Fast Strong-Coupled Multi-Harmonic Finite Element Simulation

Herbert De Gersem, Hans Vande Sande, Kay Hameyer
Katholieke Universiteit Leuven, Dep. ESAT, Div. ELEN
Kardinaal Mercierlaan 94, B-3001 LEUVEN, Belgium
Phone: +32-16-321020; Fax: +32-16-321985
Email: Herbert.DeGersem@esat.kuleuven.ac.be
www: http://www.esat.kuleuven.ac.be/elen/elen.html

Electrical devices submitted to a periodic excitation and behaving stationary, can be modelled efficiently in the frequency domain. To simulate models with complicated geometries, nonlinear materials and eddy current effects, the finite element method are commonly used. The time-harmonic finite element method assumes sinusoidal time dependence and represents all field quantities by phasors. The multi-harmonic finite element method (MHFEM), also called the harmonic balanced finite element method [1], is a natural extension of the time-harmonic approach. MHFEM applies truncated Fourier series representing the field quantities at each degree of freedom of the finite element discretisation. Several harmonic signals are combined within the same finite element simulation. Higher harmonics raise in the model by the excitation, the presence of nonlinear materials or motional effects. The application of this approach to technical devices of engineering importance is extremely expensive, both in memory and computation time [2]. This is the reason why MHFEM is not widespread. The aim of this work is to solve this numerical issue in order to generally enable the application of MHFEM to technical designs and optimisations.

The partial differential equation describing the magnetic field in terms of the magnetic vector potential, is transformed into the frequency domain by replacing all multiplications by convolutions. After applying finite element discretisation, a large and sparse convolution operator is obtained. An appropriate matrix representation is invoked retaining the symmetry of the operator. This is exploited for the iterative solution of the operator equation by a Krylov subspace method. A Successive Overrelaxation preconditioner is used to enhance the convergence of the Krylov subspace solver. To perform the solution of the small-sized Toeplitz systems corresponding to the diagonal entries of the convolution operator, the Levinson algorithm [3] is adapted to the particular form of the convolution operator.

To resolve nonlinear material characteristics, a fixed point iteration is applied. Out of an intermediate solution, new material properties are determined by inverse and discrete fast Fourier transforms. Adaptive relaxation is applied to the nonlinear loop. As a result of this approach, the overall solver is much faster when compared to the strong coupled approach relying upon a real-equivalent matrix presented in [1] and the decoupled relaxed approach presented in [2].

The method is applied to simulate the harmonic contents of the magnetic field in a electrical transformer under no-load conditions. The high flux densities give raise to significant saturation. The nonlinearity of this effect creates odd higher harmonic fields. The scaling of the computation time with respect to the number of harmonics involved in the simulation, is studied.

The implementation presented here incorporates fast algorithms for the solution of the operator equation, fast inversion of spectral data, relaxed fixed point iterations and fast updates of material properties. The acceleration achieved by this implementation, enables the application of the multi-harmonic finite element method to realistic and technically relevant models.

[1] S. Yamada, K. Bessho and J. Lu, "Harmonic balance finite element method applied to nonlinear AC magnetic analysis", *IEEE Transactions on Magnetics*, Vol. 25, No. 4, July 1989, pp. 2971-2973.
[2] J. Driesen, G. Deliége, T. Van Craenenbroeck and K. Hameyer, "Implementation of the harmonic balance FEM method for large-scale saturated electromagnetic devices", in *Software for Electrical Engineers*, A. Konrad and C.A. Brebbia, eds, WIT Press, Southampton, 1999, pp. 75-84.
[3] G.H. Golub and C.F. Van Loan, *Matrix Computations*, 2nd ed., The Johns Hopkins University Press, Baltimore, 1989.