windings. Foil windings are recently applied in transformers and establish a very particular behavior. Their incorporation in field-circuit coupled models requires a novel foil conductor model to be developed.

II. FOIL CONDUCTORS

Recently, electrical devices with foil windings are designed and manufactured. Particularly for distribution transformers, this type of windings is very effective [Fig. 1(a)]. The current distribution in the foils spontaneously compensates for asymmetries in the high voltage windings. As consequences, axial forces at short-circuit operation are minimized, a more homogeneous heat distribution is achieved and the manufacturing process is simplified when compared to conventional windings.

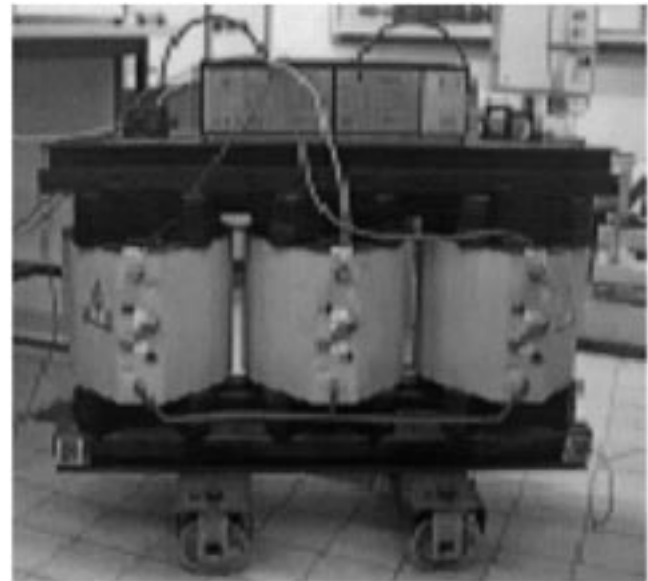
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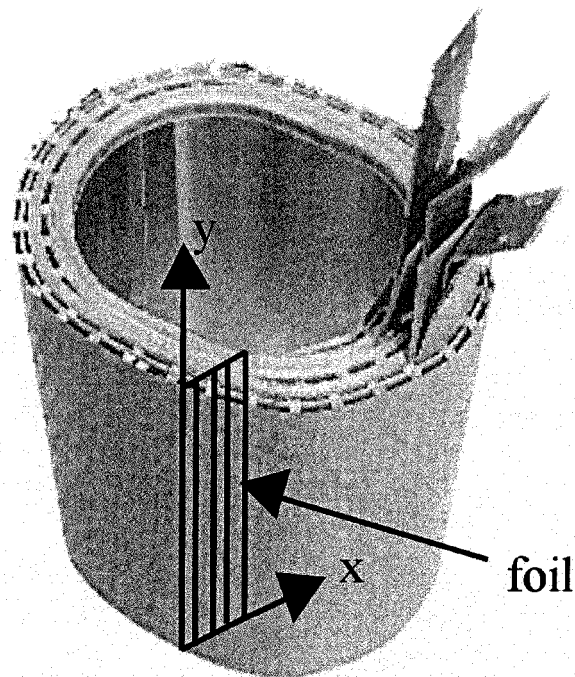
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(a)



(b)

Fig. 1. (a) Foil-winding distribution transformer. (b) Foil winding (Pauwels Trafo Belgium N.V.).

