Comparison of Time-Harmonic and Transient Finite Element Models for Asynchronous Machines

H. De Gersem, R. Mertens, K. Hameyer
Katholieke Universiteit Leuven, Dep. EE (ESAT), Div. ELEN
Kardinaal Mercierlaan 94, B-3001 Heverlee, BELGIUM
Herbert.DeGersem@esat.kuleuven.ac.be

Abstract

Time-harmonic simulations with and without considering the motion term, are compared to transient simulations for the case of induction machines. As a result, it can be stated that neglecting the spatial harmonic due to rotor slotting has more impact than neglecting the higher harmonic components related to the stator slotting.

1 Introduction

The stationary behaviour of induction machines is commonly simulated with the time-harmonic [1] or the transient finite element method [2]. Transient models suffer from excessive computation times whereas time-harmonic models deal with substantial approximations. In this paper, a motional time-harmonic approach is applied to reduce the number of assumptions related to the slotting of the device. Moreover, a comprehensive study of the impact of the approximations required for time-harmonic simulation is established.

2 Finite element models

The magnetodynamic field obeys the differential equation

$$\nabla \times \left(\nu \nabla \times \mathbf{A}\right) - \sigma \mathbf{v} \times \left(\nabla \times \mathbf{A}\right) + \sigma \frac{\partial \mathbf{A}}{\partial t} = -\sigma \nabla V \qquad (1)$$

with $\mathbf{A}$ the magnetic vector potential, V the applied voltage, $\mathbf{v}$ the velocity, ν the reluctivity and σ the conductivity. The transient model uses a linear one-step method for the time discretisation of all varying quantities. In the case $\mathbf{A}$ and V are sinusoidally varying with time, they can be represented by the phasors $\underline{\mathbf{A}}$ and $\underline{V}$. If $\mathbf{v}$, ν and σ remain constant, the time derivative in (1) can be replaced by $j\omega$ with ω the electrical pulsation. The geometry is discretised by linear triangular finite elements. The electrical supply and the external impedances are modelled by integral relations and added as algebraic circuit equations to the overall system matrix [3].

3 Modellisation of asynchronous rotational devices

A part of a rotational device is said to be uniform if the geometry is invariant with respect to the mechanical or electrical variation. In Fig. 1a, both the rotor and the stator are uniform whereas in Fig. 1b, only the rotor is uniform. The geometry of a squirrel-cage induction machine is non-uniform (Fig. 1c). Asynchronous operation is achieved when the mechanical angular velocity ω_m of the rotor differs from the angular velocity of the stator flux density wave. The relative difference in velocity is characterised by the slip $s = \frac{\omega - \lambda\omega_m}{\omega}$ where λ is the pole pair number of the stator winding.

Transient modellisation combined with a moving-mesh technique, e.g. the moving band technique, considers the relative motion of the rotor teeth with respect to the stator teeth. If the magnetic field is fixed to a reference frame moving with the rotor, the motion term in (1) vanishes. Transient simulation enables the study of the entire harmonic contents of the stationary machine behaviour and is in that sense general.

In a motional time-harmonic simulation, the rotor is assumed to be uniform with respect to the angular motion. As a consequence, harmonics due to the rotor slotting are neglected.

It is possible to invoke a non-motional time-harmonic solver by relying upon a *slip transformation*. If all phenomena in the rotor are assumed to happen at slip frequency or, equivalently, the air gap field may be considered as a spatial sine, an appropriate transformation of the rotor coordinate system leads to

$$\nabla \times \left(\nu \nabla \times \underline{\mathbf{A}}\right) + j\omega s \sigma \underline{\mathbf{A}} = -\sigma \nabla \underline{V} . \qquad (2)$$

The rotor behaves as if it has the conductivities $s\sigma$ and as if it is excited at the same frequency as the stator [4].

