Comparison of a combined Time-Harmonic - Transient finite element analysis of fast transient oscillations with laboratory measurements

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Abstract-- In high voltage systems it is important to know if hazardous fast transient oscillatory regimes occur. Important as well is the knowledge of the damage that can be caused by such regimes. A test circuit around a Tesla transformer is built for repetitive excitation of the high voltage equipment in accelerated aging tests. The combined time-harmonic - transient finite element method using a function based approach, where the magnetic model of the Tesla transformer is coupled with an external electric circuit, is used to simulate the behaviour of the test circuit. Simulations are verified with measurements and show good agreement.

***Index Terms*--Fast transient oscillations, Tesla transformer, Time-Harmonic - Transient finite element analysis.**

I. INTRODUCTION

A circuit built around a Tesla transformer easily generates a high voltage with a high frequency (HF-HV). The working principle of this transformer is briefly described in section II. The generated HF-HV is useful for tests where the test object is part of the active circuit [1] [2] and to examine e.g. the high frequency behaviour of certain insulating materials such as oil immersed paper and PVC. The results of these tests can guide to an improved system design [3], a better understanding of the possible damage and to reduced manufacturing costs. The complete test circuit is therefore built up in our laboratory as a part of a research project [2]. This paper focuses on the numerical aspects of the Tesla transformer circuit. A comparison between analytical formulas, the finite element approach and laboratory measurements is performed.

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This test circuit can also be used for educational purposes. A good methodology to educate students concerning the numerical aspects of the simulation tools is by paying particular attention to the differences between computational results and measured data obtained by practical laboratory sessions. In this way, a better understanding of the scientific background of the analytical formulas and the approximations made in the different calculation techniques can be obtained. This is not always possible due to the high costs and/or complexity of certain electromagnetic devices. The circuit that was built around the Tesla transformer, has a lot of advantages, which make it easily accessible and usable for the teaching combining theoretical and practical sessions. The most important advantages are:

- The geometry of the Tesla transformer is simple, in fact it is an axisymmetric problem. It is possible to calculate all the elements of the Tesla transformer by analytical formulas to create an equivalent circuit.
- All the materials used are magnetically linear (copper, paper, oil and transformer wood). There are no materials with hysteresis or saturation effects. Therefore no simplifications are required in the non-linear behaviour during the calculations. The magnetic permeability, as the most important material property, is well known.
- Due to the use of simple and common circuit elements such as inductances, capacitances and resistors, the implementation of the electrical circuit in the finite element software is straight forward. Due to this simplicity, it is for an technical outsider easy to understand in which way the Tesla transformer circuit works and reacts on certain disturbances (tuning, short circuit at secondary or open secondary, …).

II. TESLA TRANSFORMER PRINCIPLE: RESONATING AIR TRANSFORMER

The Tesla transformer concept is based on the energy transmission between two magnetically coupled circuits. Both the circuits are built like an RLC-circuit [4]. The air-coupling is performed by building the inductances (L_1 and L_2) of each circuit together like a single phase transformer (Fig. 1). The first circuit consists of a capacitor C_1 and inductance L_1. The second resonating circuit is formed by L_2, which is air-coupled with L_1 and the overall capacity C_{tot} of the measurement circuit (C_{d1} and C_{d2}) and the capacity of the test object C_2 [2]. The two circuits are theoretically tuned when (1) is satisfied. The resonance frequency f_2 of the secondary circuit is then (2):

$$L_1 \cdot C_1 = L_2 \cdot C_{tot} \tag{1}$$

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{1}{L_2 C_{tot}} - \frac{R_2^2}{4L_2^2}} \tag{2}$$

L_1, C_1 and R_1 are the main circuit elements of the primary or initiating circuit. L_2, C_{tot} and R_2 are the elements of the secondary or high voltage circuit. U_{DC} and R_{DC} are the DC charging voltage respectively charging resistance. Switch S is necessary to control the HF-HV.

