Higher order material spaces to represent non-linear material characteristics and source terms in FEM-computations

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***Abstract* — A new approach to represent the local change of material properties or source terms due to their non-linear characteristics is presented here. The material function is given in terms of simple basis functions, allowing a higher order representation within the area covered by the finite element. This approximation as a higher order material space has advantages for the treatment of saturable ferromagnetic materials and temperature dependencies of conductive and permanent magnetic materials. It becomes possible to treat the material function space and the field solution as a coupled problem.**

INTRODUCTION

The finite element method is often used in computational magnetics with non-linear dependencies of the source term or material constant. Based on the field solution, A non-linear material tensor is determined. Traditionally this tensor, being part of a coefficient in the describing electromagnetic field equation, is assumed to be constant (zero order, therefore associated with a *1st* order error) within the element's span. Hence, these terms, with a local field solution represented by FEM functions with the order of *N*, have an error order smaller than *N+1* [1]. This yields lower accuracy when compared to pure linear equations. It can be proved that the materials should be represented with an order of at least *N-1* to maintain the same approximation accuracy.

MATERIAL SPACES

It is possible to enhance the order of the material tensor by defining it as a weighted sum of simple basis functions. These functions can be of the same form as the finite element shape functions used for the FEM-field solution. For instance, the 2D transient magnetic field modelled by means of the magnetic vector potential, is described by [2]:

$$\nabla\cdot\left(\upsilon(A)\nabla A\right)-\sigma(T)\frac{\partial A}{\partial t}=-\sigma(T)\Delta V_s-\upsilon(A)\nabla M(A,T) \quad (1)$$

The non-linear material coefficients are written by :

- Ferromagnetic materials: $\upsilon(A)=\sum_k N_k\upsilon_k(A)$ (2)
- Thermal dep. conductivities: $\sigma(T)=\sum_k N_k\sigma_k(T)$ (3)
- Temperature and magnetic field dependent permanent magnet characteristics : $M(A,T)=\sum_k N_k M_k(A,T)$ (4)

(To determine the temperature dependencies, a coupled problem has to be solved [3].)

By using a Galerkin approach, the FEM matrices can be calculated. For example, the matrix H_k for a triangular element with surface Ω_e originating from the transient eddy current term, when first order elements for material and field are used, is given in (5). The use of this higher order approach yields a more accurate joule- and eddy-current loss distribution.

$$H_k=\frac{\Omega_e}{\Delta t}\begin{bmatrix} \sigma_1/10+\sigma_2/30+\sigma_3/30 & \sigma_1/30+\sigma_2/30+\sigma_3/60 & \sigma_1/30+\sigma_2/60+\sigma_3/30 \\ & \sigma_1/30+\sigma_2/10+\sigma_3/30 & \sigma_1/60+\sigma_2/30+\sigma_3/30 \\ (symm.) & & \sigma_1/30+\sigma_2/30+\sigma_3/10 \end{bmatrix} \quad (5)$$

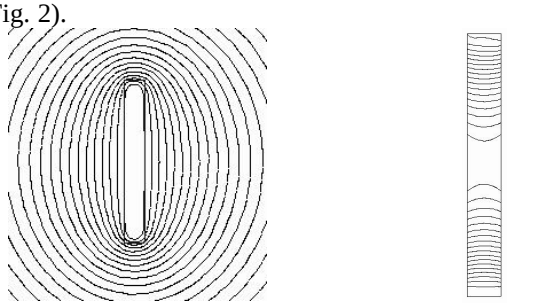
To assess the non-linearity in the iterative solution algorithm, whether it is due to a problem-own or a coupled problem dependency, several methods are possible:

1. *Point-collocation*: Assess the non-linearity using the nodal solutions.
2. *Least-squares method*: Minimise the quadratic error.
3. *Galerkin approach*: Use test functions.

The last two listed approaches require a (sparse) system solution. The size of this system depends on the number of elements having non-linear materials.

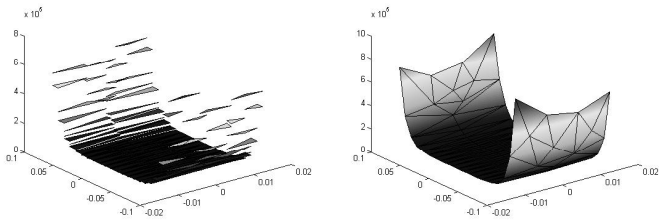
APPLICATION

To briefly illustrate the advantage of this approach, the magnetic and thermal field of a busbar are calculated for a coupled problem definition (Fig. 1). The classical zero order approach material approach and the novel first order material space method with first order functions are used (Fig. 2).



magnetic field *thermal field*

Fig. 1: Magnetic and thermal field solution.



0th order material space *1st order material space*

Fig. 2: 0th and 1st order material spaces.

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