INTRODUCING MAGNETOSTRICTION INTO COUPLED MAGNETO-MECHANICAL SYSTEMS

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Abstract

In non-rotating electric machinery like transformers, magnetostriction is an important source of deformation, and hence of vibrations and noise. This paper indicates how the magnetostrictive characteristics of the materials, e.g. $\lambda(B)$, can be introduced into the coupled magneto-mechanical finite element system. A cascade-procedure is used to determine the magnetostrictive forces and the total deformation of a small iron core coil. The predictions of the numerical method will be verified with experimental magnetostriction measurements.

1. Introduction: Magnetostriction

An important source of vibrations and noise in non-rotating electric machinery (transformers, linear motors, actuators, iron core coils, …) is the deformation caused by magnetostriction (MS). The incorporation of the magnetostrictive effect in the (numerical) design process is usually impaired since detailed data on the magnetic material behaviour are hard to obtain. Moreover, material data are often given for specific conditions (e.g. MS in rolling direction and in transverse direction only) which are not necessarily met under real operating conditions, due to rotational magnetization and the non-homogeneity of technical materials. Versatile experimental methods to obtain technical data on MS effects, permeability, losses, etc. are reviewed in [1]. Once the MS behaviour of the material is known, it has to be incorporated in the magnetic and mechanical analysis. A coupled magneto-mechanical finite element model has been presented earlier [2], and here we will illustrate how to expand this model to take MS effects into account once the material behaviour is known, e.g. in a $\lambda(B)$ format (i.e. magnetostrictive strain λ as a function of magnetic flux density B). The material data account for the link between MS and the magnetic system. The coupling between MS and the mechanical system can be understood as follows.

2. Magnetostriction and the Mechanical System

The stress occurring inside a magnetostrictive body due to the presence of a magnetic field together with mechanical boundary conditions (clamping, pinning, …) is found as follows. First, the body is considered to be free (no clamping) and the magnetostrictive strain λ due to the magnetic field is calculated, resulting in new dimensions for the body. The body is then deformed in order to fit the mechanical boundary conditions. The forces needed to fit the body into the clamping give the internal stresses occurring in the body. This approach is similar to the one commonly used to calculate thermal stresses [3].

This means that, next to the purely elastic deformation a, a second partial deformation a_{ms} due to MS needs to be considered, the sum of both giving the total deformation $a^{*}=a+a_{ms}$. In the case the body is clamped on all sides for example, the total deformation remains zero $a^{*}=0$, since the MS deformation a_{ms} is compensated by the elastic deformation $a=-a_{ms}$. This approach allows us to still define the elastic energy in the body as [2]

$$U=\frac{1}{2}a^{\mathrm{T}}Ka\,, \tag{1}$$

even when $a^{*}=0$ (K is the mechanical stiffness matrix). The undeformed 2D geometry is defined by ($x_{i,0}$, $y_{i,0}$) and the total deformation is given by $x_i=x_{i,0}+u_{i,ms}+u_i$, $y_i=y_{i,0}+v_{i,ms}+v_i$ with $a_i=[u_i\ v_i]^{\mathrm{T}}$ and $a_{i,ms}=[u_{i,ms}\ v_{i,ms}]^{\mathrm{T}}$.

3. The Expanded Magneto-Mechanical System

In [2], the magnetic and mechanical systems were combined using

$$\begin{bmatrix} M & D \\ C & K \end{bmatrix}\begin{bmatrix} A \\ a \end{bmatrix}=\begin{bmatrix} T \\ R \end{bmatrix}, \tag{2}$$

where M is the magnetic stiffness matrix, A contains the unknown magnetic vector potentials, T is the magnetic source term vector and R represents external forces (no reluctance or MS forces). To take the MS deformation a_{ms} into account, the system (2) needs to be expanded:

$$\begin{bmatrix} M(A,a,a_{ms}) & D & 0 \\ C(A,a,a_{ms}) & K(a_{ms}) & 0 \\ 0 & 0 & 1/\lambda(A) \end{bmatrix}\begin{bmatrix} A \\ a \\ a_{ms} \end{bmatrix}=\begin{bmatrix} T \\ R \\ L \end{bmatrix}, \tag{3}$$

where the dependencies of M, C and K on A, a and a_{ms} are indicated. The coupling term will be discussed further on. Let's consider the system (3):

- The third equation in (3) allows us to go from MS strain $\lambda(A)$ (found using $\lambda(B)$ and $B(A)$) to MS deformation a_{ms} using a distance measure L, e.g. with respect to a fixed point (clamping or symmetry point).

- The second equation in (3) represents the mechanical system, solving for elastic deformation a. The stiffness matrix K is a function of a_{ms} since the MS deformation determines the boundary conditions and the starting geometry $(x_{i,0}+u_{i,ms}, y_{i,0}+v_{i,ms})$ for the mechanical calculation, as explained above. The term CA represents reluctance forces.
- The first equation in (3) represents the magnetic system, where the magnetic stiffness matrix M depends on A in the non-linear case (saturation), and also depends on the total deformation $a^*=a+a_{ms}$.

