WEAK MAGNETOMECHANICAL COUPLING USING LOCAL MAGNETOSTRICTION FORCES

Koen Delaere, Ward Heylen*, Ronnie Belmans, Kay Hameyer

Katholieke Universiteit Leuven, ESAT-ELEN, Kardinaal Mercierlaan 94, B3001 Leuven, Belgium, kay.hameyer@esat.kuleuven.ac.be.
**Katholieke Universiteit Leuven, Mechanics Dept.-PMA, Celestijnenlaan 200C, B3001 Leuven, Belgium, ward.heylen@mech.kuleuven.ac.be.*

ABSTRACT

The magnetic material's deformation caused by magnetostriction is represented by an equivalent set of mechanical forces, giving the same deformation to the material as magnetostriction does. This is done in a way similar to how thermal stresses are usually incorporated into stress analysis. The resulting magnetostriction force distribution can be superposed onto other force distributions (external mechanical forces, magnetic forces) before starting the mechanical deformation or vibration analysis. This procedure is incorporated into a weakly coupled cascade solving of the magnetomechanical problem.

1. INTRODUCTION

An important source of vibrations and noise in electric devices, rotating as well as non-rotating, are the deformations caused by magnetostriction (MS). These MS deformations can be of the same order of magnitude as the deformations caused by reluctance forces (Maxwell stresses) on the iron-air interface [1]. The incorporation of the MS effect in the numerical design process is usually impaired since detailed data on the magnetic material behaviour are difficult to obtain. Versatile experimental methods to obtain all needed technical data on MS effects, permeability, losses, etc. are reviewed in [2]. Once the MS behaviour of the material is known, it has to be incorporated in the magnetic and mechanical analysis. The coupled magnetomechanical finite element model [3] is briefly reviewed and it is illustrated how to use this model to take the MS material characteristic into account by a set of MS forces. The MS material characteristic, e.g. in $\lambda(B)$ form (MS strain λ as a function of magnetic flux density B) is assumed to be known.

II. THE MAGNETOMECHANICAL SYSTEM

Both magnetostatic and elasticity finite element methods are based upon the minimisation of an energy function. The total energy E of the linear electromechanical system is the sum of the *elastic energy U* stored in a body with deformation a [4] and the *magnetic energy W* stored in a magnetic system with vector potential A [5]:

$$E = U + W = \frac{1}{2} a^T K a + \frac{1}{2} A^T M A \,, \tag{1}$$

where K is the mechanical stiffness matrix and M is the magnetic 'stiffness' matrix. Considering the similar form of the energy terms in (1), the following system of equations represents the numerically coupled magnetomechanical system:

$$\begin{bmatrix} M & D \\ C & K \end{bmatrix} \begin{bmatrix} A \\ a \end{bmatrix} = \begin{bmatrix} T \\ R \end{bmatrix}, \tag{2}$$

where T is the magnetic source term vector and R represents external forces (not of electromagnetic origin). The partial derivatives of the total energy E with respect to the unknowns $[A \;\; a]^T$ identify with the combined system (2):

$$\frac{\partial E}{\partial A} = MA + \frac{1}{2} a^{T} \frac{\partial K}{\partial A} a = T \,, \tag{3}$$

$$\frac{\partial E}{\partial a} = \frac{1}{2} A^{\mathrm{T}} \frac{\partial M}{\partial a} A + K a = R \,. \tag{4}$$

Using (2), (3) and (4), the coupling terms D and C are recognised as

$$D = \frac{1}{2} a^{\mathrm{T}} \frac{\partial K}{\partial A} \,, \tag{5}$$

$$C = \frac{1}{2} A^{\mathrm{T}} \frac{\partial M}{\partial a} \,. \tag{6}$$

The coupling term D is related to the elastic energy U by

$$D a = \frac{1}{2} a^{\mathrm{T}} \frac{\partial K}{\partial A} a = \frac{\partial U}{\partial A} \tag{7}$$

and represents the increase in elastic energy U due to an increase in the magnetic field A, with deformation a held constant. This is caused by magnetostriction in the following way:

- Imagine an element with deformation a_0 and flux density B_0.
- When the flux density in the element increases to $B_0+\Delta B$, the element expands (or shrinks) to $a_0+\Delta a$ due to magnetostriction (no external stresses need to be applied, $\Delta U = 0$, $\Delta a \neq 0$).
- In order to find the elastic energy change ΔU due to ΔB but for *constant* deformation, the element needs to be shrunk (or expanded) back to its original deformation $a_0=a_0+\Delta a-\Delta a$. The work done by external stresses to go back from $a_0+\Delta a$ to a_0 is exactly (7) (now $\Delta a = 0$, but $\Delta U \neq 0$, $a \neq 0$ and so $D \neq 0$).