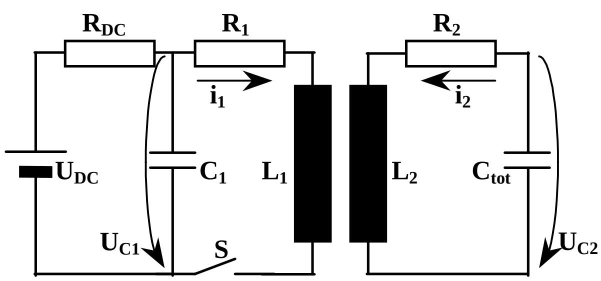

Fig. 1. Basic Tesla transformer circuit

When switch S is open, a DC voltage (U_{DC}) will charge capacitor C_1 up to its maximum value. Closing S initiates a RLC resonance in the primary circuit by discharging C_1 through inductance L_1. Resistance R_1 is the winding resistance of the primary inductor. Switching results in a high frequency oscillating damped sine wave. When the two circuits are tuned according to (1), the sine wave is transferred to the secondary circuit and initiates also a damped oscillating wave. The magnitude of the second wave is due to the transformer principle much higher than the magnitude of the first one. The test object, which is part of the secondary circuit, is subjected to this HF-HV.

To measure the frequency and magnitude of the voltage, a digital oscilloscope is used. By displaying the waveforms on the oscilloscope, it is easy to control the different quantities and to compare them with the results obtained by the simulations. In LABVIEW© a program is written to store the necessary waveforms digitally.

III. Modeling the Test Circuit by Different Methods

A. Analytical solution

To solve the circuit in Fig. 1, two equations are required when the charging circuit is neglected. The unknowns are the current i_1 (for the primary circuit) and i_2 (for the secondary circuit). Employing Kirchoff's Voltage Law for each circuit yields a system of two differential equations:

$$R_1 i_1 + L_1 \frac{di_1}{dt} - M\frac{di_2}{dt} + \frac{1}{C_1}\int_0^t i_1 dt = 0 \tag{3}$$

$$R_2 i_2 + L_2 \frac{di_2}{dt} - M\frac{di_1}{dt} + \frac{1}{C_2}\int_0^t i_2 dt = 0 \tag{4}$$

The coefficient M represents the mutual inductance between the two coils. With M the coupling factor k can be defined by (5):

$$k = \frac{M}{\sqrt{L_1 \cdot L_2}} \tag{5}$$

The coupling factor k represents the "quality" or "efficiency" of the energy transmission between the primary and secondary circuit. In a power transformer this factor is close to unity, because there is a good magnetic coupling due to the iron core and the construction. In general, the transmission of energy can be written by (6), taken into account the coupling factor k and the number of turns N_1 of the primary coil and secondary coil N_2 respectively. Applying a voltage U_1 to the primary, results in a voltage U_2 at the secondary.

$$U_2 = k \cdot \frac{N_2}{N_1} \cdot U_1 \tag{6}$$

In case of a Tesla transformer, the coupling factor is not negligible. There is no iron core and it is well known that a coupling factor between two coils in air is closer to zero than to unity. Therefore, it is necessary to measure or calculate the k factor. Measurements can be performed by applying an AC voltage to the primary or secondary and measuring the voltage at the open secondary coil, respectively the primary. By using (6) it is easy to have a good estimation of k. In our case, it was chosen to calculate the coupling factor k by using two Time Harmonic finite element calculations. The first one consists of applying a voltage of $u_1 = 1\angle 0°$ V to the primary coil and calculating the primary current i_1 and secondary voltage u_2 (the secondary is an open circuit). The resistance R_1 of the primary turns is also calculated and using (7), both the inductance L_1 and the mutual inductance M between the two coils can be calculated ($\omega = 2\pi f$, with f the frequency).

$$\begin{aligned} u_1 &= R_1 i_1 + j\omega L_1 i_1 \\ u_2 &= j\omega M i_1 \end{aligned} \tag{7}$$

The second calculation applies a voltage u_2 to the secondary coil and calculates the secondary current i_2 and the voltage across the primary coil u_1. The results of this calculation are the resistance of the secondary turns R_2, the inductance L_2 and the mutual inductance M, which is the same as the one obtained from (7). The calculations are performed at a single frequency. This is sufficient because the time harmonic solver does not take the capacity between the turns and the capacity to earth into account. This means that the frequency dependency of M is neglected. The calculations can be verified by measurements as the same laboratory tests can be done. The results show a good agreement, TABLE 1.

