

Finite Element Analysis of Electrical Machine Vibrations caused by Lorentz, Maxwell and Magnetostriction Forces

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Abstract - In order to numerically predict the stator vibration spectrum, the mechanical behaviour of the stator (mode shape) is correlated with the force distribution occurring inside the machine. This force distribution is found considering the coupling between the magnetic and the mechanical finite element systems. This coupling results in a finite element expression containing both the Lorentz forces and the Maxwell forces. Using a weak coupling approach, magnetostriction forces are added to the force distribution. The numerically predicted stator vibration spectrum of a synchronous machine is compared to stator surface measurements.

Keywords - Maxwell stress, Lorentz force, magnetostriction, modal analysis, low-noise design

I. INTRODUCTION

The main source of acoustic noise of electric machines are the radial stator vibrations. Although this deformation is mainly caused by radial reluctance forces on the stator teeth (Maxwell magnetic stress), magnetostriction effects can contribute significantly [LAFT]. In order to be able to compute stator deformation, *local* force expressions are needed. Here, a nodal force expression is derived based upon the coupled magneto-mechanical finite element model, which covers both Lorentz force and Maxwell stress. The magnetostriction effect is represented by a set of nodal forces giving rise to the same deformation as the magnetostriction would. This total force distribution is calculated using the magnetic field solutions for all relevant rotor positions. For a constant rotor speed, the forces are obtained as a function of time and the spectrum of the force distribution is computed.

Using a 2D mechanical finite element (FE) model of the stator, the undamped stator mode shapes are computed taking both iron yoke and copper coils into account. When damping is neglected, the total equation of motion of the stator decomposes into the individual modal equations of motion (modal decomposition). The generalized force (*mode participation factor*) acting upon a particular stator mode is found as the correlation between the force distribution and the mode shape. These modal equations of motion are then solved in the frequency domain giving the individual modal spectra. The total vibration spectrum at a specific location on the stator is found by applying the inverse modal decomposition to the modal spectra. The numerically predicted spectra are compared with accelerometer measurements of the stator vibration spectrum on different points of the stator of a synchronous machine in generator mode.

II. THE COUPLED MAGNETO-MECHANICAL SYSTEM

Both magnetostatic and elasticity FE methods are based upon the minimisation of an energy function. The total energy E of the electromechanical system consists of the *elastic energy* U stored in a body with deformation a [ZIEN] and the *magnetic energy* W stored in a linear magnetic system with vector potential A [SILV]:

$$E = U + W = \frac{1}{2} a^T K a + \frac{1}{2} A^T M A, \quad (1)$$

where K is the mechanical stiffness matrix and M is the magnetic 'stiffness' matrix. Considering the similar form of these energy terms, the following system of equations represents the numerically coupled magneto-mechanical system:

$$\begin{bmatrix} M & D \\ C & K \end{bmatrix} \begin{bmatrix} A \\ a \end{bmatrix} = \begin{bmatrix} T \\ R \end{bmatrix}, \quad (2)$$

where T is the magnetical source term vector and R represents external forces other than those of electromagnetic origin. Setting the partial derivatives of total energy E with respect to the unknowns $[A \ a]^T$ to zero, the combined system (2) with $T=0, R=0$ is retrieved:

$$\frac{\partial E}{\partial A} = M A + \frac{1}{2} a^T \frac{\partial K(A)}{\partial A} a = 0, \quad (3)$$

$$\frac{\partial E}{\partial a} = K a + \frac{1}{2} A^T \frac{\partial M(a)}{\partial a} A = 0. \quad (4)$$

The coupling term D can be used to represent magnetostrictive effects in an analysis using strong coupling but will not be considered here, so that $D=0$ and $T=MA$. Rearranging the mechanical equation (4) into

$$K a = -\frac{1}{2} A^T \frac{\partial M(a)}{\partial a} A = F_{em}, \quad (5)$$

reveals a means to calculate the forces F_{em} internal to the magneto-mechanical system. These magnetic forces are computed from vector potential A and the *partial derivative* of the magnetic stiffness matrix M with respect to deformation a . These forces F_{em} are also found by applying the virtual work principle to the magnetic energy W for a virtual displacement a [COUL][REN]:

