LOCAL MAGNETOSTRICTION FORCES FOR FINITE ELEMENT ANALYSIS

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Introduction: Magnetostriction

An important source of vibrations and noise in non-rotating electric machinery (transformers, linear motors, actuators, iron core coils, …) is the deformation caused by magnetostriction (MS). The incorporation of the magnetostrictive effect in the (numerical) design process is usually impaired since detailed data on the magnetic material behaviour are hard to obtain. Versatile experimental methods to obtain all needed technical data on MS effects, permeability, losses, etc. are reviewed in [1]. Once the MS behaviour of the material is known, it has to be incorporated in the magnetic and mechanical analysis. A coupled magneto-mechanical finite element model has been presented earlier [2], and here we will illustrate how to expand this model to take MS material behaviour into account, e.g. $\lambda(B)$, magnetostrictive strain λ as a function of magnetic flux density B.

The Expanded Magneto-Mechanical System

First, next to the purely elastic deformation a, a second partial deformation a_{ms} due to MS is considered, the sum of both giving the total deformation $a^*=a+a_{ms}$. In the case the body is clamped on all sides for example, the total deformation remains zero $a^*=0$, since the MS deformation a_{ms} is compensated by the elastic deformation $a=-a_{ms}$. This approach allows us to still define the elastic energy in the body as [2]

$$U=\frac{1}{2}a^{\mathrm{T}}Ka\,, \tag{1}$$

even when $a^*=0$ (K is the mechanical stiffness matrix).

In [2], the magnetic and mechanical systems were combined using

$$\begin{bmatrix} M & D \\ C & K \end{bmatrix}\begin{bmatrix} A \\ a \end{bmatrix}=\begin{bmatrix} T \\ R \end{bmatrix}, \tag{2}$$

where M is the magnetic stiffness matrix, A contains the unknown magnetic vector potentials, T is the magnetic source term vector and R represents external forces (no reluctance or MS forces). To take the MS deformation a_{ms} into account, the system (2) needs to be expanded:

$$\begin{bmatrix} M(A,a,a_{ms}) & D & 0 \\ C(A,a,a_{ms}) & K(a_{ms}) & 0 \\ 0 & 0 & 1/\lambda(A) \end{bmatrix}\begin{bmatrix} A \\ a \\ a_{ms} \end{bmatrix}=\begin{bmatrix} T \\ R \\ L \end{bmatrix}, \tag{3}$$

where the dependencies of M, C and K on A, a and a_{ms} are indicated. The coupling terms C and D will be discussed further on. Let's consider the system (3):

- The second equation in (3) represents the mechanical system, solving for elastic deformation a. The stiffness matrix K is a function of a_{ms} since the MS deformation determines the boundary conditions and the starting geometry ($x_{i,0}+u_{i,ms}$, $y_{i,0}+v_{i,ms}$) for the mechanical calculation. The term CA represents reluctance forces.
- The first equation in (3) represents the magnetic system, where the magnetic stiffness matrix M depends on A in the non-linear case (saturation), and also depends on the total deformation $a^*=a+a_{ms}$. The term Da in (2) can be recognised as [2]

$$Da=\left.\frac{\partial U}{\partial A}\right|_{a}, \tag{4}$$

representing the change in elastic energy U due to a magnetic field change, while keeping the deformation constant. This term can be calculated analytically before-hand:

$$Da = \Delta t\, E \frac{5/4-\nu}{1-\nu^2}\; \lambda(A) \frac{\partial \lambda(A)}{\partial A}, \tag{5}$$

where E and ν are the Young and Poisson modulus, and Δ and t are the element area and thickness. In calculating (5), the transverse MS λ_t was assumed to be $\lambda_t = -\lambda/2$ [4]. Expression (5) can be interpreted as a current $I_{ms}=Da$ and shifted to the right-hand side (source term). The same can be done with the term CA representing reluctance forces ($F_{rel}=CA$):

$$\begin{bmatrix} M(A,a,a_{ms}) & 0 & 0 \\ 0 & K(a_{ms}) & 0 \\ 0 & 0 & 1/\lambda(A) \end{bmatrix} \begin{bmatrix} A \\ a \\ a_{ms} \end{bmatrix} = \begin{bmatrix} T + I_{ms}(A) \\ R + F_{rel}(A,a,a_{ms}) \\ L \end{bmatrix}. \tag{6}$$

- The third equation in (3) can be converted into an elasticity equation and incorporated in the second (mechanical) equation: the mechanical stiffness matrix K^e for an element gives, after multiplication with the MS displacement a_{ms}, the nodal *magnetostriction forces* $F^e_{ms}=K^e a_{ms}$ that can be added to the external forces R and the reluctance forces F_{rel}. This cannot be done for the whole mesh at once, because the different displacements a_{ms} due to MS in all elements surrounding a node, *cannot* be summed. The forces F^e_{ms} however, *can* be summed. This gives a new system that can be solved for total deformation a^* directly:

$$\begin{bmatrix} M(A,a^*) & 0 \\ 0 & K(a^*) \end{bmatrix} \begin{bmatrix} A \\ a^* \end{bmatrix} = \begin{bmatrix} T + I_{ms}(A) \\ R + F_{rel}(A,a^*) + F_{ms}(A,a^*) \end{bmatrix}. \tag{7}$$

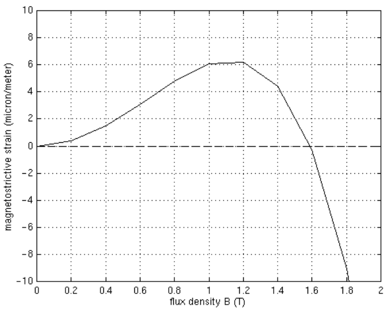

Fig.1 Magnetostrictive material characteristic $\lambda(B)$ for 3% Si Fe, non-oriented.

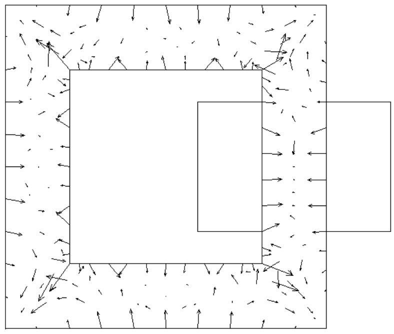

Fig.2 Nodal forces caused by magnetostriction of the iron core.

The MS leads to an additional source term I_{ms} and magnetostrictive forces F_{ms}. Fig.1 shows the MS characteristic $\lambda(B)$ of a typical core material and Fig.2 shows the nodal force distribution F_{ms} due to magnetostriction inside the iron core. This force distribution can be readily used for deformation or vibration calculation.

<u>References</u>

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