

Krylov Subspace Methods for Harmonic Balanced Finite Element Methods

Herbert De Gersem, Stefan Vandewalle, Kay Hameyer

Alternating current electrical energy transducers in steady-state regime are commonly simulated by linearised time-harmonic finite element (FE) models. As phenomena at higher harmonics, due to power electronic supply or ferromagnetic saturation, have an increasing technical importance, transient or harmonic balanced FE (HBFEM) simulation is becoming more and more important. The application of the latter is however not widespread, mainly because of the expensive solution of the corresponding matrix system. Several implementations rely upon a weakly coupled time-harmonic scheme or a real-valued splitting of the system matrix. Here, a different, coupled approach is presented.

Field quantities are denoted by p -forms, i.e. a set of p complex numbers, representing the values of p harmonic components. Because only saturation is considered, the material properties, represented by q -forms, can be quantified by q real values. The convolution of a p -form with a q -form yields a p -form. The system matrix, obtained after applying the space discretisation, consists of q -forms whereas the righthandside and the vector of degrees of freedom consist of p -forms. A division in the time domain corresponds to the solution of a Toeplitz system in the frequency domain. The implementation of HBFEM is based on objects and operators assigned to p - and q -forms.

Here, HBFEM is equipped with a Krylov subspace solver, adjusted to the particular nature of the discretised magnetodynamic partial differential equation (PDE). To retain the symmetry and positive definiteness of the underlying PDE in the system matrix, it is not split into real-valued or complex-valued equivalents. Instead, the Lanczos procedure for symmetric operators is applied to the strongly coupled system directly. For problems without eddy currents, the system is positive definite. Hence, the Conjugate Gradient method is applicable. For eddy current simulation, the symmetry is exploited by a symmetric Quasi-Minimal Residual algorithm.

The approaches are applied to the systems arising from the model of a transformer featuring severe ferromagnetic saturation and the model of an induction furnace. The scaling of the computation time with respect to the extension of the number of harmonics, is studied. The method outperforms the existing approaches considerably.