TABLE 1
VERIFICATION OF CALCULATIONS OBTAINED FROM THE TIME HARMONIC SOLVER WITH LABORATORY MEASUREMENTS

	Calculation[1]	Measurement	
	@ 50 kHz	Value	Frequency
Mutual Inductance [mH]	17.4	18.4	50 kHz
		18.3	500 Hz
		17.4	50 Hz
Primary Inductance L_1 [μH]	31.6	33.1	1 kHz
Primary Resistance R_1 [mΩ]	22.5	24.9	1 kHz
Secondary Inductance L_2 [mH]	6.10	6.28	1 kHz
Secondary Resistance R_2 [Ω]	5.04	5.90	1 kHz

[1]Note that the calculations of TABLE 1 are done at a single frequency of 50 kHz and the measurements at different frequencies.

The easiest method for solving the system of (3) and (4) is transforming the two equations into a system of four first order differential equations. This can be done by using the voltages over the capacitors as additional variables.

$$\begin{aligned} y_1 &= -\frac{1}{C_1}\int_0^t i_1 dt \qquad & y_3 &= i_1 \\ y_2 &= -\frac{1}{C_2}\int_0^t i_2 dt \qquad & y_4 &= i_2 \end{aligned} \tag{8}$$

Equations (8) in combination with (3) and (4) finally gives:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & L_1 & -M \\ 0 & 0 & -M & L_2 \end{bmatrix} \cdot \frac{d}{dt} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\frac{1}{C_1} & 0 \\ 0 & 0 & 0 & -\frac{1}{C_2} \\ 1 & 0 & -R_1 & 0 \\ 0 & 1 & 0 & -R_2 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \tag{9}$$

As starting conditions the voltage U_{DC} is applied over the capacitor C_1, no voltage over C_2 and both the currents i_1 and i_2 are zero (10).

$$y(0) = \begin{bmatrix} U_{DC} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{10}$$

Writing this in matrix-vector notation results in the notation found in literature [5].

$$A \cdot \dot{y} = B \cdot y + C \tag{11}$$

Solving this system of differential equations (11) can numerically be performed with various methods. There are two basic approaches: the One Step Methods and the Multi-Step Methods [6]. In general the numerical solution of an initial value problem of the form (12) is sought.

$$y' = f(x, y) \text{ with } y(x_0) = y_0 \tag{12}$$

1) Euler's first order method

The method of Euler is the most simple one step method based on a truncated Taylor series. The method truncates the Taylor series after the first order term,

$$y_{n+1} = y_n + h \cdot f(x_n, y_n) \tag{13}$$

with h a fixed or normalized step size $h = x_{n+1} - x_n$. This is called an explicit method as the function evaluation of the derivative depends only on the solution at $x = x_n$.

2) Predictor-Corrector method

In the predictor step, a first prediction of the solution is made using the one step Euler method of (13). This solution will be corrected in the corrector step as much as necessary to lower the error between two succesive approximative solutions as much as possible. The used correction step is given by (14):

$$y_{n+1} = y_n + h \cdot \left(\alpha \cdot f(x_n + h, y_{n+1}) + (1-\alpha) \cdot f(x_n, y_n)\right) \tag{14}$$

Various difference schemes can be used by changing the value of the parameter α in the recurrence relation (14). All the simulations are performed using a time-stepping scheme with $\alpha = 2/3$ (Galerkin Method).

