Strong magnetomechanical FE coupling using local magnetostriction forces*

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Abstract. The magnetic material's deformation caused by magnetostriction is represented by an equivalent set of mechanical forces, giving the same deformation to the material as magnetostriction does. This is done in a way similar to how thermal stresses are usually incorporated into stress analysis. The resulting magnetostriction force distribution can be superposed onto other force distributions, like the magnetic force distribution. These two force distributions are the key ingredients of a numerically strong coupling of the magnetic and the mechanical finite element systems.

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1 Introduction

An important source of vibrations and noise in electric devices, rotating as well as non-rotating, are the deformations caused by magnetostriction. These magnetostrictive deformations can be of the same order of magnitude as the deformations caused by reluctance forces (Maxwell stresses) on the iron-air interface [1]. The incorporation of magnetostriction in the numerical design process is usually impaired since detailed data on the magnetic material behaviour are difficult to obtain. Versatile experimental methods to obtain all needed technical data on magnetostriction, permeability, losses, etc. are reviewed in [2]. Once the magnetostrictive behaviour of the material is known, it has to be incorporated in the magnetic and mechanical analysis. The strong-coupled magnetomechanical finite element (FE) model [3] is briefly reviewed and it is illustrated how to expand this model to take magnetostriction into account. The magnetostriction material characteristic, *e.g.* in $\lambda(B)$ format (magnetostrictive strain λ as a function of magnetic flux density B), is assumed to be known.

2 The magnetomechanical system

Both magnetostatic and elasticity FE methods are based upon the minimisation of an energy function. The total energy E of the magnetomechanical system consists of the magnetic energy W stored in a linear magnetic system with vector potential A, and the elastic energy U stored in a body with deformation a:

$$E = W + U = \frac{1}{2}A^T M A + \frac{1}{2}a^T K a, \tag{1}$$

where K is the mechanical stiffness matrix and M is the magnetic 'stiffness' matrix. Considering the similar form of the energy terms (1), a good candidate to represent the magnetomechanical system is

$$\begin{bmatrix} M & D \\ C & K \end{bmatrix} \begin{bmatrix} A \\ a \end{bmatrix} = \begin{bmatrix} T \\ R \end{bmatrix}, \tag{2}$$

where T is the magnetic source term vector and R represents external forces. The partial derivatives of the total energy E with respect to the unknowns $[A\ a]^T$ identify with the combined system (2):

$$\frac{\partial E}{\partial A} = MA + \frac{1}{2}a^T \frac{\partial K}{\partial A} a = 0, \tag{3}$$

$$\frac{\partial E}{\partial a} = \frac{1}{2}A^T \frac{\partial M}{\partial a} A + Ka = 0. \tag{4}$$

Using (2), (3) and (4), the coupling terms C and D are recognised as

$$C = \frac{1}{2}A^T \frac{\partial M}{\partial a}, \tag{5}$$

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$$D = \frac{1}{2} a^T \frac{\partial K}{\partial A} \,. \tag{6}$$

3 Magnetic forces

The magnetic stiffness matrix M is a function of permeability μ and geometry x. The geometry depends on the displacement a by $x = x_0 + a$, so that $\partial M / \partial a \neq 0$. Rearranging the mechanical equation (4) into

$$Ka = -\frac{1}{2} A^T \frac{\partial M}{\partial a} A = -CA = F_{\mathrm{mag}}, \tag{7}$$

reveals a means to calculate the magnetic forces F_{mag} internal to the magnetomechanical system. For the nonlinear case, $M(a)$ becomes $M(A,a)$ and magnetic energy W is given by the integral

$$W = \int_0^A T^T \mathrm{d}A = \int_0^A A^T M(A,a) \mathrm{d}A, \tag{8}$$

where $T{=}MA$ and $M^T{=}M$ was used. The force expression (7) now becomes

$$F_{\mathrm{mag}} = -\frac{\partial W(A,a)}{\partial a} = -\int_0^A A^T \frac{\partial M(A,a)}{\partial a} \mathrm{d}A. \tag{9}$$

The partial derivative $\partial M / \partial a$ and the integral (9) are found analytically using the shape functions and the magnetization characteristic of the material, *e.g.* $\nu(B^2)$, as explained in detail in [3].

