#### Exact Tracking of Linear Synchronous Reluctance Servodrive Servodrive

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#### ABSTRACT

Today's conventional machine tool feed axes drive solution consists of a rotational motor and the mechanical converter which converts rotational motion into translational. Since the mechanical elements introduce backlash and elasticity in the system, better performance of the feed drive could be achieved by the linear motor with appropriate servo control. The design of the input-output linearizing control of a linear synchronous reluctance motor is described in the paper. Input-output linearizing control combined with the tracking controllers provides high tracking performance of a ser vodrive. The performances of the servodrive system are evaluated through the comparison of results obtained by the classical cascade control and the input-output linearizing control.

# 1 INTRODUCTION

The task of machine tool feed drives is to move and position the machine axes. In general, the feed axes perform linear movement. Today's conventional drive solution consists of a rotational motor attached to the lead screw with a lead screw nut, which converts rotational motion into translational motion. The disadvantage of this system is that the mechanical elements posses backlash and elasticity limiting machining speed and positioning accuracy. These problems could be overcome with the use of linear motors in feed drives. Linear motors have several benefits: they are simple and stiff allowing high speeds and accurate positioning. However, there exists strong interaction between the machining process and the direct drives, which can degrade the workpiece surface nish. Therefore, to exploit the direct linear drives for machining applications, their servo-control must provide high accuracy tracking and dynamic stiffness  $[1]$ , [2].

Due to their favourable dynamic characteristics, simple construction and transparent control structure, Linear Synchronous Reluctance (LSR) motors are appropriate for use in high performance servo drives [3]. A frequent approach used in servomotor control design isbased on the standard cascade methods with or without compensation of the back EMFs. In [4] we have shown, for the rotational motor, that the inputoutput linearizing control combined with the tracking controllers can provide exact tracking of the reference tra jectories and high disturbance rejection. The similar control design for the LSR motor is described in this paper. The control design procedure is based on the nonlinear dynamic model of the motor, therefore, the LSR motor model is given first. The model of the LSR motor is coupled and nonlinear, which makes this motor difficult to control. Since the highest possible tracking performance and high disturbance rejection are required, the input-output linearizing control design [5] is used. The result of the input-output linearization is linearized, decoupled but unstable model of the LSR machine.

The system is stabilized by the state tracking controllers. The performances of the servodrive system are evaluated through the comparison of the tracking obtained by the classical cascade control and the input-output linearizing control. The comparison of the results has shown that the input-output linearizing control provides better performances of LSR ser vodrive than the classical cascade structure. Moreover, if the linearized and decoupled system is controlled by the tracking controllers, the exact tracking of the reference tra jectories can be achieved. It should be noted that the on-line implementation complexity is almost the same for both controllers.

# 2 TWO AXIS LSR MOTOR MODEL

The LSR motor construction [3] is schematically shown in Fig.1. This is a single-sided, short primary, moving primary LSR motor.



Fig. 1. One-sided, short primary, moving primary LSR motor

The two axis model of a LSR motor is described by following equations:

$$
\begin{array}{rcl}\n\frac{di_d}{dt} & = & -\frac{R}{L_d}i_d + \frac{\pi}{\tau_p}v \frac{L_q}{L_d}i_q + \frac{u_d}{L_d} \\
\frac{di_q}{dt} & = & -\frac{R}{L_q}i_q - \frac{\pi}{\tau_p}v \frac{L_d}{L_q}i_d + \frac{u_q}{L_q} \\
\frac{dv}{dt} & = & \frac{\pi}{\tau_p m}(L_d - L_q)i_d i_q - \frac{1}{m}F_l = k i_d i_q - \frac{1}{m}F_l \quad \text{as} \\
\frac{dx}{dt} & = & v\n\end{array}
$$
\n(1)

where  $u_d$ ,  $u_q$  and  $i_d$ ,  $i_q$  are the  $d$ -q reference frame voltages and currents,  $R$  is the Ohmic resistance of primary,  $L_d$  and  $L_q$  are the direct- and the quadrature axis inductances,  $\tau_p$  is the primary pole pitch, m is the mass of the primary,  $x$  and  $v$  are the position and the speed of the primary with respect to the secondary and  $F_l$  is the load force including friction force  $fv$ .  $f$ is the coefficient of friction.