3) Multi-step method

As multi-step method the four step Adams-Bashforth method is chosen.

$$\begin{aligned} y_n = y_{n-1} + h \cdot (&55 \cdot f(x_{n-1}, y_{n-1}) - 59 \cdot f(x_{n-2}, y_{n-2}) \\ &+ 37 \cdot f(x_{n-3}, y_{n-3}) - 9 \cdot f(x_{n-4}, y_{n-4}))/24 \end{aligned} \tag{15}$$

A typical problem of implementing a four step method is the calculation of the values during the first three steps. Different techniques are possible. The most simple technique is to use a one step approach in the first step, in the second step a two step method and in the third step a three step Adams-Bashforth method.

$$
\begin{aligned}
y_1 &= y_0 + h \cdot f(x_0, y_0) \\
y_2 &= y_1 + h \cdot \left(3 \cdot f(x_1, y_1) - f(x_0, y_0)\right)/2 \\
y_3 &= y_2 + h \cdot \left(23 \cdot f(x_2, y_2) - 16 \cdot f(x_1, y_1) \right. \\
&\qquad \left. + 5 \cdot f(x_0, y_0)\right)/12
\end{aligned} \tag{16}
$$

4) *Runge-Kutta method*

The fourth order Runge-Kutta method is a special form of the one-step methods. This method uses also a truncated Taylor series without calculating the higher derivatives. The four traps Runge-Kutta method means that there are four function evaluations used in the final increment function.

$$
\begin{aligned}
k_1 &= f(x_0, y_0) \\
k_2 &= f(x_0 + h/2, y_0 + h * k_1/2) \\
k_3 &= f(x_0 + h/2, y_0 + h * k_2/2) \\
k_4 &= f(x_0 + h, y_0 + h * k_3)
\end{aligned} \tag{17}
$$

$$
y = y + h \cdot (k_1 + 2 \cdot k_2 + 2 \cdot k_3 + k_4)/6 \tag{18}
$$

B. *Time-Harmonic - Transient-Finite element solution*

To simulate the behaviour of the test circuit with the finite element method (FEM), the combined Time-Harmonic - Transient method using a function based approach is taken [7], [8]. The magnetic finite element model of the Tesla transformer is coupled with an external electrical circuit model. This method uses a one-step time-stepping scheme. To reduce the total computation time, a time-harmonic solution is taken as start solution for the transient method [9].

Due to the presence of the switch S, which is replaced by a thyristor TH in the laboratory setup (see Fig. 2 and Fig. 3), two calculations are required because is at this moment it is only possible to vary the parameters of the external electric circuit with respect to time. The first calculation determines the point where the current i_1 passes through zero. This is the point at which the thyristor extinguishes. In the second calculation the thyristor is modeled as a variable resistor. Due to the function based approach it is simple to suddenly increase the value of the resistor to simulate the opening of the switch (thyristor). A step function is used, which at each time step is evaluated and the resistance value is changed accordingly.

With a more appropriate function description, i.e. to vary a parameter not only with respect to the time, only one transient calculation is required in order to save computation time.

IV. Simulations

In total there are 5 different simulations and laboratory measurements done for three different configurations of the Tesla circuit. The first circuit is the basic circuit of a Tesla transformer as shown in Fig. 1. The Tesla transformer is once tuned and once not tuned (2 simulations). The second series of simulations (2 simulations) is done on the circuit shown in Fig. 2 (simplified Tesla circuit). Again, the first time with a tuned Tesla transformer and the second simulation and measurement on a free running Tesla transformer (not tuned). Note that this time switch S is replaced by a thyristor TH. This means that only the first half wave cycle of the primary current will flow in the primary circuit. The secondary will be ignited through this wave, and proceeds like an exponentially damped sine wave independent of the charging of C_1 because TH is an open circuit.