$$F_{em} = -\frac{\partial W}{\partial a} = -\frac{\partial}{\partial a} \left[\frac{1}{2} A^T M(a) A \right], \quad (6)$$

where vector potential A has to remain unchanged (constant flux) [ODEN]. For the non-linear case, the matrix M is a function of magnetic field and displacement: $M(A,a)$. The magnetic energy W is now given by the integral

$$W = \int_0^A T^T dA = \int_0^A A^T M dA, \quad (7)$$

where $T=MA$ and $M^T=M$ was used. The force expression (6) now becomes

$$F_{em} = -\frac{\partial W}{\partial a} = -\int_0^A A^T \frac{\partial M(A,a)}{\partial a} dA. \quad (8)$$

Note that adding a constant to A indeed does not change the value of the integral. The partial derivative $\partial M/\partial a$ is derived explicitly using the analytical shape functions and the magnetization characteristic of the material, e.g. $v(B^2)$, as explained in detail in [DELMagdeburg].

III. MAXWELL AND LORENTZ FORCES

The expression (8) for the force F_{em} was derived in a general fashion, not focussing on permeability interfaces or regions with current. Any permeability interfaces will contribute greatly to the $\partial M/\partial a$ summation over a node that lies on the interface and will yield the same value as the Maxwell stress. Elements with current density will affect the vector potential profile in such a way that, when (8) is used, exactly the Lorentz force acting on that element is revealed. The power of expression (8) is that both forces are found in one single procedure.

Fig.1 shows a conductor with current I in a uniform external magnetic field B_e , but shielded by a ring of magnetic material. The Lorentz force per meter on the conductor without shielding is $F_{lor}=Ix B_e$. With shielding the Lorentz force is $F_s=Ix B_s$ where B_s is the (much smaller) homogeneous field at the conductor after shielding. Figure 2 shows the magnetic field using a very large flux line density so that the small field at the conductor becomes visible. The field shown in Fig.2 is the sum of the homogeneous field B_s and the field of the conductor current itself. Fig.3 shows the results of the force expression (8). Fig.4 shows a detail of Fig.3 around the conductor. The sum of all the nodal forces on the conductor gives exactly $F_s=Ix B_s$. The

sum of the nodal forces on the shielding ring gives $F_M = F_{tot} - F_s = I \times B_e - I \times B_s$ so that the total force on the ring-conductor system again gives F_{tot} .

Fig.6 shows the force distribution obtained for one pole of a six-pole synchronous machine in generator mode for the magnetic field shown in Fig.5 for two rotor positions. The forces on the stator are mostly due to the Maxwell part of (8).

IV. MAGNETOSTRICTION FORCES

There are several so-called *magneto-mechanical effects*, i.e. effects where the mechanical deformation or stress changes the magnetization $\mu_0 M$ in the material:

- The most important effect is the well-known *magnetostriction effect* $\lambda(B)$ pertaining to the strain λ of a piece of magnetized material.
- The *inverse magnetostriction* effect is the dependency of the magnetization $\mu_0 M$ on the tensile or compressive stress σ . Since stress influences magnetization, it will also influence the magnetostriction itself and turn the $\lambda(B)$ characteristic into a $\lambda(B, \sigma)$ dependency.
- A smaller effect is the ΔE effect: change of effective Young's modulus due to magnetization.
- Normally there is no relevant volume change due to magnetostriction [JILES], but under some conditions and for some materials there is. This effect is referred to as *volume magnetostriction*.

The last two effects will not be considered here since they are only of secondary importance for the usual engineering applications [HIRSINGER]. Fig.7 shows magnetostriction characteristics for non-oriented 3% SiFe as a function of flux density and stress. Fig.8 shows magnetostriction characteristics of M330-50A which has different behaviour parallel and perpendicular to the rolling direction (anisotropic material).

When the $\lambda(B)$ characteristic is known, the strain λ^e and the displacement a_{ms}^e of every finite element can be found using the magnetic field solution. This displacement a_{ms}^e is then represented by a set of mechanical forces F_{ms} (here referred to as *magnetostriction forces*) using the element's mechanical stiffness matrix: $F_{ms} = K a_{ms}$. There will usually be forces parallel and perpendicular to the flux density vector (even for isotropic material) since the mechanical Poisson modulus is about 0.3 while magnetostriction keeps the volume constant (Poisson modulus of 0.5). Fig.9 shows the magnetostriction forces for the magnetic field of Fig.5. Fig.9a shows the forces obtained using the isotropic non-oriented 3% SiFe and Fig.9b shows the forces for the anisotropic M330-50A. Fig.10 makes a comparison between Maxwell stresses and magnetostriction forces for this machine. It can be seen that both force components will contribute about equally to the stator deformation.