The term Da in (2) can be recognised as [2]

$$Da = \left.\frac{\partial U}{\partial A}\right|_a , \tag{4}$$

representing the change in elastic energy U due to a magnetic field change, while keeping the deformation constant. This term can be calculated analytically beforehand:

$$Da = \Delta t\, E \frac{5/4-\nu}{1-\nu^2}\ \lambda(A)\frac{\partial\lambda(A)}{\partial A}, \tag{5}$$

where E and ν are the Young and Poisson modulus, and Δ and t are the element area and thickness. In calculating (5), the transverse MS λ_t was assumed to be $\lambda_t= -\lambda/2$ [4]. Expression (5) can be interpreted as a current $I_{ms}=Da$ and shifted to the right-hand side (source terms). The same can be done with the term CA representing reluctance forces ($F_{rel}=CA$):

$$\begin{bmatrix} M(A,a,a_{ms}) & 0 & 0 \\ 0 & K(a_{ms}) & 0 \\ 0 & 0 & 1/\lambda(A) \end{bmatrix} \begin{bmatrix} A \\ a \\ a_{ms} \end{bmatrix} = \begin{bmatrix} T+I_{ms}(A) \\ R+F_{rel}(A,a,a_{ms}) \\ L \end{bmatrix}. \tag{6}$$

4. Magnetostrictive forces

The third equation in (6) can be converted into an elasticity equation and incorporated in the second equation: the mechanical stiffness matrix K^e for an element gives, after multiplication with the MS displacement a_{ms}, the nodal MS forces $F^e_{ms}=K^e a_{ms}$ that can be added to the external forces R and the reluctance forces F_{rel}. This cannot be done for the whole mesh at once, because the different displacements a_{ms} due to MS in all elements surrounding a node, *cannot* be summed. The forces F^e_{ms} however, *can* be summed. This gives a new system that can be solved for total deformation a^* directly:

$$\begin{bmatrix} M(A,a^*) & 0 \\ 0 & K(a^*) \end{bmatrix} \begin{bmatrix} A \\ a^* \end{bmatrix} = \begin{bmatrix} T+I_{ms}(A) \\ R+F_{rel}(A,a^*)+F_{ms}(A,a^*) \end{bmatrix}. \tag{7}$$

5. Cascade Approach

The system (7) can be solved using a cascade procedure:

1. solve $MA=T$ to give an initial guess A^0, for the undeformed case $a^{*0}==0$,
2. find all a^k_{ms} using $\lambda(B(A^{k-1}))$ $\rightarrow a^k_{ms}$
3. solve all $K^e(a^{*k-1})a^k_{ms} = F^{e,k}_{ms}$ and sum $\rightarrow F^k_{ms}$
4. solve $K(a^{*k-1})\, a^{*k} = R - F_{rel}(A^{k-1},a^{*k-1})+F^k_{ms} \rightarrow a^{*k}$
5. solve $M(A^{k-1},a^{*k})\, A^k = T - I_{ms}(A^{k-1})$ $\rightarrow A^k$
6. repeat from 2 until $\Delta a^*<\varepsilon(a^*)$.

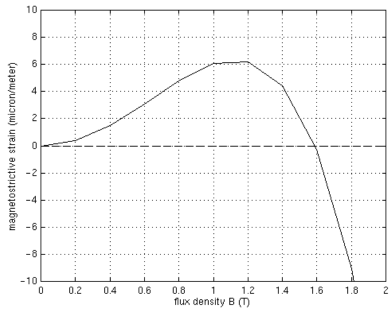

Fig.1 Magnetostrictive material characteristic for 3% Si Fe, non-oriented.

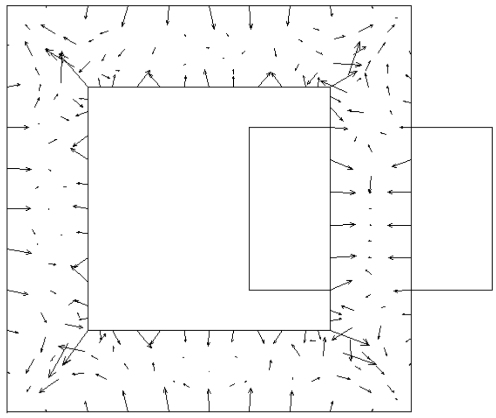
Fig.2 Nodal forces caused by magnetostriction of the iron core.

These techniques are used to analyse the MS effects on a small iron core with coil. Fig.1 shows the MS characteristic $\lambda(B)$ of the core material, which can be both positive and negative (expanding and shrinking). Fig.2 shows the nodal force distribution F_{ms} due to magnetostriction inside the iron core. This force distribution can be readily used for deformation calculation.

6. Acknowledgement

The authors are grateful to the Belgian "Fonds voor Wetenschappelijk Onderzoek Vlaanderen (FWOV)" for its financial support; Koen Delaere has a FWOV scholarship. The authors thank the Vlaamse Wetenschappelijke Stichting for its financial support and the Belgian Ministry of Scientific Research for granting the IUAP No.P4/20 on Coupled Problems in Electromagnetic Systems. The Research Council of the K.U.Leuven supports the basic numerical research.

7. References

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