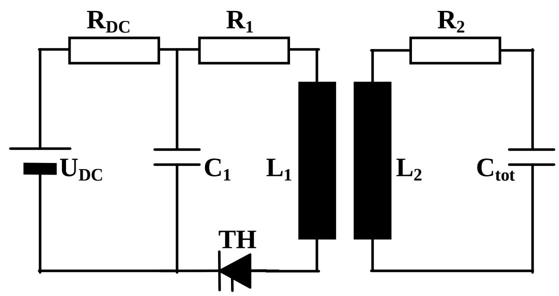

Fig. 2. Second test circuit: Simplified Tesla circuit

The last set of simulation and measurement is done on the circuit described in [2]. A simplified circuit is drawn in Fig. 3 (extended Tesla circuit). During the tests, the primary and secondary circuit are tuned. C_3 is a decoupling capacitor preventing that the HF-HV wave would penetrate to the AC-main supply.

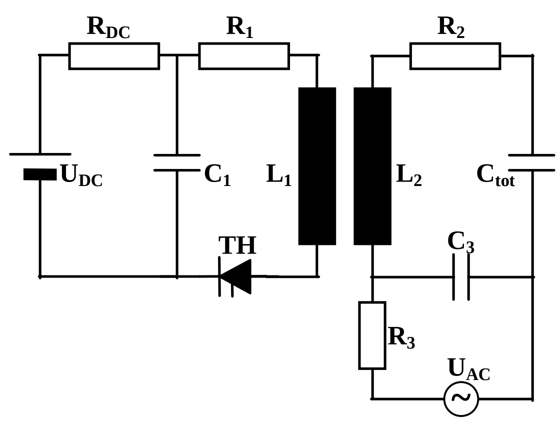

Fig. 3. Third test circuit: Extended Tesla transformer circuit

V. Results of the different methods

Only a comparison of the results obtained for the second and third test circuit will be published. In both cases, the Tesla transformer is perfectly tuned. The reason why only these results will be discussed is due to the more practical importance of these two test cases in comparison with the other three. The results for the remaining three test cases are similar and will lead to the same conclusions.

Measured and calculated values of the voltage across C_{tot}

for a tuned Tesla transformer circuit of Fig. 2 are resumed in TABLE 2. The DC voltage applied to U_{DC} equals 500 V. For an easy and accurate comparison, the peak to peak voltage is measured between the second positive peak and the first negative peak. Also the frequency is measured for the first period of the damped oscillating sine wave. Finally, the last column of the table gives the used time step for each different method. In case of measurements, the sampling rate of the oscilloscope is displayed into the last column.

TABLE 2
COMPARISON SIMULATIONS AND MEASUREMENTS FOR THE SIMPLIFIED TESLA CIRCUIT

Method	$V_{peak/peak}$ [V]	Frequency [kHz]	Time Step [s]
One Step	6613	39.56	2.0e-08
Multi-Step	6440	39.62	2.0e-08
Runge-Kutta	6050	39.65	2.0e-08
Predictor-Corrector	6470	39.62	2.0e-08
FEM	6205	39.42	1.25e-08
Measurement	5210	39.06	2.0e-08

Fig. 4 gives the first 100 µs of the voltage across C_{tot} for each different method (analytical, measurement and FEM). In case of the analytical calculations, a plot is given for one method only. From TABLE 2 it can be seen that the values are close together In this case the results obtained with a predictor corrector method are plotted. The horizontal and vertical scaling of the three plots is the same.