A foil winding consists of a conductive foil wound around a support [Fig. 1(b)]. An alternating current applied to the foil conductor, is redistributed toward the tips of the conductor but not in the perpendicular direction [Fig. 2(a)] while the insulation

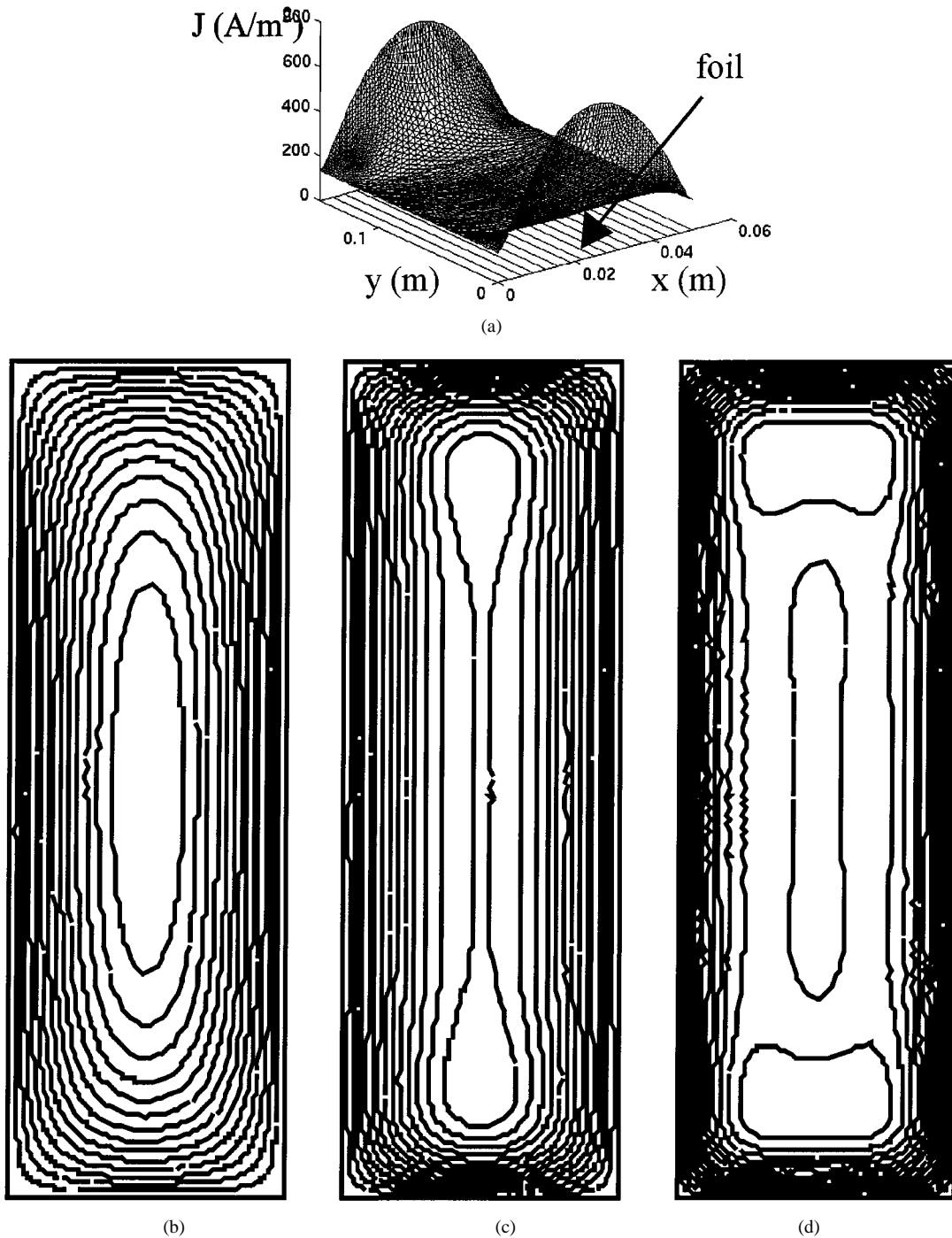


Fig. 2. (a) Current distribution in the foil winding and magnetic flux lines within (b) a stranded, (c) a foil and (d) a solid conductor.

enables a voltage gradient in the perpendicular direction. In contrast with massive bars and common wire windings, a foil winding experiences a skin effect only in the direction of the foils (Fig. 2). An accurate simulation of this unidirectional skin effect is important as it is responsible for local heating effects and the consecutive deterioration or even breakdown of the insulation material [3].