The term D is used to represent MS in a *strong coupling* scheme [6], but will not be considered further, since here MS is dealt with by a weak coupling procedure. The weak coupling ($D \approx 0$) suffices for the commonly used materials with MS in the range of $\lambda \sim 10^{-6}$ m/m, but strong coupling is needed for materials with higher λ.

The coupling term C (6) represents the dependency of magnetic parameters in M on the mechanical displacement a. Right-multiplying C by vector potential A immediately renders all magnetic forces F_{mag} (Lorentz forces as well as reluctance forces):

$$-CA = -\frac{1}{2} A^{\mathrm{T}} \frac{\partial M}{\partial a} A = -\frac{\partial W}{\partial a} = F_{mag} \,, \tag{8}$$

where the last equation expresses virtual work. The approach presented here is extended to the non-linear magnetic case in [3].

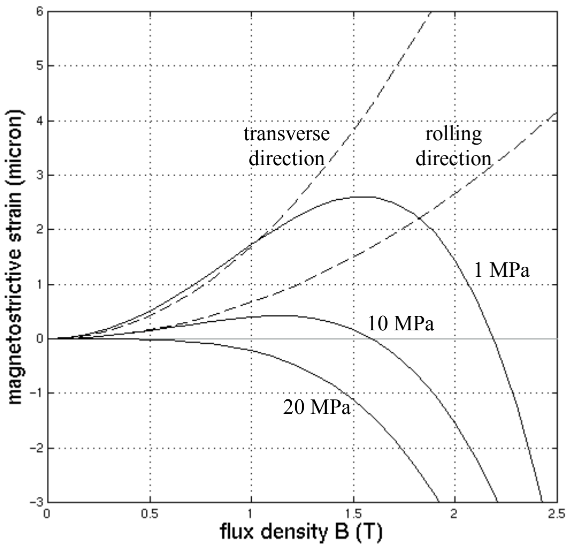


Fig. 1. Magnetostriction characteristics of non-oriented 3% SiFe (solid lines, as a function of tensile stress) and M330-50A (dashed lines, for rolling and transverse direction).

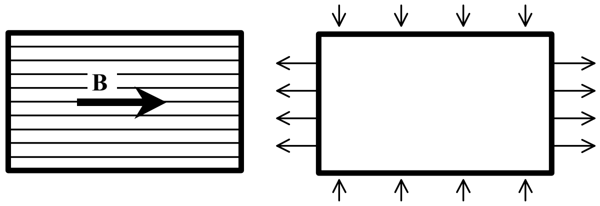


*Fig. 2. The set of forces (right) representing the strain caused by magnetostriction due to the magnetic field **B** (left), consists of a set forces parallel to **B** and a set forces perpendicular to **B**.*

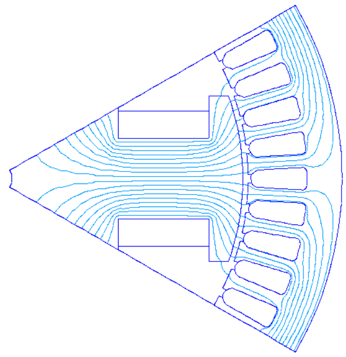


Fig. 3. Magnetic field for one pole of a six-pole synchronous machine.

3. MAGNETOSTRICTION FORCES

3.1 Concept

When the coupling term D is not used ($D = 0$) and the magnetic forces $F_{mag} = -CA$ are shifted to the right hand side of (2), the system becomes uncoupled:

$$\begin{bmatrix} M & 0 \\ 0 & K \end{bmatrix} \begin{bmatrix} A \\ a \end{bmatrix} = \begin{bmatrix} T \\ R + F_{mag} \end{bmatrix}. \tag{9}$$

Now MS effects are built into the analysis using a force distribution F_{ms} that can be added to R and F_{mag}. By *magnetostriction forces* F_{ms} we indicate the set of forces that induces the same strain in the material as the magnetostriction effect does. This approach is similar to the use of thermal stresses due to heating [7]. In calculating thermal stresses, the thermal expansion of the free body (no boundary conditions) is calculated based on the temperature distribution, and the thermal stresses are found by deforming the expanded body back into its original shape (back inside the original boundary conditions). In calculating MS forces, the expansion of the free body (no boundary conditions) due to magnetostriction is calculated based on the magnetic flux distribution, and the MS forces are found as the reaction to the forces needed to deform the expanded body back into its original shape (or back inside the original boundary conditions).

