The simulation of magnetic problems with combined fast and slow dynamics using a transient time-harmonic method[1]

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Abstract. An interesting method based on a variable splitting into two time-scales, the 'transient time harmonic method' is proposed allowing to compute transient phasor solutions of problems involving slow, close to quasi-static and fast dynamics simultaneously, yielding stiff properties. The time-step of the dynamic problem can be chosen larger than the fundamental time interval, resulting in an 'envelope' model for the problem with the small time constant. The derivation of the FEM matrices is discussed. Examples, including a transformer operating a slow varying load and a transient coupled electromagnetic-thermal problem, are discussed.

Résumé. Une méthode intéressante, appelée 'méthode transitoire-harmonique' est proposée. Elle permet le calcul de problèmes contenant à la fois des phénomènes rapides et lents, voire quasi-statiques, conduisant à des propriétés rigides. Le pas de temps du problème dynamique est supérieur à la période fondamentale, ce qui conduit à un 'modèle enveloppe' du problème ayant la plus faible constante de temps. La dérivation des matrices FEM est discutée. Des exemples, comprenant un transformateur fournissant une charge lentement variable et un problème électromagnétique couplé au champ thermique, sont discutés.

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1 Introduction

Traditionally, electromagnetic devices operating on a definite fundamental frequency are simulated by either a transient computation or a frequency domain method, such as the time-harmonic or harmonic balance method. The choice between the methods is made, based on the fact whether a non-repetitive phenomenon or the steady-state is studied. The transient time-step size is related to the time constant of the dynamic phenomenon. The time- or multi-harmonic approaches implicitly assume periodic solutions, written in terms of a single sinusoid or a set of superposed harmonic functions.

However, problems may arise when simulating models with combined fast and slow dynamics. Such an example is a coupled thermal-magnetic problem, with time constants related to the period of the fundamental electrical supply frequency (< 1 sec.) and thermal time constants (> 1 hour). These types of problems have a mathematical stiff nature and therefore require special integration techniques [1].

An alternative interesting method, the transient time-harmonic approach, is proposed here to tackle this type of problems. This approach, actually forming a bridge between the pure transient method and the steady-state assuming methods, is presented in this paper.

2 Method Derivation

2.1 Steady state frequency domain methods

The well-known 2D equation describing the transient magnetic field evolution in terms of the magnetic vector potential, is [2]:

$$\nabla \cdot (\upsilon \nabla(\boldsymbol{A})) - \sigma(\boldsymbol{T})\frac{\partial \boldsymbol{A}}{\partial t} = -\sigma(\boldsymbol{T})V \qquad (1)$$

with:
- $\boldsymbol{A}$ magnetic vector potential
- υ magnetic reluctivity tensor, possibly dependent on the magnetic field for ferromagnetic materials
- σ electrical conductivity
- $\boldsymbol{T}$ temperature

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V source voltage

The applicable boundary conditions are Dirichlet (parallel field lines) and Neumann (perpendicular field lines). The initial conditions depend on the problem, for instance a zero field or a steady state field calculated separately. The position dependence of the field variables is not indicated explicitly, but is assumed implicitly.

This equation is transformed into a real-valued matrix equation, using the Finite Element Method (FEM). The transient field solution is calculated as a sequence of consecutive partial solutions, one time-step Δt apart. To obtain a stable method, Δt needs at least to be smaller than half of the fundamental period (due to aliasing effects). Additionally, problem specific stability limits have to be considered. These boundaries are determined by the fastest phenomenon in the field, requiring the shortest time-step.

Equation (1) is transformed to the time-harmonic equation [1] by applying a Fourier transformation (which also yields the Harmonic Balance methods [3]) or by simply substituting:

$$\boldsymbol{A}(t) = \underline{\boldsymbol{A}} \cdot e^{j\omega t} \tag{2}$$

Hence, the assumed steady-state solution is entirely described by the complex phasor $\underline{\boldsymbol{A}}$. The time dependency is completely described by the exponential term, the only dynamic phenomenon left in steady-state at the fundamental frequency.

2.2 Transient time-harmonic method

However, if the complex phasor solution is allowed to change in time, but evolving with a slower dynamic behaviour than the fundamental frequency phenomenon, an alternative method is obtained. In fact, two time scales are separated by this type of variable splitting [1], thereby avoiding problems due to stiffness. Eq. (2) is replaced by:

$$\boldsymbol{A}(t) = \underbrace{\underline{\boldsymbol{A}}(t)}_{\substack{dynamic\\ phenom.}} \cdot \underbrace{e^{j\omega t}}_{\substack{quasi-\\ harmonic}} \tag{3}$$

involving the time dependent complex phasor solution $\underline{\boldsymbol{A}}(t)$. This phasor can be interpreted as an 'envelope' around the solution (e.g. in Fig. 1 for an exponential decay).