III. COUPLED FIELD-CIRCUIT SIMULATION

A. Magnetic Model

Eddy currents are considered by the quasistatic subset of Maxwell's equations. Most of the induced phenomena occur in

a surface layer of the conductors. The skin depth $\delta = 1/\sqrt{\pi f \sigma \mu}$ depends upon the conductivity σ , the permeability μ of the material and the applied frequency f . Here, a 2-D model excited by a single frequency is used to demonstrate the foil conductor model. This does not alter the generality of the theory presented here. The governing differential equation is

$$-\nabla \cdot (\nu \nabla A_z) = J_z, \quad (1)$$

with A_z and J_z the z -components of the magnetic vector potential and the current density and $\omega = 2\pi f$ the pulsation.

B. Solid Conductor Model

The voltage drop ΔV_{sol} applied to a massive conductor is constant on its cross-section Ω_{sol} . Submitted to a time-varying magnetic field, the current density is

$$J_{sol}(x, y) = \frac{\sigma}{\ell_z} \Delta V_{sol} - j\omega\sigma A_z(x, y), \quad (2)$$

with ℓ_z the length of the 2-D model. The *solid conductor model* consists of the magnetic model (1) with current density (2), coupled to the external electric circuit via the integral relation for the total current:

$$I_{sol} = \int_{\Omega_{sol}} J_{sol}(x, y) d\Omega. \quad (3)$$

C. Stranded Conductor Model

In many devices, windings with a considerable number of turns, connected in series, are used. The extents of the wires are smaller than the skin depth corresponding to the applied frequencies. However, the extents of the cross-section of the entire winding may exceed δ . The treatment of each separate wire as a solid, would require the geometrical details of all wires to be resolved in the mesh. Moreover, one unknown and one integral relation per wire have to be added to the coupled system of equations. This increases the size of the system to be solved and therefore spoils the efficiency of the simulation. Instead, a modeling assumption is granted. As δ exceeds the dimension of each of the wires, the current density may be assumed to be constant within the cross-section of each wire and, because the wires are connected in series, within the cross-section of the entire winding. The current density J_{str} is related to the current I_{str} supplying the stranded conductor, by

$$J_{str} = \frac{N_{str}}{\Delta_{str}} I_{str} \quad (4)$$

with N_{str} the number of turns and Δ_{str} the surface of the cross-section Ω_{str} of the entire winding, including all wires, insulation material and gaps in between them.

The voltage drop is not constant within Ω_{str} as the wires experience the electromotive force due to the time-varying induction field. The summation of voltages corresponding to the winding is replaced by an integral averaging the voltage gradient over the cross-section of the stranded conductor:

$$\nabla V_{av} = \frac{1}{\Delta_{str}} \int_{\Omega_{str}} \left(\frac{J_{str}}{f_{str}\sigma} + j\omega A_z(x, y) \right) d\Omega, \quad (5)$$

with f_{str} the fill factor accounting for the correction of the overall conductivity made necessary by the presence of insulation and gaps. The total voltage drop along the winding

$$\Delta V_{str} = N_{str} \frac{1}{\Delta_{str}} \int_{\Omega_{str}} \ell_z \left(\frac{J_{str}}{f_{str}\sigma} + j\omega A_z(x, y) \right) d\Omega \quad (6)$$

ouples the circuit to the magnetic model (1) with current density (4). This approach yields the *stranded* (or *filamentary*) *conductor model* for a winding.

D. Classical Approaches for Foil Windings

Up to now, foil conductors have been treated within FE simulation by the solid or stranded conductor model or by a series-connection of solid conductor models. The former approaches provide too much or not enough freedom to approximate the skin effect, whereas the latter requires the designer to treat each foil as a separate geometrical entity and circuit element. Moreover, since the cross-section of each foil is a long rectangle, attaining well-shaped FEs requires a very fine mesh. Although the voltages along neighboring foils do not differ that much, the field-circuit coupled model incorporates one unknown voltage per foil. The huge number of degrees of freedom, both in the FE model and in the external circuit, results in an inefficient simulation.