For FE models, this can be done on an element by element basis. The midpoint (center of gravity) of the element is considered to be fixed. The MS deformation of the element, i.e. the displacement of the nodes with respect to the midpoint, is found using the element's flux density B^e and the $\lambda(B)$ characteristic of the material. If a set of $\lambda(B,\sigma)$ characteristics are given, one has to be chosen for the appropriate value of tensile stress.

3.2 Strain for Isotropic Materials

Fig.1 shows a typical MS characteristic for isotropic 3% SiFe (solid lines) as a function of tensile stress. For isotropic materials, the local xy-axis of the element are rotated so that the flux density vector **B** coincides with the local x-axis. The strains λ_x and λ_y in the local frame are then given by

$$\begin{aligned} \lambda_x &= \lambda \\ \lambda_y &= \lambda_t = -\lambda/2 \\ \lambda_z &= \lambda_t = -\lambda/2 \end{aligned} \tag{10}$$

where $\lambda=\lambda(B)$ is the MS strain in the direction of **B** (x-direction) and λ_t is the MS strain in the transverse direction (y and z-directions). Usually, magnetostriction will leave the total volume and density unchanged [8], so that $\lambda_y = \lambda_z = -\lambda_x/2$. This volume invariance is equivalent to a magnetostrictive 'Poisson modulus' of 0.5, which is bigger than the mechanical Poisson modulus of about 0.3. Therefore, when the MS deformation is represented by a set of mechanical forces in the direction of the vector **B**, there is always a set of forces perpendicular to **B** to correct this difference in

Poisson modulus (Fig.2). In a 2D *plane strain* analysis, the thickness (z-direction) of the material has to remain constant and an additional tensile z-stress needs to be applied in order to obtain $\lambda_z = 0$. This adjusts the values (10) to

$$\begin{aligned} \lambda_x &= \lambda + \nu\lambda_t \\ \lambda_y &= \lambda_t + \nu\lambda_t \\ \lambda_z &= \lambda_t - \lambda_t = 0 \end{aligned} \tag{11}$$

where ν is the mechanical Poisson modulus of the material and $\lambda_t = -\lambda/2$.

3.3 Strain for Anisotropic Materials

Fig.1 shows a typical MS characteristic for anisotropic M330-50A steel (dashed lines) for rolling direction and transverse direction. For anisotropic materials, the flux density vector is decomposed into a B_x and a B_y component in the element's local xy-axis, arranged so that the x-axis coincides with the rolling direction, and the y-axis with the transverse direction. The rolling direction curve $\lambda_{\mathrm{RD}}(B)$ is then used with B_x as input, and the perpendicular direction curve $\lambda_{\mathrm{PD}}(B)$ with B_y as input, giving

$$\begin{aligned} \lambda_x &= \lambda_{\mathrm{RD}}(B_x) - \lambda_{\mathrm{PD}}(B_y)/2 \\ \lambda_y &= \lambda_{\mathrm{PD}}(B_y) - \lambda_{\mathrm{RD}}(B_x)/2 \\ \lambda_z &= -\lambda_{\mathrm{RD}}(B_x)/2 - \lambda_{\mathrm{PD}}(B_y)/2 \end{aligned} \tag{12}$$

A similar correction as above can be made for the plane strain case.

3.4 Displacement

Still working in the local xy-axis, the strains λ_x, λ_y are converted into a displacement $a_{ms} = (a_{x,i}, a_{y,i})$ considering the midpoint of the element (x_m, y_m) as fixed:

$$\begin{bmatrix} a_{x,i} \\ a_{y,i} \end{bmatrix} = \begin{bmatrix} x_i - x_m \\ y_i - y_m \end{bmatrix} \begin{bmatrix} \lambda_x \\ \lambda_y \end{bmatrix}, \; i=1,2,3 \tag{13}$$

where i indicates the three element nodes with co-ordinates (x_i, y_i).