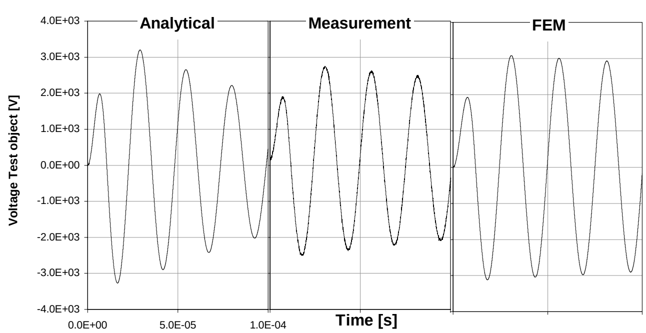

Fig. 4. Comparison of the voltage waves obtained from the different methods

The same tests and simulations are performed for the extended Tesla circuit of Fig. 3. The peak to peak value of the voltage across C_{tot}, the frequency and the time step for the different methods are given in TABLE 3. In case of the FEM method, there are two time steps given. As long as the thyristor is opened, only the 50 Hz AC voltage U_{AC} will be noticeable at C_{tot}. Because this 50 Hz voltage is of less importance, the time step can be chosen less accurate (5.0e-05 s) to reduce computation time. Once the thyristor is fired, the oscillatory regime starts. Due to the higher frequency, the time step becomes smaller as long as required (1.25e-08 s).

TABLE 3
COMPARISON SIMULATIONS AND MEASUREMENTS FOR THE EXTENDED TESLA CIRCUIT

Method	$V_{peak/peak}$ [V]	Frequency [kHz]	Time Step [s]
One Step	DIV	DIV	4.0e-08
Multi-Step	6520	39.06	4.0e-08
Runge-Kutta	6494	39.12	4.0e-08
Predictor-Corrector	6140	39.22	4.0e-08
FEM	6086	39.19	5.0e-05 / 1.25e-08
Measurement	5310	40.00	4.0e-07

A plot of the full sine wave (50 Hz AC voltage and the HF-HV) obtained from the FEM solution and the measurements are shown in Fig. 5. The FEM solution is plotted as a dotted line, the measurements as a solid line. In the upper left corner, a zoom of the HF-HV FEM solution and in the lower right corner a zoom of the HV-HV measured wave is drawn. In both cases the figures starts at 5.0e-03 seconds, the top of the first half wave cycle of the 50 Hz AC mains.

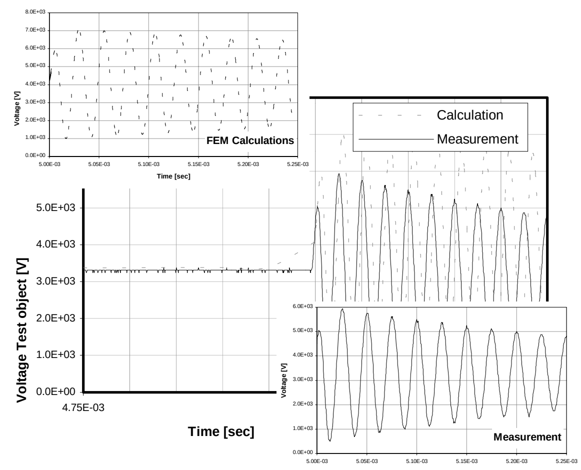

Fig. 5. Comparison of the voltage waves obtained from the FEM solution (upper left corner) and measurements (lower right corner)

VI. CONCLUSIONS

By using the results form TABLE 2 and TABLE 3, the following conclusions can be made.

The peak to peak value of the voltage U_{Ctot} obtained from the Time-Harmonic - Transient solver shows a good agreement with the voltages obtained from the analytical calculations. They agree within 1%, as both methods use the same simplifications and assumptions. This is also true for the frequency. Even the most rudimentary one step method gives a good agreement for the frequency and in comparison with the other analytical methods for the peak to peak voltage as well.

In comparison with the analytical and FEM approximations, the measurements show a good agreement for the frequency but not for the peak to peak voltage. Those differences in amplitude have not only to be attributed to the small differences between the calculated and measured values of the components. Other reasons are:

- The analytical and FEM methods do not consider the capacity between the turns and the capacity to earth of the secondary coil.
- The system used for measuring the wave form has a certain accuracy and is not free of noise and undesired reflections and can be affected by the surrounding.
- The measuring systems also affects the test circuit.

For all the different test circuits and for all the different simulations which are performed, the calculations with both methods, the Time-Harmonic - Transient solver and the analytical methods, show a good agreement with the wave shapes obtained from measurements. In all the cases, the calculated voltage wave have the same shape as the measured voltage wave. The simulations give a good idea of the expected phenomena.

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