# **Comparison of Motional and Nonmotional Time-Harmonic Finite Element Simulations of Solid Rotor Single-Phase Induction Machines**

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*Abstract* **- A comparison is done between the forces obtained using a motional and a nonmotional time-harmonic finite element formulation for a solid rotor single-phase induction machine. The accuracy of the approximative nonmotional formulation is acceptable except for velocities close to the synchronous speed.** 

#### I. MOTIONAL FORMULATION

The 2D time-harmonic motional formulation of the magnetic field in a uniform conductive body moving at the velocity **v** and excited by the voltage  $V$  is

$$
-\nabla \cdot (\mathbf{v} \nabla A_z) + \sigma \mathbf{v} \cdot \nabla A_z + j \omega \sigma A_z = \frac{\sigma}{\ell_z} V \tag{1}
$$

with  $A<sub>z</sub>$  the *z*-component of the magnetic vector potential,  $\omega$ the pulsation,  $\ell_z$  the depth of the model, v the reluctivity and  $\sigma$  the conductivity. This formulation is applied to compute the speed-torque characteristic of the single-phase induction machine presented in [1] as the TEAM30 benchmark model (Fig. 1 and Fig. 2).

## II. NONMOTIONAL FORMULATION

If the magnetic field is assumed to be sinusoidally distributed along the air gap, it is possible to decompose the alternating field into two sinusoidal waves of equal magnitude with opposite directions of rotation. Furthermore, in the case of rotating fields, a nonmotional description is obtained by multiplying the conductivities of the moving bodies by the slip, e.g. the relative difference of the angular velocity of the rotor to the rotating air gap field. The torques simulated for the slips corresponding to both rotating fields, are superposed to obtain the global torque. Fig. 3 shows that severe cancellation errors may occur in the neighbourhood of the synchronous speed of 377 rad/s. The difference between the simulated torques of the two studied approaches is also caused by neglecting the higher spatial harmonics in the air gap field in the nonmotional approach. Nonmotional simulations have the advantage that they yield symmetric system matrices and avoid the typical numerical oscillations occurring in motional solutions for high velocities.

#### III. CONCLUSIONS

The nonmotional formulation predicts the torque generated by the solid rotor single-phase induction machine up to an acceptable accuracy (maximum error 10%). If a large harmonic content of the air gap field is expected or if accurate results are required in the neighbourhood of the synchronous speed, a motional formulation is appropriate.



Fig. 1. Real and imaginary part of the magnetic flux in a single-phase induction machine, simulated by a motional time-harmonic formulation.



Fig. 2. Torque-speed characteristic of the single-phase motor  $(x =$  analytical solution [1]).



Fig. 3. Relative difference between the motional and nonmotional timeharmonic simulations.

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#### **REFERENCES**

[1] K.R. Davey, "Analytic analysis of single- and three-phase induction motors", *IEEE Transactions on Magnetics*, Vol. 34, No. 5, September 1998, pp. 3721-3727.