Finite Element Based Expressions for Lorentz, Maxwell and Magnetostriction Forces

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Abstract − **To numerically predict the vibration spectrum of an electrical device, the force distribution inside the machine is required. The numerical coupling between the magnetic and the mechanical finite element system is used to derive a finite element based force expression covering both Lorentz and Maxwell forces. The magnetostriction effect is represented by a set of nodal forces causing the same deformation as magnetostriction does. All nodal forces are then summed to obtain the total force distribution.**

1. Introduction

Although stator deformation is mainly caused by radial reluctance forces on the stator teeth, magnetostriction effects can contribute significantly to stator deformation [1]. In order to compute stator deformation, a *local* force expression is needed. Based upon the coupled magneto-mechanical finite element (FE) model, a nodal force expression is derived which covers both Lorentz force and Maxwell stress. The magnetostriction effect is represented by a set of nodal forces, in a similar way as thermal stresses are usually handled.

2. Coupled Magneto-Mechanical System

Both magnetostatic and elasticity FE methods are based upon the minimisation of an energy function. The total energy E of the electromechanical system consists of the elastic energy U stored in a body with deformation a and the magnetic energy W stored in a linear magnetic system with vector potential A:

$$E = U + W = \frac{1}{2}a^T K a + \frac{1}{2}A^T M A \qquad (1)$$

where K is the mechanical stiffness matrix and M is the magnetic stiffness matrix. Considering the similar form of these energy terms, the following system of equations represents the numerically coupled magneto-mechanical system:

$$\begin{bmatrix} M & D \\ C & K \end{bmatrix} \begin{bmatrix} A \\ a \end{bmatrix} = \begin{bmatrix} T \\ R \end{bmatrix} \qquad (2)$$

where T is the magnetical source term vector and R represents external forces. Setting the partial derivative of total energy E with respect to displacement a to zero,

the mechanical equation of the system (2) with $R=0$ is retrieved:

$$\frac{\partial E}{\partial a} = K a + \frac{1}{2}A^T \frac{\partial M(a)}{\partial a}A = 0 \qquad (3)$$

Rearranging the mechanical equation (3) into

$$Ka = -\frac{1}{2}A^T \frac{\partial M(a)}{\partial a}A = -CA = F_{em} \qquad (4)$$

reveals a means to calculate the forces F_{em} internal to the magneto-mechanical system. For the non-linear case, $M(a)$ becomes $M(A,a)$ and magnetic energy W is given by the integral

$$W = \int_0^A T^T dA = \int_0^A A^T M(A,a)dA \qquad (5)$$

where $T=MA$ and $M^T=M$ was used. The force expression (4) now becomes

$$F_{em} = -\frac{\partial W(A,a)}{\partial a} = -\int_0^A A^T \frac{\partial M(A,a)}{\partial a}dA \qquad (6)$$

The partial derivative $\partial M / \partial a$ is derived explicitly using the analytical shape functions and the magnetization characteristic of the material, e.g. $\nu(B^2)$, as explained in detail in [2].

3. Maxwell and Lorentz Forces

Expression (6) for the force F_{em} was derived in a general fashion, not focussing on permeability interfaces or regions with current. The power of expression (6) is that Lorentz forces and Maxwell stress (usually considered separately) are found in one single procedure. Fig.1a shows a conductor with current I in a uniform external magnetic field B_e, shielded by a ring of magnetic material ($\mu_r \gg 1$). The Lorentz force per meter on the conductor without shielding is $F_{tot}=IB_e$. With shielding the Lorentz force is $F_s=IB_s$ where B_s is the (much smaller) homogeneous external field at the conductor. Fig.1b shows the magnetic field inside the ring using a very large number of flux lines so that the small field at the conductor becomes visible. The field shown in Fig.1b is the sum of the homogeneous field B_s and the field of the conductor current itself. Fig.2a shows the force distribution (6) with a more detailed view of the forces on the conductor in Fig.2b. The sum of the nodal forces on the conductor gives exactly $F_s=IB_s$.
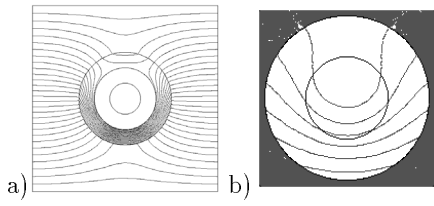
Figure 1: a) Conductor shielded from external field by permeable ring, b) detail: magnetic field inside ring.
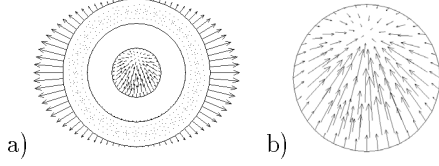