IV. FOIL CONDUCTOR MODEL

A. Modeling Assumptions

Consider a winding of N_{foil} foils, wound around a vertical axis. The foil regions have an extent $h_y = y_2 - y_1$ in the y -direction and a small extent $h_x = x_2 - x_1$ in the x -direction. In the applications considered here, h_y exceeds the skin depth δ whereas h_x/N_{foil} is smaller than δ . The foil current I_{foil} is the same in all foils. The voltage drop ΔV is constant within the cross-section of a single foil. The current density may depend upon x and y .

B. Continuum Model

The current density in the foil winding is

$$J_{foil}(x, y) = \frac{\sigma}{\ell_z} \Delta V(x) - j\omega\sigma A_z(x, y). \quad (7)$$

The continuum representation of a foil conductor replaces the discrete foil winding by a fictitious continuous wound filamentary sheet. The *foil conductor model* is based on the continuum modeling assumptions:

- 1) The sheet current density

$$Z_{foil} = \int_{y_1}^{y_2} J_{foil}(x, y) dy = \frac{N_{foil}}{h_x} I_{foil} \quad (8)$$

remains constant in the x -direction;

- 2) The voltage drop $\Delta V(x)$ is constant in the y -direction.

The total voltage drop ΔV_{foil} along the foil conductor is obtained similarly as in the case of the stranded conductor, by a homogenization procedure:

$$\Delta V_{foil} = N_{foil} \frac{1}{h_x} \int_{x_1}^{x_2} \ell_z \Delta V(x) dx. \quad (9)$$

The continuum model couples the 2-D magnetodynamic partial differential equation (1) with the current density (7) to the 1-D integral equation following from (8),

$$\frac{\sigma f_{foil}}{\ell_z} \Delta V(x) - \int_{y_1}^{y_2} j\omega\sigma A_z(x, y) dy - \frac{N_{foil}}{h_x} I_{foil} = 0, \quad (10)$$

with f_{foil} the fill factor of the foil winding, and the 0-D integral relation (9).

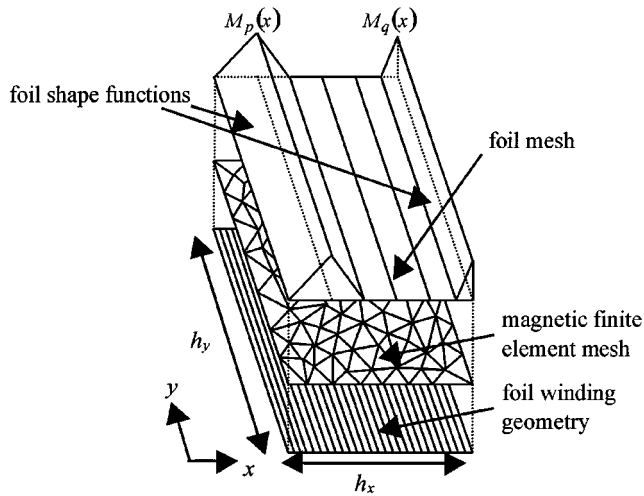


Fig. 3. Cross-section of the foil winding, magnetic mesh, foil element mesh and 2 foil form functions.