The interpretation of the coupling term D for magnetostrictive materials will be investigated in Section 5. For materials without magnetostriction, the term D vanishes since the mechanical stiffness matrix K is a function of Young modulus, Poisson modulus and geometry only. For $D = 0$, and with the magnetic forces $F_{\mathrm{mag}} = -CA$ shifted to the right hand side of (2), the system becomes

$$\begin{bmatrix} M & 0 \\ 0 & K \end{bmatrix} \begin{bmatrix} A \\ a \end{bmatrix} = \begin{bmatrix} T \\ R + F_{\mathrm{mag}} \end{bmatrix}. \tag{10}$$

4 Magnetostriction forces

4.1 Concept

Now magnetostriction is built into the analysis using a force distribution F_{ms} that can be added to R and F_{mag}. By magnetostriction forces F_{ms} we indicate the set of forces that induces the same strain in the material as the magnetostriction effect does. This approach is similar to the use of thermal stresses due to heating [5]. In calculating thermal stresses, the thermal expansion of the free body (no boundary conditions) is calculated based on the temperature distribution, and the thermal stresses are found by deforming the expanded body back into its original shape (back inside the original boundary conditions). In calculating magnetostriction forces, the expansion of the free body due to magnetostriction is calculated based on the magnetic flux density, and the magnetostriction forces are found as the reaction to the forces needed to deform the expanded body back into the original boundary conditions.

For FE models, this can be done on an element by element basis. The midpoint (center of gravity) of the element is considered to be fixed. The magnetostrictive deformation of the element, *i.e.* the displacement of the nodes with respect to the midpoint, is found using the element's flux density B_{e} and the $\lambda(B)$ characteristic of the material. If a set of $\lambda(B,\sigma)$ characteristics are given, one has to be chosen for the appropriate value of tensile stress.

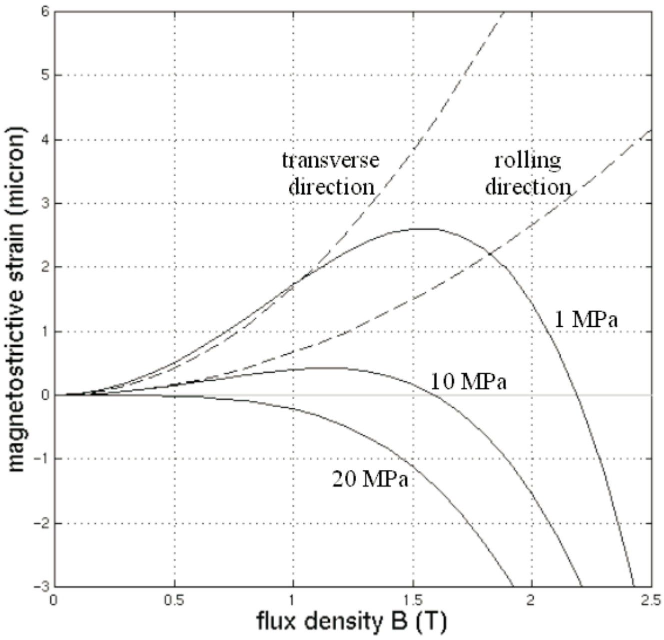


Fig. 1. Magnetostrictive material characteristics of non-oriented 3% SiFe (solid lines, as a function of tensile stress) and M330-50A (dashed lines, for rolling and transverse direction).