### 3 LINEAR SYNCHRONOUS RELUCTANCE MOTOR CONTROL DESIGN

The model of the LSR motor is coupled and nonlinear which makes this motor difficult to control. Since the highest possible tracking performance and high disturbance rejection are required, the input-output linearizing control [5] is chosen for the servodrive. In order to derive the control law the motor model is first rearranged and written in the matrix form:

$$
\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{G}\mathbf{u} + \mathbf{F}\boldsymbol{\xi} \tag{2}
$$

$$
y = Cx \tag{3}
$$

where  $\mathbf{F}\xi$  is the perturbed part of the nominal system and x and u are the state and the input vector, respectively:

$$
\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} i_d \\ i_q \\ v \\ x \end{bmatrix} \quad \mathbf{u} = \begin{bmatrix} u_d \\ u_q \end{bmatrix}
$$

C is the output vector:

$$
\mathbf{y} = \mathbf{C}\mathbf{x} = \begin{bmatrix} x_1 \\ x_4 \end{bmatrix} = \begin{bmatrix} i_d \\ x \end{bmatrix}
$$

and  $G$  and  $f(x)$  are:

$$
\mathbf{G} = \begin{bmatrix} \mathbf{g_1} & \mathbf{g_2} \end{bmatrix} = \begin{bmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}
$$

$$
\mathbf{f}(\mathbf{x}) = \begin{bmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \\ f_3(\mathbf{x}) \\ f_4(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} -\frac{R}{L_d}i_d + \frac{\pi}{\tau_p}v\frac{L_q}{L_d}i_q \\ -\frac{R}{L_q}i_q - \frac{\pi}{\tau_p}v\frac{L_d}{L_q}i_d \\ \frac{\tau_p}{\pi m}(L_d - L_q)i_d i_q \\ v \end{bmatrix}
$$

Since the load force  $F_l$  cannot be directly measured and consequently cannot be used in the control input calculation, it is excluded from the nominal part of  $\sim$ the motor model and will be later considered as an external disturbance.

In the next step input-output linearization of the nominal system is done [5]. The nominal part of the LSR motor model is written in terms of higher derivatives of outputs  $y_1$  and  $y_2$ :

$$
\begin{bmatrix}\n\dot{y}_1 \\
\dot{y}_2\n\end{bmatrix} = \begin{bmatrix}\n-\frac{R}{La}i_d + \frac{\pi}{\tau_p} v \frac{L_q}{La} i_q \\
k[\frac{\pi}{\tau_p} v (\frac{L_q}{La} i_q^2 - \frac{L_d}{L_q} i_d^2) - \frac{R}{La} i_d i_q - \frac{R}{L_q} i_d i_q\n\end{bmatrix} + \begin{bmatrix}\n\frac{1}{La} & 0 \\
\frac{k}{La} i_q & \frac{k}{La} i_d\n\end{bmatrix} \begin{bmatrix}\nu_d \\
u_q\n\end{bmatrix} = \mathbf{D} + \mathbf{E} \mathbf{u}
$$
\n(4)

The obtained model is clearly still nonlinear and coupled. Therefore the control voltage u is chosen as:

$$
\mathbf{u} = \mathbf{E}^{-1}(-\mathbf{D} + \mathbf{v}) = \mathbf{I}(\mathbf{x}, \mathbf{v})
$$
 (5)

where  $\mathbf{v} = |v_d| v_q$  T is the new system input and  $\mathbf{E}$   $\mathbf{v}$  D and  $E^{-1}$  are:

$$
\mathbf{E}^{-1} = \begin{bmatrix} L_d & 0\\ -L_q \frac{i_q}{i_d} & \frac{L_q}{k} \frac{1}{i_d} \end{bmatrix} \tag{6}
$$

$$
\mathbf{E}^{-1}\mathbf{D} = \begin{bmatrix} -Ri_d + \frac{\pi}{\tau_p} v L_q i_q \\ -Ri_q - \frac{\pi}{\tau_p} v L_d i_d \end{bmatrix}
$$
\n(7)

After inserting  $(5)$  into  $(4)$ , the following closed-loop system is obtained:

$$
\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = \mathbf{D} + \mathbf{E} \mathbf{u}
$$
  
=  $\mathbf{D} + \mathbf{E} \left( -\mathbf{E}^{-1} \mathbf{D} + \mathbf{E}^{-1} \mathbf{v} \right) = \begin{bmatrix} v_d \\ v_q \end{bmatrix}$  (8)

Now the input-output behavior of the system  $(8)$  is linear, while input-state variables relation is still nonlinear. This nonlinear relation is eliminated by selecting a new set of state variables  $\mathbf{z} = |z_1 \, z_2 \, z_3 \, z_4|$  Tintroduced by the nonlinear transformation  $z = T(x)$ , defined as:

$$
\mathbf{T}(\mathbf{x}) = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} i_d \\ x \\ v \\ k i_d i_q \end{bmatrix}
$$
(9) Fi

Equations (5) and (9) represent so called input transformation  $\mathbf{u} = \mathbf{I}(\mathbf{x}, \mathbf{v})$  and the state transformation  $z = T(x)$ .