V. STATOR MODE SHAPES AND MODE PARTICIPATION FACTORS

Using the 2D mechanical stiffness matrix K and mass matrix M_m of the stator, the undamped 2D stator mode shapes are found, some of which are shown in Fig.11. The modes are calculated taking mass and stiffness of the yoke iron and the stator coil copper into account. For a given force pattern f^α , occurring for rotor position α , and a given mode shape ϕ_i , the mode participation factor Γ_i^α is defined as [THOM]:

$$\Gamma_i^\alpha = \frac{\phi_i^T f^\alpha}{\phi_i^T M_m \phi_i} . \quad (9)$$

The j^{th} element of the vector ϕ_i is the displacement of the i^{th} mode shape at the j^{th} node, while the j^{th} element of the vector f^α is the force on the j^{th} node due to the magnetic field for rotor position α . Fig.12 shows the mode participation factors (MPF) of several modes for rotor positions from 0° to 360° . The MPF usually contain both a DC- and an AC-component.

VI. STATOR VIBRATION SPECTRUM

The vibration of the stator is governed by

$$M_m \ddot{u} + C_m \dot{u} + K u = f(t) , \quad (13)$$

where $u(t)$ is the nodal displacement and $f(t)$ is the force distribution acting on the stator, $f(t) = f^\alpha$ for $\alpha = 2\pi nt/60$. M_m , C_m and K are the mechanical mass, damping and stiffness matrices respectively. Using the modal decomposition

$$u = Pq, \quad (14)$$

with P the modal matrix containing a selected set of N stator mode shapes and q the vector of generalised modal co-ordinates, (13) is transformed into

$$P^T M_m P \ddot{q} + P^T C_m P \dot{q} + P^T K P q = P^T f(t), \quad (15)$$

where all terms were premultiplied by P^T . Only when the mechanical damping C_m is assumed to be proportional ($C_m = \alpha K + \beta M_m$), the system of equations (13) can be decoupled into [MEIR]

$$\ddot{q}_i + 2\zeta_i \omega_i \dot{q}_i + \omega_i^2 q_i = \Gamma_i(t), \quad i = 1..N, \quad (16)$$

where ω_i is the mode's circular eigenfrequency and ζ_i is the modal damping factor. Here damping is neglected ($\zeta_i = 0$). Note that the modal decomposition indeed transforms the force $f(t)$ into the MPF $\Gamma_i(t)$, $i = 1..80$, as prescribed by (9). From (9), the MPF are known as a function of rotor position, the rotor speed n allows us to find the MPF as a function of time. The individual modal equations (16) are solved in the frequency domain after applying a discrete Fourier transformation to $q_i(t)$ and $\Gamma_i(t)$:

$$Q_i(k\Delta\omega) = \frac{\Gamma_i(k\Delta\omega)}{\omega_i^2 - (k\Delta\omega)^2}. \quad (17)$$

The individual complex spectra Q_i of the relevant modes are composed again into the actual stator displacement spectra U using the modal composition (14). Fig.13 compares the measured stator acceleration to the spectrum predicted using the 2D magnetic and mechanical finite element models. The correspondence is encouraging.

VII. CONCLUSION

Using 2D mechanical and magnetic finite element models, a relatively reliable numerical prediction of the vibration spectrum of the synchronous machine is obtained. The force distribution occurring for all relevant rotor positions is correlated to the 2D mode shapes of the induction machine stator, yielding *mode participation factors* as a function of time. A finite element expression for local electromagnetic forces is presented covering both Maxwell and Lorentz forces. Magnetostriction forces are introduced as those forces causing the same mechanical deformation as magnetostriction would. The global equation of motion of the stator is decomposed into modal equations of motion which are solved separately in the frequency domain. The spectral information of the modes is transformed back into spectral acceleration information for the stator surface. Considering the use to 2D models, relatively good agreement is obtained between predicted and measured vibration spectra.

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