4.2 Strain for isotropic materials

Figure 1 shows a typical magnetostriction characteristic for isotropic 3% SiFe (solid lines) as a function of tensile stress σ. For isotropic materials, the local xy-axes of the element are chosen so that the flux density vector $\boldsymbol{B}$ coincides with the local x-axis. Usually, magnetostriction will not change the total volume and density [4], so that the strains in the local frame are given by

$$\begin{aligned} \lambda_x &= \lambda \\ \lambda_y &= \lambda_t = -\lambda/2 \\ \lambda_z &= \lambda_t = -\lambda/2 \end{aligned} \tag{11}$$

where $\lambda = \lambda(B)$ is the magnetostrictive strain in the direction of $\boldsymbol{B}$ and λ_{t} is the magnetostrictive strain in the transverse directions. The volume invariance is equivalent to a magnetostrictive 'Poisson modulus' of 0.5, which is bigger than the mechanical Poisson modulus of about 0.3.

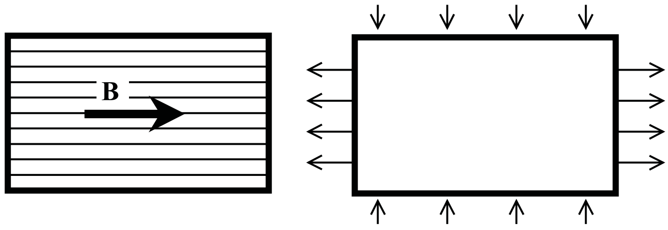


Fig. 2. Set of forces (right) representing the strain caused by magnetostriction due to the magnetic field $\boldsymbol{B}$ (left), consists of a set parallel and a set perpendicular to the flux vector.

Therefore, when magnetostriction is represented by a set of mechanical forces, there is always a set of forces in the direction of $\boldsymbol{B}$ and a set perpendicular to $\boldsymbol{B}$ to correct this difference in Poisson modulus (Fig. 2).

In a 2D plane strain analysis, the thickness (z-direction) of the material has to remain constant and an additional z-stress needs to be applied in order to obtain $\lambda_z = 0$. This adjusts the values (11) to

$$\begin{aligned} \lambda_x &= \lambda + \nu\lambda_\mathrm{t} \\ \lambda_y &= \lambda_\mathrm{t} + \nu\lambda_\mathrm{t} \\ \lambda_z &= \lambda_\mathrm{t} - \lambda_\mathrm{t} = 0 \end{aligned} \tag{12}$$

where ν is the Poisson modulus of the material.

4.3 Strain for anisotropic materials

Figure 1 shows a typical magnetostriction characteristic for anisotropic M330-50A steel (dashed lines) for rolling direction and transverse direction. For anisotropic materials, the flux density vector is decomposed into a B_x and a B_y component in the element's local xy-axes, arranged so that the x-axis coincides with the rolling direction, and the y-axis with the transverse direction. The rolling direction curve $\lambda_\mathrm{RD}(B)$ is then used with B_x as input, and the perpendicular direction curve $\lambda_\mathrm{PD}(B)$ with B_y as input, giving

$$\begin{aligned} \lambda_x &= \lambda_\mathrm{RD}(B_x) - \nu\lambda_\mathrm{PD}(B_y) \\ \lambda_y &= \lambda_\mathrm{PD}(B_y) - \nu\lambda_\mathrm{RD}(B_x) \\ \lambda_z &= -\nu\lambda_\mathrm{RD}(B_x) - \nu\lambda_\mathrm{PD}(B_y). \end{aligned} \tag{13}$$

A similar correction as above can be made for plane strain.

4.4 Displacement and force

Still working in the local xy-axes, the element's strains λ_x^e, λ_y^e are converted into three nodal displacements $a_{\mathrm{ms},i}^\mathrm{e} = (a_{x,i}^\mathrm{e}, a_{y,i}^\mathrm{e}), i = 1,2,3$ considering the midpoint of the element $(x_\mathrm{m}^\mathrm{e}, y_\mathrm{m}^\mathrm{e})$ as fixed:

$$\begin{bmatrix} a_{x,i}^\mathrm{e} \\ a_{y,i}^\mathrm{e} \end{bmatrix} = \begin{bmatrix} x_i - x_\mathrm{m}^\mathrm{e} \\ y_i - y_\mathrm{m}^\mathrm{e} \end{bmatrix} \begin{bmatrix} \lambda_x^\mathrm{e} \\ \lambda_y^\mathrm{e} \end{bmatrix}, \tag{14}$$

where i is the index for the three element nodes (x_i, y_i).