Figure 2: a) Force distribution (6) for shielding problem, b) detail: force distribution on conductor.

The sum of the nodal forces on the shielding ring gives $F_M = F_{tot} - F_s = IB_e - IB_s$ so that the total force on the ring-conductor system again gives F_{tot} [3].

4. Magnetostriction Forces

Effects where mechanical deformation or stress changes the magnetization $\mu_0 M$ in the material or vice versa, are called magneto-mechanical effects. The most important is the *magnetostriction effect* $\lambda(B)$, giving the strain λ of a piece of material due to its magnetization. The *inverse magnetostriction* effect is the dependency of the magnetization $\mu_0 M$ on the applied tensile stress σ. Since stress influences magnetization, it will also influence the magnetostriction itself and turn the $\lambda(B)$ characteristic into a $\lambda(B, \sigma)$ dependency [4].

Fig.3 shows the $\lambda(B, \sigma)$ characteristic for isotropic non-oriented 3% SiFe. Using the magnetic field solution B, the strain λ_e of every element is found and
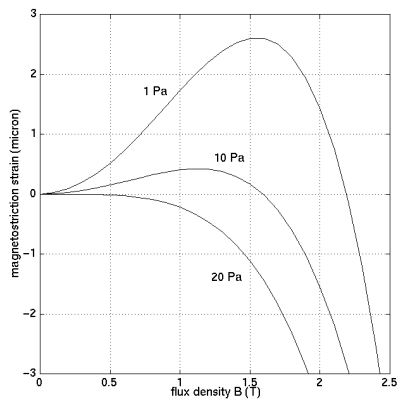

Figure 3: Magnetostrictive material characteristics of non-oriented 3% SiFe as a function of tensile stress.
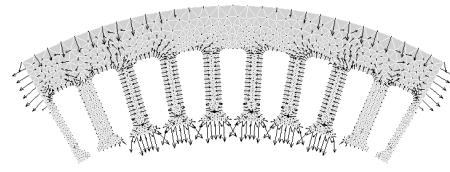

Figure 4: Total force distribution on one pole of a six-pole synchronous generator.

converted to a displacement a_{ms}^e by considering the element's midpoint as fixed. Next, the displacement a_{ms}^e is represented by a set of nodal forces F_{ms}^e (here referred to as *magnetostriction forces*) using the element's mechanical stiffness matrix K^e: $F_{ms}^e = K^e a_{ms}^e$.

Fig.4 shows the total force distribution on one pole of a six-pole synchronous machine (using isotropic 3% SiFe) obtained by summing the Lorentz, Maxwell and magnetostriction force distributions. The forces obtained using (6) primarily act on the stator teeth tips, while the magnetostriction forces want to decrease the stator circumference. In the full paper, it is also explained how to apply this technique to anisotropic materials with direction dependent $\lambda(B)$ characteristics.

5. Conclusion

A FE based expression for local electromagnetic forces is presented, covering both Maxwell and Lorentz forces. Magnetostriction forces are introduced as those forces causing the same mechanical deformation as magnetostriction does. The total nodal force distribution is obtained as the sum of Lorentz, Maxwell and magnetostriction forces.

Acknowledgement

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