Electrodynamic Finite Element Model Coupled to a Magnetic Equivalent Circuit

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***Résumé* - Un champ électrodynamique est couplé à un circuit magnétique équivalent. Pour le problème électrodynamique, on utilise une formulation en potentiel vecteur électrique, dicrétisée par éléments finis. Les inconnues du circuit équivalent sont les flux et les forces magnétomotrices. La matrice du système couplé est mixte et hybride. La méthode est appliquée à la simulation de la distribution des courants induits dans un matériau laminé, et au calcul des pertes d'un système de chauffage diélectrique.**

***Abstract* - An electrodynamic field is coupled to a magnetic equivalent circuit. The electrodynamic problem is formulated by the electric vector potential and discretised with finite elements. The magnetic lumped parameter model is described in terms of unknown fluxes and magnetomotive forces. The coupled system matrix has a mixed and hybrid nature. The method is applied to simulate eddy current distributions in laminated material and losses in a dielectric heater.**

I. INTRODUCTION

As electric and magnetic phenomena are linearly related to each other, the discrete coupling of both fields is easily realized in one system matrix. Quasi-static electric fields are characterised by linear material characteristics and current distributions following clearly determined paths through the conductive parts in the model. The magnetic fields, however, suffer from arbitrary flux paths and non-linear material properties. As a consequence, technical devices are efficiently and accurately modelled by a magnetic finite element model coupled to an electric lumped parameter description. The technical importance of this kind of hybrid coupling schemes is reflected by the large efforts found in literature to optimise field-circuit coupling simulation techniques [1].

There are also devices, e.g. laminated materials, induction furnaces and dielectric heaters for which this assumptions are not true. The electrodynamic field requires an accurate description whereas the magnetic field can be represented by a magnetic equivalent circuit. This paper considers field-circuit coupled electrodynamic-magnetic models.

II. ELECTRODYNAMIC MODEL

The continuity of the current density $\mathbf{J}$ and Ampère's law for the magnetic field strength $\mathbf{H}$ are applied by defining the electric vector potential $\mathbf{T}$ by $\mathbf{J} = \nabla \times \mathbf{T}$ and the magnetic scalar potential φ by $\mathbf{H} = \mathbf{T} - \nabla\varphi$. The combination of the constitutive relations with Faraday-Lenz's law yields the governing differential equation

$$\nabla \times \left(\rho \nabla \times \mathbf{T}\right) + j\omega\mu\mathbf{T} = j\omega\mu\nabla\varphi \,. \qquad (1)$$

ω is the pulsation, ρ the resistivity and μ the permeability.

The magnetomotive force (MMF) V_m along a flux path $\ell_1 - \ell_2$ is defined by

$$V_m = \int_{\ell_1}^{\ell_2} \left(-\nabla\varphi\right) d\ell \,. \qquad (2)$$

The geometry and the excitation of the devices considered here, permit a 2D or axisymmetric discretisation. In that case, the current is in the plane (*x*-*y*-plane or *r*-*z*-plane respectively) whereas the flux and the electric vector potential are perpendicular to the plane.

III. MAGNETIC EQUIVALENT CIRCUIT

The magnetic equivalent circuit (MEC) consists of flux sources, MMF sources, reluctances, magnetic inductors and the magnetic branches embedded in the electrodynamic model (Fig. 1). A permeable part in the electrodynamic FE model is excited by the MMF V_m. If the magnetic branch is part of the MEC, the MMF across the branch, V_m, serves as an extra unknown. The additional integral relation

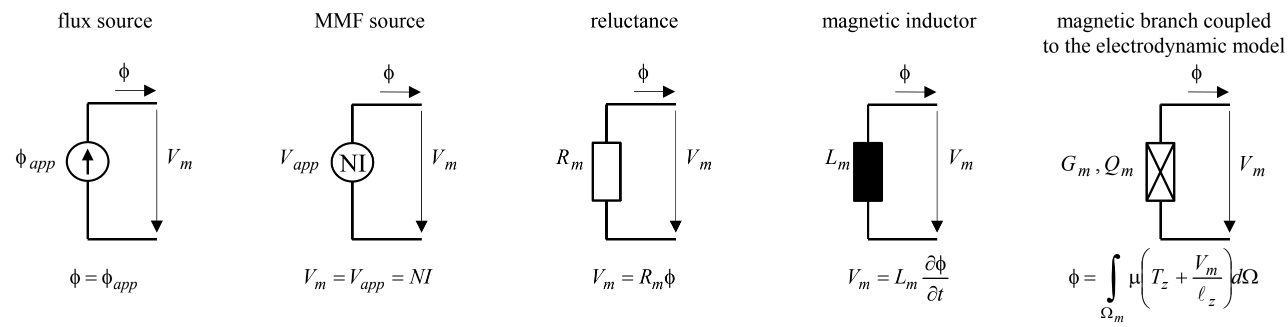

Fig. 1: Magnetic circuit elements.