The block diagram of the input-output linearizing control is shown in Fig. 2 where M denotes the LSR machine model. The state-space model of this system in terms of state variables z is:

$$
\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_d \\ v_q \end{bmatrix}
$$
 (10)

$$
\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}
$$
 (11)



Fig. 2. Block diagram of the input-output linearizing control

The block diagram of the system (10), (11) is shown in Fig. 3. This system is linearized, decoupled and unstable. It is stabilized by the tracking controllers.

The direct axis current controller is given by (12) where  $y_{1r}$  and  $y_1$  denote the reference and the actual primary current  $\sqrt{i_d}$ 



Fig. 3. Decoupled and linearized closed-loop system

$$
v_d = \dot{y}_{1r} + K_{d0} \left( y_{1r} - y_1 \right) + K_{d1} \int_0^t \left( y_{1r} - y_1 \right) d\tau \tag{12}
$$

The position controller is given by (13) where  $y_{2r}$  and  $y_2$  denote the reference and the actual position of the primary:

$$
v_q = \ddot{y}_{2r} + K_{q2} (\ddot{y}_{2r} - \ddot{y}_2) + K_{q1} (\dot{y}_{2r} - \dot{y}_2) + K_{q0} (y_{2r} - y_2) + K_{q1} \int_0^t (y_{2r} - y_2) d\tau
$$
 (13)

(10) The term y denotes the acceleration of the primary which cannot be measured. In the control scheme the estimated acceleration  $\ddot{y}_2$ , obtained by the observer similar to the one described in [4], was used.

#### 4 RESULTS

The experimental verification of the proposed tracking control was performed on the laboratory equipment shown in Fig. 4 and described in [6]. Elements of the experimental system are: a three-phase  $LSR$ motor; a modified inverter Allen Bradley 1336S-B050 (37 kW, up to 77 A); an embedded controller Innovtive Integratin SBC32C that includes a Texas Instruments floating point DSP TMS320C32 (60 MHz, 60 MFLOPS), 4 independent 200 kHz A/D converters, 4 independent 200 kHz D/A converters and 512 KBytes of RAM; two current measurement chains (LEM LT $-100$  S) and position measurement chain (linear scale Iskra TELA TGM1 with interpolation module SIM 110 BX1); a host PC with the installed development environment. The smooth reference trajectories for the position  $x_r$  and the speed  $v_r$  are generated from the kinematic model and are shown in Fig. 5.

The results obtained by the clasical cascade control are presented first. The cascade control structure include PI- currents and speed controllers and the Pposition controller. The time behavior of the tracking (position) error  $x_r - x$ , speed error  $v_r - v$ , machine currents  $i_d$  an  $i_q$  and voltages  $u_d$  and  $u_q$  during the movement defined in Fig. 5 are shown in Figs. 6a) to 6e). The  $d$ -axis reference current is set to the constant



Fig. 4. Experimetal system



Fig. 5. Reference position  $x_r$  and speed  $v_r$ 

value  $i_{dr} = 8$  A in order to obtain the maximal force in the nonsaturated operation. The absolute value of the maximum tracking error is near to 2:0 mm while the steady state position error is zero. The controller settings with the LSR motor model data are given in Table 1.

The results of the input-output linearizing tracking control are shown in Figs. 7a) to 7e) in the same sequence as in the case of the cascade control. It is obvious, that the tracking error in this case is much smaller, but the motor voltages  $u_d$  and  $u_q$  are higher and less smooth. The magnitudes of both model currents  $i_d$  and  $i_q$  remain on the same level as in the case of cascade control. The tracking controller settings are given in Table 1.

# 5 CONCLUSION

The input-output linearizing tracking control of the LSR servomotor drive was designed in order to achieve the highest possible accuracy in trajectory tracking. The proposed control strategy requires only moderate



Fig. 6. Tracking experiment results obtained by the cascade control

computational effort in the real-time application. If it is combined with the tracking controller and only the nominal part of the system is considered it can provide exact position tracking. The tracking capability of the proposed input-output linearizing control and classical cascade control is compared experimentally. Much better tracking performances are obtained by the use of input-output linearizing control than by





the use of cascade control.

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Fig. 7. Tracking experiment results obtained by the input- -output linearizing control

by B. Hribernik, University of Maribor and T.A. Lipo, University of Wisconsin.

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