The mechanical element stiffness matrix K^e yields, after multiplication with the magnetostrictive displacement a_ms^e of the nodes, the nodal magnetostriction forces

$$F_\mathrm{ms}^\mathrm{e} = K^\mathrm{e} a_\mathrm{ms}^\mathrm{e}. \tag{15}$$

Equation (15) has to be performed element by element (using K^e) and not for the whole mesh at once (using the global matrix K), because the N different displacements $a_{\mathrm{ms},ij}, j = 1...N$, due to magnetostriction in the N elements surrounding node i, should not be summed. The magnetostriction forces are now introduced in (10) giving

$$\begin{bmatrix} M & 0 \\ 0 & K \end{bmatrix} \begin{bmatrix} A \\ a \end{bmatrix} = \begin{bmatrix} T \\ R + F_\mathrm{mag} + F_\mathrm{ms} \end{bmatrix}. \tag{16}$$

The force distribution F_ms or the total distribution $F_\mathrm{mag} + F_\mathrm{ms}$ can also be used for any other kind of post-processing based on force distributions, *e.g.* calculating mode participation factors with stator mode shapes [3].

5 The coupling term $\partial U/\partial A$

The term D in (2) is related to magnetostriction: Da represents the change in elastic energy U due to a change dA (with corresponding change in magnetic field dB), with deformation a held constant:

$$Da \sim \frac{\partial U}{\partial A}, \tag{17}$$

where $\sim$ anticipates to the fact that $\partial U/\partial A$ will turn out to have terms independent of a. Imagine an element with deformation a_0 and flux density B_0. When the flux density in the element increases to $B_0 + \Delta B$, the element expands to $a_0 + \Delta a$ due to magnetostriction (no external stresses need to be applied, so $\Delta U = 0, \Delta a \neq 0$). In order to find the elastic energy change ΔU due to ΔB but for constant deformation, the element needs to be shrunk back to its original deformation a_0. The external work done to go back from $a_0 + \Delta a$ to a_0 is stored in ΔU and allows us to find $\Delta U/\Delta B$. For an isotropic material under plane stress, $\partial U/\partial A$ is

$$\frac{\partial U}{\partial A} = \Delta t E \frac{5/4 - \nu}{1 - \nu^2} \lambda(A) \frac{\partial \lambda(A)}{\partial A}, \tag{18}$$

where Δ and t are element area and thickness and E and ν are Young and Poisson modulus. By expanding the area Δ in terms of x_0 and a (linked by $x = x_0 + a$) as

$$\Delta(x_0 + a) = x_0^T d_1 x_0 + x_0^T d_2 a + a^T d_3 a, \tag{19}$$

(18) can be rewritten as

$$\frac{\partial U}{\partial A} = x_0^T D_1 x_0 + x_0^T D_2 a + a^T D_3 a. \tag{20}$$

Since $a \ll x_0$, the third term in (20) can be neglected. The first term in (20) does not depend on displacement a and should be put on the right hand side of the coupled