C. Foil Elements

The foil conductor model requires an additional 1-D discretization for the voltage:

$$\Delta V(x) = \sum_{q=1}^{n_f} \Delta V_q M_q(x). \quad (11)$$

$M_q(x)$ is a foil shape function (FSF) and n_f is the number of FSFs. A FSF has a constant value in the y -direction and has a local support in the x -direction (Fig. 3). The support of a FSF is a long rectangle and has thus a similar shape as a single foil itself. A FSF, however, may overlap several foils and the FSFs may have different supports in the x -direction. The voltages along N_f foils are represented by n_f unknowns ΔV_q . If N_f is large and the voltages do not vary too rapidly along x -cross-section of the foil winding, choosing $n_f \ll N_f$ offers a significant reduction of the number of unknowns in the model while retaining an acceptable accuracy. The foil mesh, i.e., the set of all FSFs, does also not coincide with the magnetic mesh. The magnetic mesh has only to resolve the magnetic field whereas, the foil mesh takes care of the voltage drop. As a consequence, the magnetic mesh can be kept significantly smaller. Moreover, meshing the 2-D geometry is not influenced by the large differences in the dimensions introduced by the foils.

D. Galerkin Approach

A system of algebraic equations is obtained by applying the weighted residual method to the system of differential and integral equations. The Galerkin approach applies the shape functions as weighting functions as well. The FE shape functions are applied to (1) with current density (7):

$$\int_{\Omega} \left(-\nabla \cdot (v \nabla A_z) + j\omega \sigma A_z - \frac{\sigma f_{foil}}{\ell_z} \Delta V \right) N_i d\Omega = 0. \quad (12)$$

Similarly, the FSF's are applied to weight (10):

$$\int_{x_1}^{x_2} \left(\frac{\sigma f_{foil}}{\ell_z} \Delta V - \int_{y_1}^{y_2} j\omega \sigma A_z dy - \frac{N_{foil}}{h_x} I_{foil} \right) M_p dx = 0. \quad (13)$$

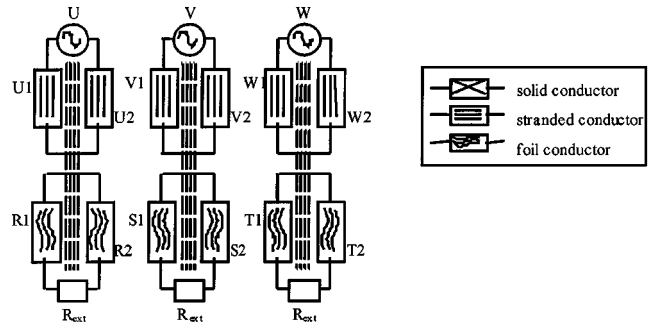


Fig. 4. External circuit of a three-phase foil-winding transformer.

The integral relation (9) fits within the algebraic circuit description developed in [2]. The resulting system of equations for a voltage driven foil conductor is

$$\begin{bmatrix} k_{ij} + l_{ij} & z_{iq} & 0 \\ z_{pj} & \xi g_{pq} & \xi s_p \\ 0 & \xi s_q & 0 \end{bmatrix} \begin{bmatrix} A_j \\ \Delta V_q \\ I_{foil} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \xi \Delta V_{foil} \end{bmatrix} \quad (14)$$

where

$$k_{ij} = \int_{\Omega} v \nabla N_i \cdot \nabla N_j d\Omega; \quad l_{ij} = \int_{\Omega} j\omega \sigma N_i N_j d\Omega; \quad (15)$$

$$z_{iq} = - \int_{\Omega} \frac{\sigma}{\ell_z} N_i M_q d\Omega; \quad g_{pq} = \int_{\Omega} \frac{\sigma f_{foil}}{\ell_z} M_p M_q d\Omega; \quad (16)$$

$$s_p = - \frac{N_{foil}}{\Delta V_{foil}} \int_{\Omega} M_p d\Omega. \quad (17)$$

$\xi = 1/j\omega \ell_z$ is a factor applied to symmetrize the system.

The evaluation of the integrals z_{iq} is not straightforward because $N_i(x, y)$ and $M_q(x)$ are defined on different, non-matching meshes. If applying Gauss integration, the number of points has to be chosen such that a sufficient resolution is achieved both with respect to the magnetic as the foil mesh. Here, an analytical evaluation is preferred. An auxiliary grid containing all edges and vertices of both meshes is constructed by intersecting all edges of both meshes. $N_i(x, y)$ and $M_q(x)$ are linear functions on each grid cell. z_{iq} can be computed analytically by adding the contributions of the integral for all auxiliary grid cells. The auxiliary grid is used for integral evaluation only and is not considered in the FE solution process itself.