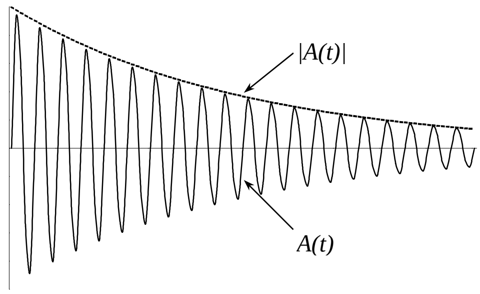


Fig. 1. Graphical interpretation of the time dependent phasor 'envelope' solution.

This methodology is suggested in circuit analysis as well to study electronic circuits with modulated signals [4], but as far as known to the authors, it has not appeared in (electromagnetic) field analysis.

The time derivative of (3) becomes:

$$\frac{d\boldsymbol{A}}{dt} = \left(j\omega \underline{\boldsymbol{A}} + \frac{\partial \underline{\boldsymbol{A}}}{\partial t} \right) \cdot e^{j\omega t} \tag{4}$$

Substitution of (3) and (4) in (1) yields the 'transient time-harmonic' partial differential equation, after eliminating the exponential term, thereby removing the fast dynamics from the equation:

$$\nabla \cdot (\upsilon \nabla (\underline{\boldsymbol{A}})) - \sigma(\boldsymbol{T}) \left(j\omega \underline{\boldsymbol{A}} + \frac{\partial \underline{\boldsymbol{A}}}{\partial t} \right) = -\sigma(\boldsymbol{T}) \underline{V} \tag{5}$$

In many systems such as electrical energy distribution applications, the pulsation ω is known and constant. However in, for instance, free oscillating system it may be an unknown, to be determined separately. Further on, it is assumed that ω is known.

The boundary conditions, as well as the initial conditions associated to the original field equation (1) are transformed into conditions for (5) by substituting (3).

This equation is transformed into FEM equations using the Galerkin method [1]. The time derivative is replaced by a single step finite difference with a Δt, relevant for the slow phenomenon time scale:

$$\begin{aligned}
&\left(\vartheta (\boldsymbol{K}_A + j\omega \boldsymbol{H}_A) + \frac{\boldsymbol{R}_A(\boldsymbol{T}(t_n))}{\Delta t} \right) \underline{\boldsymbol{A}}(t_n) \\
&= -\left((1-\vartheta)(\boldsymbol{K}_A + j\omega \boldsymbol{H}_A) - \frac{\boldsymbol{R}_A(\boldsymbol{T}(t_{n-1}))}{\Delta t} \right) \underline{\boldsymbol{A}}(t_{n-1}) \\
&+ \vartheta \underline{\boldsymbol{F}}_A(t_n) + (1-\vartheta) \underline{\boldsymbol{F}}_A(t_{n-1})
\end{aligned} \tag{6}$$

with:
- $\boldsymbol{K}$ FEM matrix associated with the diffusion term
- $\boldsymbol{H}$ FEM matrix associated with the harmonic term
- $\boldsymbol{R}$ FEM matrix associated with the transient term
- $\boldsymbol{F}$ FEM vector associated with the source term
- ϑ time weight

2.3 Computational aspects

In principle, there is no aliasing-related bound on Δt, as it is made independent of the fast phenomenon, which is represented by the oscillatory function. It is determined by the larger time constants of the slow dynamics in the problem. Therefore, time-steps spanning multiple periods of the underlying oscillation pose no problem. The complete time evolution can be reconstructed from these phasors by multiplying with the exponential function as in (3).

An extension with circuit equations [5] is made by substituting the two-term time derivative function (4) in the appropriate induction terms.

Multi-harmonic algorithms are possible as well by extending (3) to a summation of harmonic components.