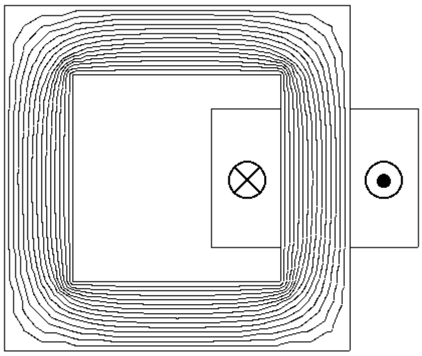
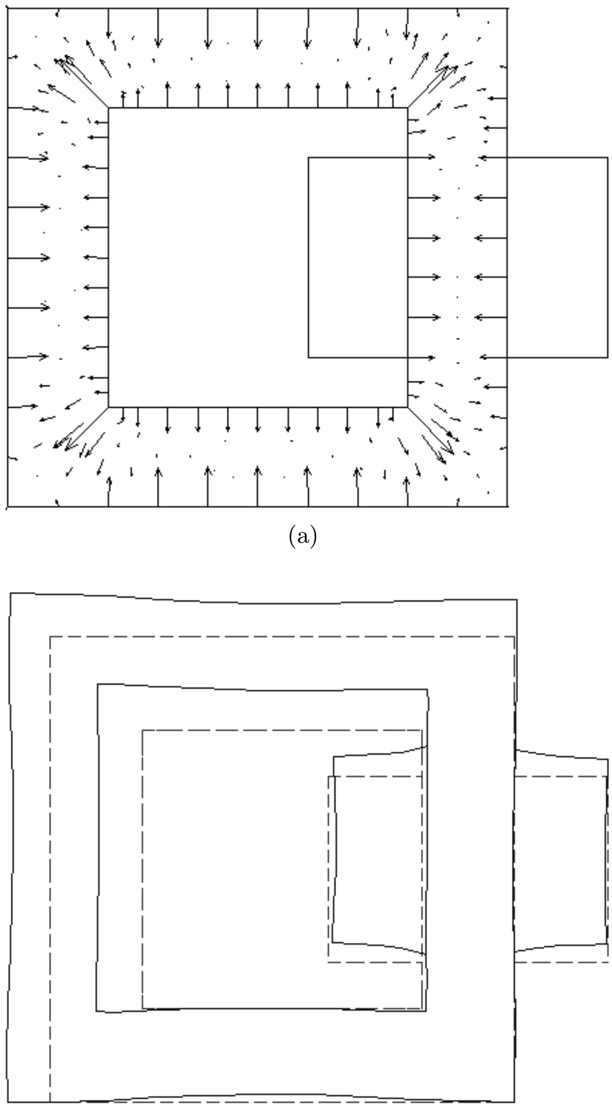


Fig. 3. Geometry and flux lines for iron core excited by current in copper coil.



(a)

(b)

Fig. 4. (a) Magnetostriction force distribution and (b) resulting deformation (magnetic forces excluded).

system (2). This term can be interpreted as an additional current density I_{ms} representing the influence of magnetostriction on the magnetic field. The second term can be identified with the term Da in (2). Interpreting (20) in this way gives

$$\frac{\partial U}{\partial A} \approx I_{\mathrm{ms}} + Da + 0, \tag{21}$$

so that the system with strong coupling becomes

$$\begin{bmatrix} M & D \\ C & K \end{bmatrix} \begin{bmatrix} A \\ a \end{bmatrix} = \begin{bmatrix} T - I_{\mathrm{ms}} \\ R + F_{\mathrm{ms}} \end{bmatrix}. \tag{22}$$

6 Example

Figure 3 shows an iron core (side 100 mm, $E = 2.2 \times 10^{11}$ Pa, $\nu = 0.3$) with flux lines due to excitation by the copper coil (0.025 A/mm^2). The iron core is mechanically fixed with a pin on its right bottom corner, and a horizontal slider on its left bottom corner. The magnetostriction of the iron core material is isotropic and follows the 'rolling direction' curve in Figure 1. Figure 4 shows the magnetostriction forces F_{ms} and the deformation caused by magnetostriction (2×10^5 times magnified). Figure 5 shows the magnetic forces (9) and the deformation caused by them (4×10^8 times magnified). For this example (no airgap), the magnetic forces are about 2000 times smaller than the magnetostriction forces. Since the magnetic forces cause the core to shrink, and the magnetostriction causes the core to expand, this example clearly differentiates between these two phenomena.

These results were obtained solving (22) using 15 steps of successive substitution with a relaxation factor 0.4.