A physical interpretation of (14) is straightforward. If each foil element is thought to correspond to a fictitious foil, each foil equation with index p relates the current through the single foil to its voltage drop and the magnetic field inside. g_{pq} represents the conductance. $[g_{pq}]$ is not diagonal because of the overlap between the FSFs. The last equation sums the voltages along all fictitious foils.

E. Foil Mesh Refinement

As the FE mesh and the FSF mesh do not coincide, adaptive mesh refinement can be applied independently. The FEs are refined based on the error estimation of the local magnetic field and the current density, yielding considerable refinement at the tips of the foils. The error estimator for the FSFs weights the voltage variation between two successive foil elements. As the

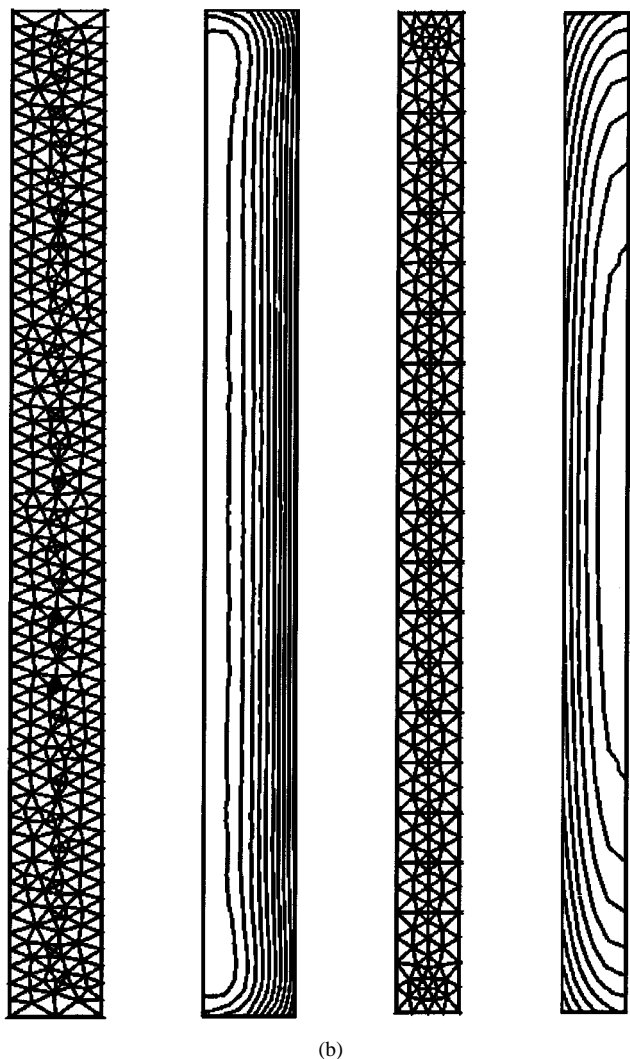
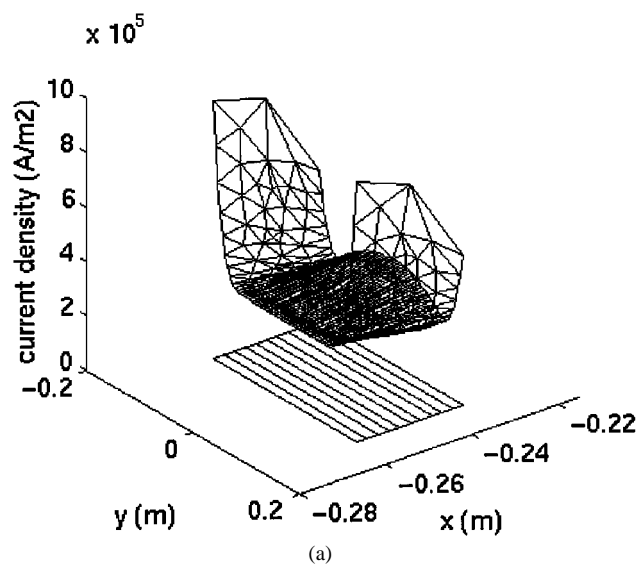


