Full Multigrid for Magnetostatics using Unstructured and Nonnested Meshes

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***Abstract* - The performance of a full multigrid scheme featuring a hierarchy of unstructured and nonnested grids is compared to that of nested multigrid and that of preconditioned Conjugate Gradients.**

I. Introduction

The basic idea of multigrid (MG) methods is to use a coarser discretisation to wipe out a low frequent error component during the iterative solution of a linear system related to a discretised partial differential problem [1]. If the concept is applied recursively, a hierarchy of finite element (FE) meshes is required. A full MG cycle is achieved if the coarser levels are also used to determine an initial estimate for the finer discretisations.

Relevant electromagnetic models usually require unstructured meshes to resolve for all geometrical details (Fig. 1). Moreover, efficient adaptive mesh refinement algorithms rely upon aspect ratio enhancing techniques, such as restoring the original geometry, moving nodes to the centroids of their supports and swapping edges to obtain local Delaunay properties. As a consequence, the full MG scheme has to deal with restriction and prolongation operators between nonnested grids.

II. Multigrid Scheme

The defect computed on a grid is prolongated to a finer grid by a projection applying the shape functions to obtain the value at interior nodes [2]. To avoid searching in the mesh during the MG cycle, a prolongation operator is constructed in advance. Only dependences larger than a certain treshold, e.g. 1%, are admitted to the prolongation stencils. The connectivity of the periodic boundary conditions has to be taken into account in the prolongation operator.

The MG cycle consists of 1 front Gauss-Seidel presmoothing step, the restriction defined by the adjoint of the prolongation, the coarse grid correction, the prolongation and 1 front Gauss-Seidel postsmoothing step. On the coarsest level an exact solve is performed. V-cycles are applied [1].

III. Application

The MG solver is applied to a model of a synchronous generator with six salient poles (Fig. 1). Enhancing the mesh during adaptation improves the convergence of the global error (Table I). The quality of the mesh has a advantageous influence on the convergence of MG (Table II). The more expensive prolongation in the case of nonnested MG does not harm the performance compared to nested MG and preconditioned Conjugate Gradients (Table II).

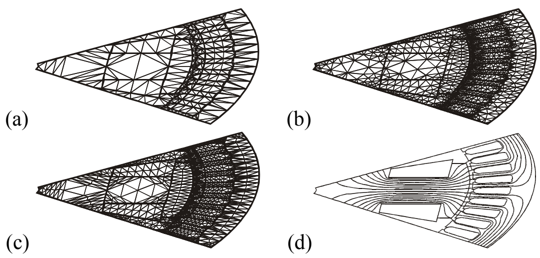

Fig. 1. (a) coarsest mesh; (b) mesh refined with aspect ratio enhancement; (c) nested refined mesh; (d) flux lines of the solution.

Table I. Comparison of the convergence of the global error of nested and nonnested mesh adaptation.

Adaptation step	Number of mesh nodes	Error nonnested mesh adaptation	Error nested mesh adaptation
0	331	2.057e-01	2.057e-01
1	1160	7.604e-02	9.154e-02
2	4501	2.730e-02	3.717e-02
3	17729	9.955e-03	1.141e-02
4	70369	3.269e-03	4.917e-03
5	280385	6.951e-04	1.339e-03

Table II. Cumulative timings (in s) (and iteration counts) of nested and nonnested MG compared to Symmetric Successive Overrelaxation (SSOR) and Incomplete Cholesky (IC) preconditioned Conjugate Gradients (CG).

Step	Nonnested MG	Nested MG	SSORCG	ICCG
0	4.80 (-)	4.80 (-)	0.25 (92)	0.29 (67)
1	5.70 (19)	6.22 (44)	0.67 (176)	1.25 (113)
2	8.39 (19)	9.72 (48)	3.29 (333)	11.00 (216)
3	19.84 (19)	27.66 (78)	26.10 (653)	142.87 (434)
4	77.69 (18)	117.19 (86)	241.09 (1286)	2220.33 (872)
5	419.90 (17)	624.69 (92)	1970.24 (2540)	-

IV. Conclusions

The nonnested full MG scheme combines a better convergence of the global error to faster iterative solutions of the linear systems when compared to nested MG. MG outperforms preconditioned CG.

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