7 Conclusions

The numerically strong coupling of the magnetic and mechanical finite element systems is based upon the magnetic force distribution and upon magnetostriction. The magnetostriction can be represented by a set of forces giving the same deformation to the material as magnetostriction does. Two additional terms $\partial U/\partial A = I_{\mathrm{ms}} + Da$ represent the influence of magnetostriction on the magnetic field itself.

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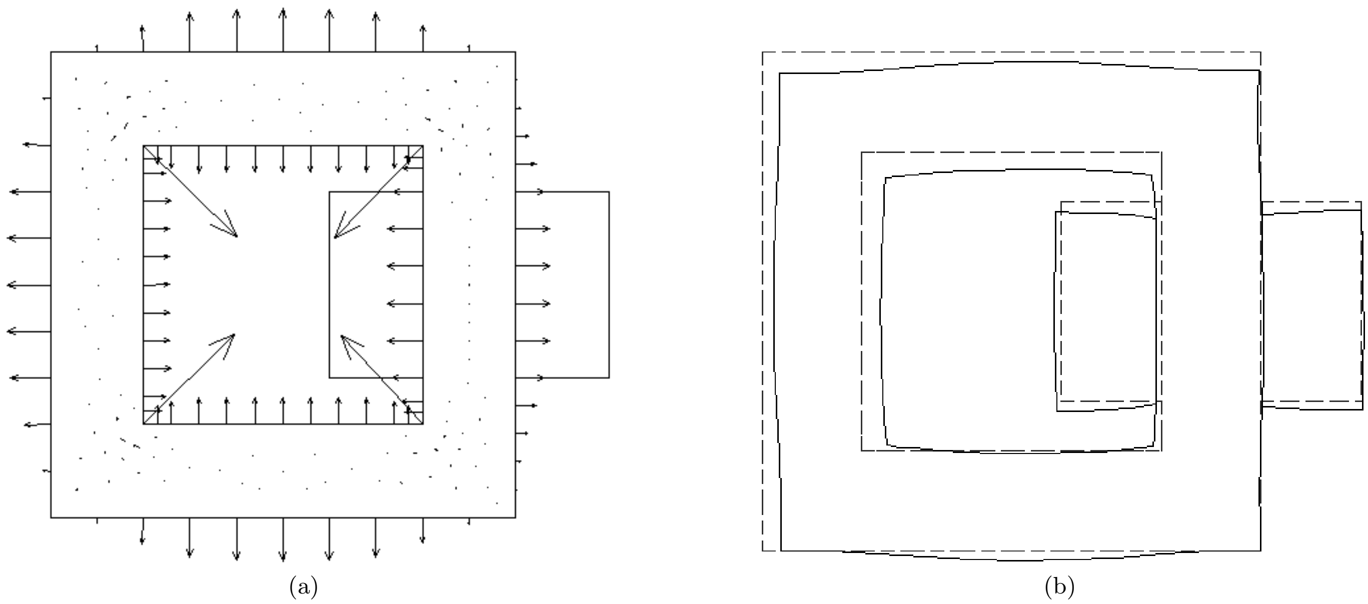

Fig. 5. (a) Magnetic force distribution and (b) resulting deformation (no magnetostriction).

References

1. L. Låftman, Ph.D. thesis, Lund Institute of Technology Sweden, 1995.
2. H. Pfützner, in *Proceedings of the Xth International Symposium on Theoretical Electrical Eningeering ISTET'99, Magdeburg, Germany, 1999*, p. 377.
3. K. Delaere *et al.*, in *Proceedings of the Xth International Symposium on Theoretical Electrical Eningeering ISTET'99, Magdeburg, Germany, 1999*, p. 417.
4. D. Jiles, *Introduction to Magnetism and Magnetic Materials* (Chapman and Hall 1991).
5. B.D. Cullity, *Introduction to Magnetic Materials* (Addison-Wesley - Series in Metallurgy and Materials, Philippines 1972).