3.5 Magnetostriction Forces

The mechanical stiffness matrix K^e for an element gives, after multiplication with the MS displacement a_{ms} of the nodes, the nodal *magnetostriction forces*

$$F_{ms}^e = K^e a_{ms}. \tag{14}$$

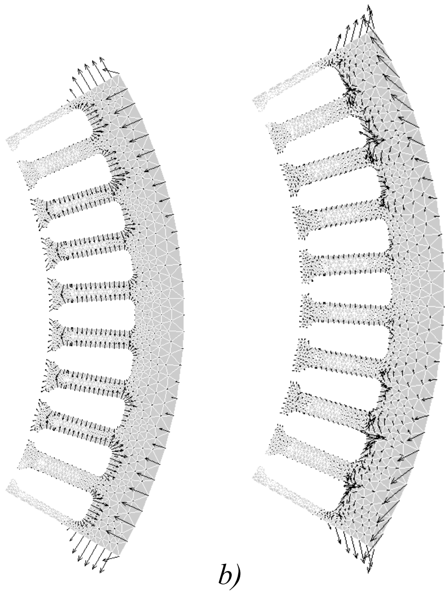
Equation (14) has to be performed element by element (using K^e) and not for the whole mesh at once (using the global matrix K), because the N different displacements $a_{ms,j}$, $j = 1...N$, due to MS in the N elements surrounding a node, should not be summed. First they are converted into MS forces and then the N forces F^e_{ms} on one node are summed.

The MS forces are now introduced in (9) giving

$$\begin{bmatrix} M & 0 \\ 0 & K \end{bmatrix} \begin{bmatrix} A \\ a \end{bmatrix} = \begin{bmatrix} T \\ R + F_{mag} + F_{ms} \end{bmatrix}. \tag{15}$$

First, the magnetic equation of system (15) is solved to give A, from which the magnetic forces F_{mag} are calculated using (8) and the MS forces F_{ms} using (14). Then the mechanical equation of (15) is solved to give deformation a due to all forces. The force distribution F_{ms} or the total distribution $F_{mag}+F_{ms}$ can also be used for any other kind of post-processing based on force distributions, e.g. calculating mode participation factors with stator mode shapes [3].

4. EXAMPLE



a) b)

Fig. 4. Magnetostriction forces on stator for a) isotropic non-oriented 3% SiFe, b) anisotropic M330-50A

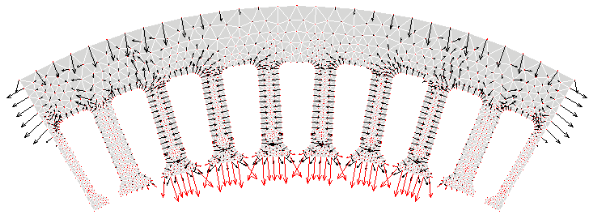


Fig. 5. Combined view of reluctance forces on stator teeth and magnetostriction forces on stator yoke and teeth sides.

Fig.3 shows the magneticfield in one pole of a six-pole synchronous machine. B_{max} in the teeth is 1.26 T corresponding to $\lambda = 2.3$ µm/m for 3% SiFe with 1 MPa tensile stress. Fig.4 shows the magnetostriction forces on the stator for the magnetic field of Fig.3: Fig.4a for a stator of isotropic non-oriented 3% SiFe and Fig.4b for a stator of anisotropic M330-50A (both materials were modelled with Young modulus $E = 2.2\ 10^{11}$ Pa and $\nu = 0.3$). In the areas of high flux density in the stator, there are MS forces parallel to the flux lines and also a set of MS forces perpendicular to the flux lines, *both* seeking to increase the circumference of the stator. In the anisotropic case, the general MS force pattern remains but the forces appear slanted, giving shear stresses in the stator. Fig.5 repeats Fig.4a but also shows the reluctance forces F_{mag} for the magnetic field in Fig.3. It can be seen that F_{ms} and F_{mag} are of the same order of magnitude (the size of the nodal force vectors on the teeth tips is 25 N).

5. CONCLUSION

Magnetostrictive deformation can be represented by a set of equivalent forces (so-called *magnetostriction forces*), much the same way as thermal expansion is translated into thermal stresses. This leads to an easy cascade solution of a weakly coupled magnetomechanical system which will suffice for analysing devices that use the common materials with normal magnetostriction ($\lambda \sim 10^{-6}$m/m).

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