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Strong coupling of magnetic and mechanical finite element analysis

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Abstract

A strong coupling between the magnetic and the mechanical finite element model is presented. The two coupling terms represent magnetic forces and magnetostriction, respectively. The coupled system is solved using a fixed point iteration (successive substitution) with relaxation. The influence of physical parameters (low or high magnetostriction) and numerical parameters (scaling factors) on the convergence is investigated.

Keywords: Finite element method; Magnetomechanical coupling; Magnetostriction; Numerical simulation

1. Introduction

The investigation of noise and vibrations of electrical machinery is based on the coupling between the magnetic field and the mechanical stator deformation. Stator deformations are caused not only by reluctance forces, but also by magnetostriction of the stator yoke [1]. The interaction between the two systems is captured in one magnetomechanical matrix which is solved at once. Next to the magnetisation characteristic of iron, the magnetostriction characteristic $\lambda(B)$ is needed. The total energy E of the magnetomechanical system is the sum of the elastic energy U and the magnetic energy W. The three unknowns on one node (vector potential, x- and y-displacement) are gathered in one vector $[Aa]^T$. This leads to the following combination of the magnetic finite element system $MA = T$ and the mechanical finite element system $Ka = R$:

$$\begin{bmatrix} M & D \\ C & K \end{bmatrix} \begin{bmatrix} A \\ a \end{bmatrix} = \begin{bmatrix} T \\ R \end{bmatrix}, \qquad (1)$$

where T is the magnetic source term vector and R represents external forces. The mechanical system is assumed to always stay in its linear range. The coupling term C is related to magnetic forces by $F_{mag} = -CA$ for linear magnetic systems, and by a similar integral expression for nonlinear magnetic systems [2].

2. Magnetostriction forces

By *magnetostriction forces*, we indicate the set of forces that induces the same strain in the material as magnetostriction does. This approach is similar to how thermal stresses are usually taken into account. For finite element models, this can be done on an element-by-element basis, where the centre of gravity of the element is used as a local fixed point. The magnetostrictive deformation of the element, i.e. the displacement of the three nodes with respect to the centre, is found using the element's flux density B^e and the $\lambda(B)$ characteristic of the material. The element's mechanical stiffness matrix K^e allows us to convert the displacements a^e_{ms} into a set of forces using $F^e_{ms} = K^e a^e_{ms}$. This approach can be used for both isotropic and anisotropic materials under plane strain or plane stress. When the flux density B_0 in an element increases to $B_0 + \Delta B$, the element wants to expand to $a_0 + \Delta a$ due to magnetostriction. In order to find the

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elastic energy change ΔU due to ΔB but for constant deformation, the element is shrunk back into its original deformation a_0. The external work done to go back from $a_0 + \Delta a$ to a_0 is stored in ΔU and allows us to find: (1) an analytical expression for Da in Eqs. (1) and (2) an equivalent current source I_{ms} representing the influence of the magnetostrictive material on the magnetic field

$$\begin{bmatrix} M & D \\ C & K \end{bmatrix} \begin{bmatrix} A \\ a \end{bmatrix} = \begin{bmatrix} T - I_{ms} \\ R + F_{ms} \end{bmatrix}. \quad (2)$$

3. Convergence of the fixed point iteration

System (2) is now solved directly using a fixed point iteration with relaxation. The convergence is tested using the model of an iron core (side 10 cm, thickness 1 cm) excited by a current coil. The current in the coil is fixed to 40 A (5.10^4 A/m^2). The mesh has 850 elements and 446 nodes, giving 1338 unknowns. The core is mechanically constrained by a pin and a slider. The core material is linear with permeability $\mu_r = 1000$. The magnetostriction of the iron core material is assumed to be isotropic and to increase quadratically as a function of flux density, where the value at $B = 2$ T will be used to indicate the severity of the magnetostriction. A zero starting solution will not lead to convergence for the fixed point iteration. To obtain a useful starting solution, the magnetic system is solved separately giving A_0. This needs to be done only once, and the starting solution $[A_0 0]^T$ can then be used for all other cases mentioned below. Inside one substitution step, the matrix system is solved using a GMRES solver and takes about 150 steps for this model. Table 1 gives an overview of the relaxation factors used and the number of steps needed to reach a solution for which the relative error is smaller than 0.1%. The relaxation factors are taken as high as possible while still obtaining convergence. The values in Table 1 are found using a magnetically linear material, in order to emphasise influence of the nonlinearity coming from magnetostriction. Table 1 also mentions the solution times needed on a HP B1000 workstation. In Table 1, it appears that for low magnetostriction, up to $\lambda(2\,\mathrm{T}) < 500\,\mu$m, the convergence is identical to the case without magnetostriction and needs only five substitution steps. Most technical materials have magnetostriction below this limit. Only special materials like Terfenol (used in actuators and linear motors-based on magnetostriction) will reach magnetostriction values of the order of 2000 μm. The higher the magnetostriction, the lower the relaxation factor than can be used to obtain convergence. For the cases, $\lambda(2\,\mathrm{T}) = 1000\,\mu$m and $\lambda(2\,\mathrm{T}) = 1500\,\mu$m the convergence can be accelerated by applying a slightly lower relaxation factor (indicated with boldface in the table). The displacement vector a needs to be scaled with a factor f in order to make sure that the vector potential A and the displacement a have the same order of magnitude, otherwise the error estimate used would not capture both the magnetic and the mechanical system. The scaling factor f is of minor importance when the core material is magnetically linear and has no magnetostriction. However, the scaling is essential when the core material is nonlinear because convergence will be obtained only for $f = 10^2$ and higher (the exact value of the scaling does not make a significant difference). For the magnetically linear case with high magnetostriction, convergence can only be obtained for $f = 10^5$ and higher. Increasing the scaling factor further up to $f = 10^{16}$ is of no influence.

Table 1
Convergence as a function of magnetostriction for linear magnetic material ($\mu_r = 1000$)

Magnetostriction $\lambda(2\,\mathrm{T})$ (μm)	Relaxation factor	Number of steps	Solution time (s)
0	1.0	5	3.44
2.5	1.0	5	3.61
25	1.0	5	3.41
250	1.0	5	3.38
1000	0.5	12	8.64
	0.6	9	7.21
	0.7	**9**	**6.50**
	0.8	11	8.04
1500	0.4	16	11.53
	0.5	**12**	**8.78**
	0.6	32	22.65
2000	0.1	67	47.89
	0.2	33	23.16
	0.3	24	17.10
2500	0.1	67	47.86

4. Conclusion

A numerically strong coupling between the magnetic and the mechanical system has been established. The coupling terms are related to magnetic forces and magnetostriction. Magnetostriction forces are derived based on the analogy with thermal stresses. The magnetomechanical system is solved using a fixed point iteration. The relaxation factors and number of steps needed to obtain a solution strongly depend on the nonlinearities in the system: the saturation characteristic and the magnetostriction characteristic of the materials. A scaling factor for the displacement vector needs to be applied in order to balance the order of magnitude of the elements in the vector of unknowns.

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