It must be noted that theoretically any transient solution can be obtained by this method, as shown in (7). In general, however, this is a computationally rather inefficient approach, due to the complex variables, than using the transient method directly. This is true, unless a smooth transition to larger time-steps (e.g. in adaptive methods) is required.

$$\boldsymbol{A}(t) = \left(\boldsymbol{A}(t) \cdot e^{-j\omega t}\right) \cdot e^{j\omega t} \tag{7}$$

3 Applications

3.1 Slowly varying sources or loads

When loads or sources are varying relatively slowly (in amplitude, phase and/or frequency) compared to their fundamental period, a transient analysis, using the proposed transient time harmonic technique, yields an efficient solution method that allows to compute the solution's time evolution at the slow rate. A regular transient method would require multiple time-steps per period.

As an example, a single magnetic field solution, simulating a ‘Power Quality’ problem, more in particular ‘flicker’ is computed. This is a rapid voltage change in which the voltage amplitude changes at a modulating frequency of >10 Hz, which is experienced as very annoying in electrical lighting as perceived by the human eye. The cause is generally a permanently varying load or supply, causing a constantly changing voltage drop in the supply impedance and transformer.

The example used here, is a single-phase transformer (Fig. 2), connected to a voltage supply at 50 Hz, operating a variable resistive load. The load value, occurring in the circuit equations, changes between 20 % and 100 % at a rate of 11 Hz. Therefore, the steady-state has a large period: the smallest common multiple of the fundamental and the flicker period. The primary supply voltage at 50 Hz is connected with a rather severe internal impedance, e.g. a long cable (Fig. 3).

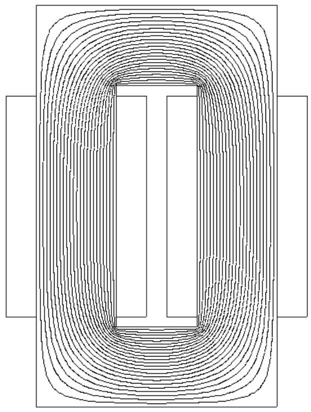


Fig. 2. Single-phase transformer model.

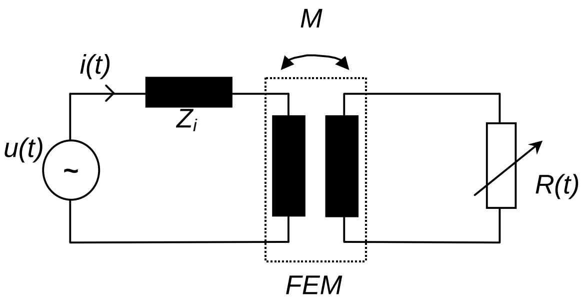


Fig. 3. Flicker simulation circuit.

Fig. 4 shows the evolution of the magnitude of the flux in the leg of the ferromagnetic transformer core. A Fourier analysis shows that this phenomenon contains a dominant (fast) 50 Hz component and a smaller (slow) flicker subharmonic plus interharmonics. A time-step of 0.02 sec, which is in fact one period of the mains frequency, is used. The flux change is limited since part of the voltage drop influences the leakage fluxes and because of the saturation level.

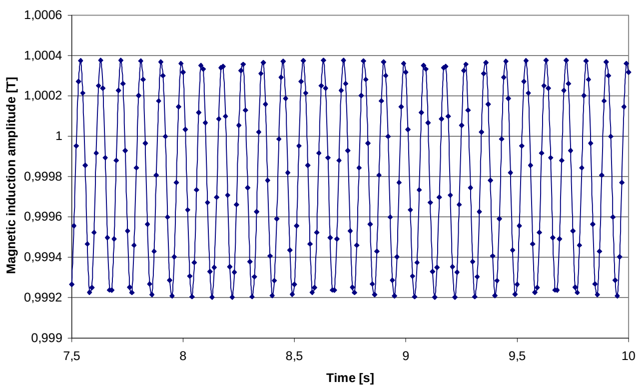


Fig. 4. Change of the magnitude of the magnetic induction in a transformer leg, during voltage flicker.

3.2 Slowly changing non-linearities, due to thermal coupling

When the time evolution magnetic problem to be solved is coupled to a thermal field (eq. (5), extended with appropriate convection constraints), due to material parameter thermal dependencies, an extremely large difference can be noticed between the magnetic (range of seconds) and the thermal time constant (range of hours). Due to the high computational costs, the standard transient field method, even in a special version for stiff problems, can obviously not be used.

$$\nabla \cdot (\lambda \nabla T) - \rho c \frac{\partial T}{\partial t} = -q \quad (8)$$