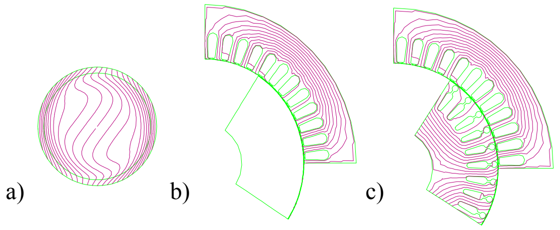
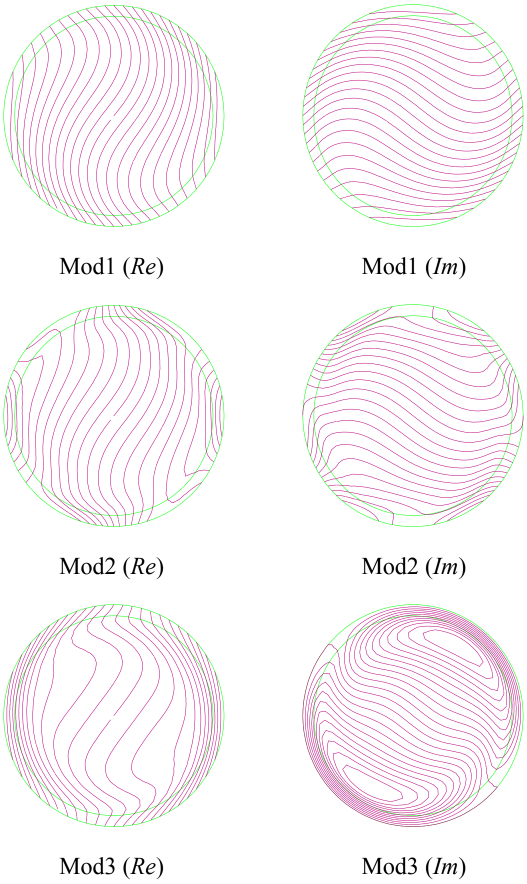


Fig. 1: Uniformity of the geometry of rotating devices: a) rotor and stator uniform; b) rotor uniform; c) non-uniform geometry.

4 Comparison between transient and time-harmonic simulation

A comparison between the transient approach, the motional time-harmonic approach and the time-harmonic technique with slip transformation, is set up using four models with an increasing number of non-uniform geometrical entities (Table I).

4.1 Rotating disk excited by a pure sine wave



Mod1 (*Re*) Mod1 (*Im*)

Mod2 (*Re*) Mod2 (*Im*)

Mod3 (*Re*) Mod3 (*Im*)

Fig. 2: Real (*Re*) and imaginary (*Im*) part of the magnetic flux lines in a rotating cylinder submitted to a pure sinusoidal rotational field (Mod1), to a sinusoidal rotational field augmented by a 5th harmonic (Mod2) and to an alternating field (Mod3).

The greatest simplification is a rotating conductive disk (Fig. 2). A pure sinusoidal rotating magnetic field is applied at the boundary of the air gap around the disk. All simulation techniques obtain a correct solution (Mod1 in Table I).

4.2 Rotating disk excited by a distorted wave

A 5th harmonic inverse travelling field with 20% of the fundamental magnitude of the magnetic vector potential, is superposed to the fundamental air gap field. The slip transformation is not applicable for this case. As a consequence, the non-motional time-harmonic model is approximate (Mod2 in Table I). The transient and motional time-harmonic solvers still return the correct solution. Induction machines with solid rotors for high speed operation and machine models with homogenized rotor properties are members of this class of devices. For these devices, the motional time-harmonic solver is a valuable low-cost alternative for the transient approach.

4.3 Rotating disk excited by an alternating field

Single-phase induction machines give rise to an alternating field in the air gap. The slip transformation is only applicable in the case of a rotating field and is thus not valid in this case. The motional time-harmonic approach is applicable for solid rotor machines (Mod3 in Fig. 2 and Table I).

4.4 Induction machine

If also the rotor slotting has to be considered, only a transient model with an appropriate time-stepping scheme reflects the true behaviour. The time-harmonic simulation technique with slip transformation, however, still provides technically reliable results for stationary operation (Mod4 in Table I). For a large range of induction machines, neglecting the rotor slotting introduces a more severe modellisation error than the slip transformation. The slip transformation for induction machines is acceptable as the harmonic contents of the air gap field is less than 5%. The motional time-harmonic approach is not applicable because the rotor is not uniform.

6 Conclusions

The comparison of transient and time-harmonic models for the simulation of stationary operating asynchronous machines, reveals that transient modellisation is indispensable if slotting effects have to be considered. The time-harmonic model with slip transformation obtains a sufficient accuracy for fundamental machine parameters. For devices with a uniform rotor submitted to a distorted air gap field, the motional time-harmonic approach yields the same result as the transient method with a lower computational cost.

Table I: Comparison of time-harmonic simulations with slip transformation (TH+slip) or motion term (TH+motion) and transient simulations of the torque

	TH+slip	TH+motion	Transient
Mod1	325 Nm *(C)*	325 Nm *(C)*	325 Nm *(C)*
Mod2	315 Nm *(A)*	205 Nm *(C)*	205 Nm *(C)*
Mod3	*(-)*	-118 Nm *(C)*	-118 Nm *(C)*
Mod4	292.6 Nm *(A)*	*(-)*	293.0 Nm*(C)*

(C) correct model
(A) approximate model
(-) model not applicable.

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