Fig. 5. (a) Foil mesh and current density in a foil conductor and (b) magnetic meshes and magnetic flux densities of a foil conductor and stranded conductor respectively.

resolution of the FSFs may not exceed the one of the real foils, foil elements smaller than true foils are excluded from refinement. The auxiliary grid for integral evaluation has to be rebuilt after each mesh refinement step.

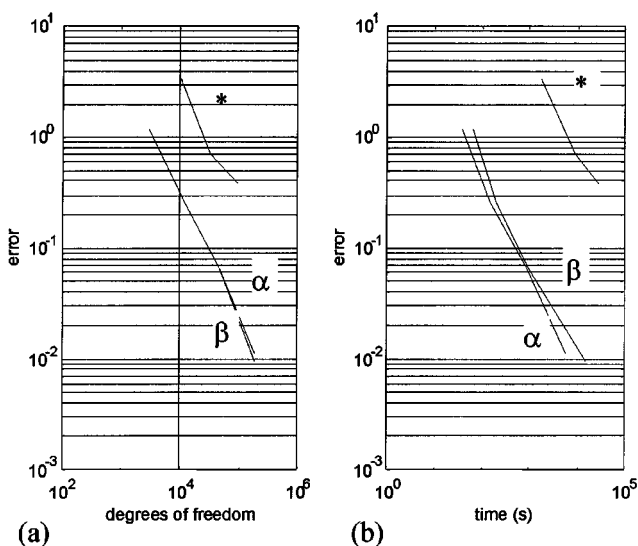


Fig. 6. Convergence of the global error with respect to (a) the number of degrees of freedom and (b) the computation time, comparison between (*) the conventional series-connection of solid conductors and the foil winding models with (α) 10 or (β) 20 foil elements per foil winding.

V. APPLICATION

The foil conductor model is applied to simulate short-circuit operation of a dry-type, three-phase foil-winding distribution transformer [Fig. 1(a)]. The secondary foil windings, each with 50 turns, are connected to additional resistors modeling the contact resistances (Fig. 4). The current distribution in the secondary winding R2 is shown in Fig. 5.

The convergence of the discretization error is compared for transformer models applying the novel foil conductor model and a conventional model considering each foil cross-section as a separate solid conductor. The reference model is obtained from the conventional one by refining the FE mesh until the relative change of the short-circuit current between two successive refinement steps is below 0.1%. It can be observed that the foil conductor models with 10 and, 20 foil elements, converge significantly faster when compared to the conventional model (Fig. 6). The time saving by applying the novel method, is two orders of magnitude. Avoiding the small regions of the individual foils to be meshed, results in a tenfold smaller mesh while keeping the same overall accuracy. The considerable reduction of the resolution due to discretizing the voltage across a foil winding, is also validated by the fact that increasing the number of foil elements does not that much increase the convergence for the meshes resolving the magnetic field. In fact, the required resolution for the foil mesh is tuned to the characteristic mesh size of the magnetic mesh. The introduction of a discretization of the foil voltage, presented here, enables a conscious choice of this resolution.

VI. CONCLUSIONS

The foil conductor model adds an additional discretization in terms of rectangular shaped foil finite elements to the magnetic model. Independent mesh generation and refinement and, as a consequence, a considerable saving of degrees of unknowns, is

offered. The foil conductor model enables the accurate simulation of a three-phase foil-winding transformer with a factor 100 times less computational efforts when compared to conventional field-circuit coupled approaches.

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