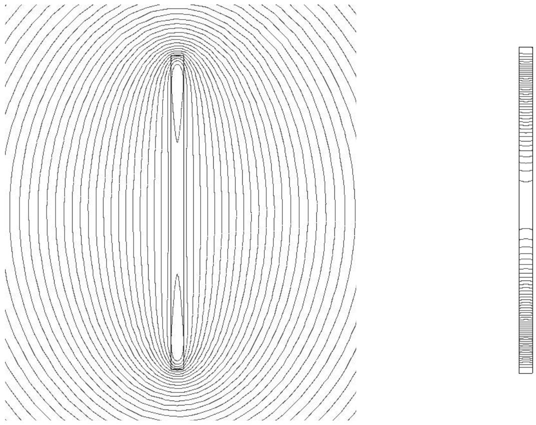
with:
- T temperature
- λ thermal conductivity
- ρ mass density
- c specific heat
- q heat source density

Often the problem is assumed to reside in some 'temporarily steady-state' for the magnetic problem. This is equivalent to neglecting the time derivative in (5) and the resulting time-harmonic equation is coupled to a transient thermal field [4]. For many applications this approach leads to a solution, but, as is experienced by the authors, unfortunately sometimes the used non-linear iteration becomes unstable due to this assumption unless unreasonably small time-steps are used. This phenomenon is encountered in the described magnetic/thermal problem when significant skin effect variations occur due to the local heating effects. In this case the transient time-harmonic method, which is theoretically more accurate, has to be applied and yields the coupled problem solution using reasonable time-steps.

Fig. 5 shows the solutions of a coupled example problem: a solid metallic conductor, with a large aspect ratio and a skin depth being smaller than the length, is driven by a voltage source and cooled by natural convection, imposing an asymmetrical convection due the heating rising air. In this case the electrical conductivity is hyperbolically temperature dependent with a parameter α:

$$\sigma(T) = \frac{\sigma_{ref}}{1 + \alpha(T - T_{ref})} \quad (8)$$

The current density is the highest at the top and the bottom of the conductor; due to local heating (final hot spot temperature = 60°C), the skin depth becomes larger and the current and loss density profile changes. When this transient problem is solved by means of block iteration involving the steady-state time-harmonic and transient thermal field, divergence is encountered even when extremely small time-steps are used (due to truncation errors). The use of the transient-time harmonic method (6) yields stable converging iteration (Fig. 6).



(a) Magnetic field solution (b) Temperature field solution

Fig. 5. Final magnetic and thermal field solution obtained for the transient coupled problem solution of a large aspect ratio solid conductor.

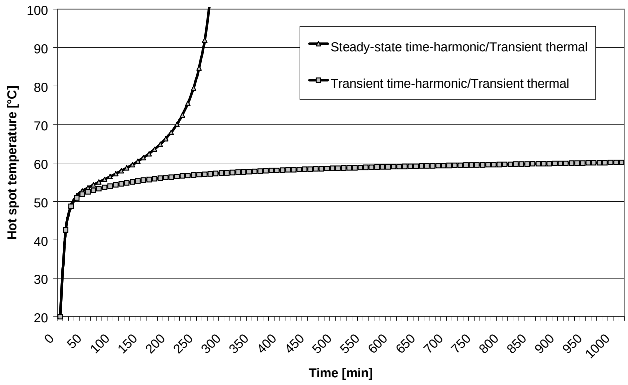


Fig. 6. Diverging (non-linear algorithm with approximating time-harmonic approach) and converging (non-linear algorithm with more correct transient time-harmonic approach).

Conclusions

An approach, the transient time harmonic method, is presented to simulate magnetic problems with combined fast and slow dynamics. This type of problems shows stiffness properties and would require complicated real-valued integration methods involving small time-steps. Here, a variable splitting yielding a separation of the fast and slow field changes is applied. The solution is calculated as the time evolution of a complex phasor. The resulting complex transient method can be advanced using large time-steps (larger than a single oscillation period), at the pace of the slow phenomenon.

Two examples illustrate the advantages of this methodology. At first, a single phase transformer supplying a variable (‘flickering’ at 11 Hz) load is simulated. The voltage drop in the supply system causes a change of the magnetic flux magnitude at the rate of the load oscillation.

Secondly, a coupled transient thermal-magnetic simulation is made. Due to the large difference in time constants, a very stiff problem is obtained. It is preferable to obtain the (thermal) solution using large time-steps. Often, the magnetic subproblem solution within these intervals is approximated using a steady-state method, but this may lead to divergence, as shown. The use of the transient time-harmonic method is theoretically more accurate and yields better converging coupled solutions.

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