$$\phi = \int_{\Omega_m} \left(\mu \frac{V_m}{\ell_z} + \mu T_z \right) d\Omega \tag{3}$$

with ℓ_z the length of the 2D model, relates the magnetic flux to the MMF and the electric vector potential distribution. The discrete form of (3) is

$$\phi = G_m V_m + \mathbf{Q_m} \mathbf{T_z} \tag{4}$$

with G_m the magnetic conductance, $\mathbf{Q_m}$ the coupling matrix and $\mathbf{T_z}$ the nodal values for the electric vector potential.

If no MMF sources and magnetic inductors occur in the MEC, a common modified nodal analysis of the circuit part is sufficient to obtain a sparse and symmetric coupled system. Here, a more general topological method is applied. The treatment is analogeous to the approach presented in [2] and ends up with a mixed description in terms of both unknown MMFs and fluxes. A tree is traced through the MEC. The tree branches are in priority MMF sources, branches coupled to the electrodynamic field and reluctances. The tree branches are modelled by unknown MMFs. To the remaining branches, unknown fluxes are assigned. The coupling mechanism preserves the sparsity and the symmetry of the matrix for all possible connections of the magnetic elements of Fig. 1.

IV. Applications

Two examples are presented. The first model consists of several iron laminates with coating material on both sides (Fig. 2). The coating material is less conductive and less permeable than the iron, preventing excessive eddy current losses but at the expence of a higher reluctance of the global magnetic flux path [3]. The MEC represents the parallel connection of all domains in the electrodynamic model, excited by a flux source (Fig. 2).

The second example is a dielectric heating device (Fig. 3). A cylindrical dielectricum is placed between two circular electrodes. Both dielectric and conductive heating effects are considered [4]. The geometry and the excitation are modelled by an axisymmetric model. As a consequence, the MEC consists of the short-circuit connection of all magnetic paths. Here, the excitation has an electric nature and is applied as a difference in electric vector potential. The combination of conductive and dielectric effects involves a complex valued resistivity in (1). If the geometrical dimensions exceed the wave length, a wave phenomenon is observed (Fig. 3).

V. Conclusions

An electrodynamic finite element model is combined with a magnetic equivalent circuit in one sparse and symmetric system matrix. The coupling scheme is applied to simulate the eddy current losses in laminated material and the total losses in a dielectric heating device.

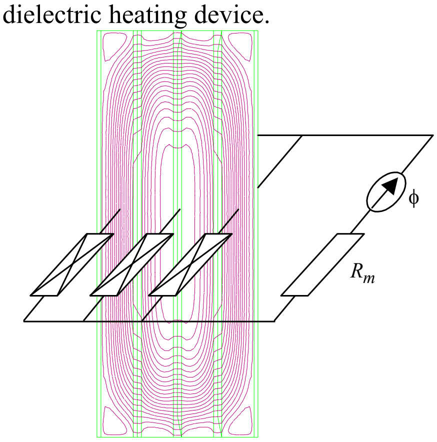

Fig. 2: 2D electrodynamic model of a laminated material combined with a magnetic equivalent circuit.

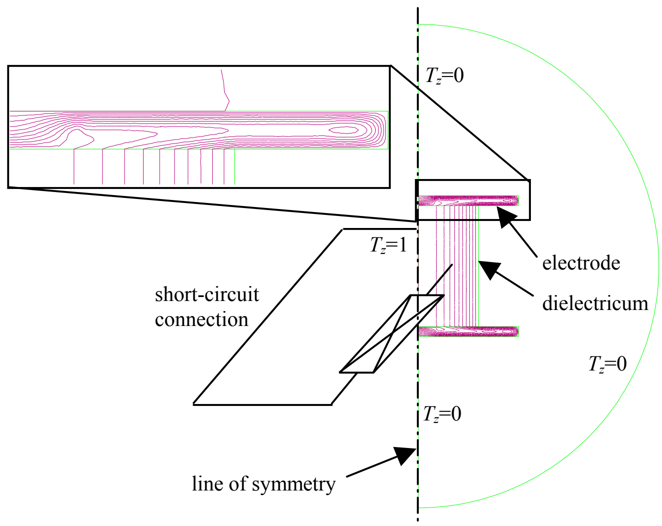

Fig. 3: 2D electrodynamic model of a laminated material combined with a magnetic equivalent circuit.

Acknowledgement

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