

# Practical Method to Determine Additional Load Losses due to Harmonic Currents in Transformers with Wire and Foil Windings

Johan Driesen, Thierry Van Craenenbroeck, Bert Brouwers, Kay Hameyer, Ronnie Belmans

Katholieke Universiteit Leuven, Dep. EE (ESAT), Div. ELEN

Kardinaal Mercierlaan 94, B-3001 Leuven, Belgium

Phone: +32/16/32.10.20 Fax: +32/16/32.19.85

e-mail: johan.driesen@esat.kuleuven.ac.be

**Abstract:** A method is presented to determine the additional load losses in transformers caused by harmonic currents. Several black-box short circuit tests at different harmonic frequencies have to be conducted on existing transformers or have to be simulated in the design stage. A 'K-factor'-related formula allowing for the estimation of the total augmented losses in the transformers is derived. This approach is also valid for transformers containing windings of the foil type. It is shown by means of measurements and simulations that the rise of the losses does not follow a squared dependency as it is the case with traditional wire winding transformers. An alternative K-factor definition to be used with these transformers seems to be necessary and is provided here.

**Keywords:** transformers, power quality problems, power system harmonics, K-factors, finite element methods.

## I. INTRODUCTION

Transformers carrying current harmonics or supplied by non-sinusoidal voltages, exhibit additional load losses, often yielding higher hot spot temperatures. Reducing the maximum apparent power, transferred by the transformer, often called derating, is required when such power quality related problems occur. To estimate the derating of the transformer, the load's K-factor may be used. This factor reflects the additional load losses and is derived for traditional wire wound transformers [1-7].

Nowadays however, alternative winding designs such as foil windings or mixed wire/foil windings are applied in transformers, such as distribution transformers. For these transformers, the standardised K-factor does not reflect the additional load losses anymore. The actual K-factor proves to be strongly construction dependent. Therefore, it is impossible to define the K-factor of the load as such and it

should be determined at the design stage of the transformer by the given load data, using field simulation methods [3], or measuring techniques such as the method proposed here. This approach is explained and illustrated in this paper using existing foil-winding transformers.

## II. ADDITIONAL LOSS FACTOR 'K'

To define the frequency dependent additional loss factor, a resistance factor ' $K_{\Delta R}$ ' is defined:

$$K_{\Delta R}(f) = \frac{R_{AC}(f) - R_{DC}}{R_{AC}(f_1) - R_{DC}} \quad (1)$$

with  $R_{DC}$  the equivalent series DC-resistance and  $R_{AC}$  the series AC-resistance. This AC resistance depends on the frequency due to the current redistribution in the winding and other additional losses. Hence, this factor indicating the relative increase of the transformer's series resistance is, a function of the considered frequency. This dependence differs to a major extent according to the construction and the placement of the windings.

Finally, the frequency dependent additional loss factor ' $K_{\Delta P}$ ' is determined by (2), which requires the knowledge of the relevant harmonics in the load current's spectrum.

$$K_{\Delta P} = \sum_{f > f_1} K_{\Delta R}(f) \left( \frac{I_f}{I_R} \right)^2 \quad (2)$$

with  $K_{\Delta P}$  additional loss factor  
 $K_{\Delta R}$  resistance factor  
 $I_f$  current at harm. frequency  $f$   
 $I_R$  rated current

To determine this factor for a given transformer, prototype or computation model, the series resistances or short circuit resistances have to be determined, either by measurement or by numerical simulation and post-processing of the resulting fields.

## III. MEASURING METHOD

The load losses and associated resistances are determined by performing a series of short circuit tests at different

frequencies, during which the transformer is considered as a 'black box'. In these tests, the no-load losses can be neglected and the transformer is supposed to behave linearly as the flux level is very low. The short circuit test is performed at the relevant harmonic frequencies. It is important that the transformer is brought to its rated operational temperature first, since the resistances are temperature dependent. In particular the resistance of the foil windings often has a higher temperature in the regions with the highest current densities.

The driving source for these tests is a rotating generator suited for higher speeds/frequencies or a set of power amplifier(s)/converters in combination with accurate signal generators both with a very low harmonic content. The sources do not have to be of the same power rating and voltage levels as the transformer, as only the losses have to be supplied during the test. When supplying the high voltage winding, the required current magnitude remains moderate. The DC-resistance is obtained by extrapolation of the low-frequency resistance values obtained from the set of AC measurements or can be measured separately.

#### IV. COUPLED ELECTROMAGNETIC-THERMAL SIMULATION METHOD

##### A. Magnetic field simulation

When applying appropriate simulation techniques, the magnetic field in and around the core is for instance obtained by using time-harmonic finite element methods (FEM). It is important that the eddy currents and leakage fields are considered in the simulation since they determine the frequency dependency of the load losses. In the finite element method, this means that conductors with dimensions larger than the skin depth, have to contain elements smaller than the skin depth. The 'air' included in the model has to be modelled sufficiently large, which can be accomplished by means of transformations [8]. To define the loads and sources and to describe the coil connections, the 2D magnetic FEM model is extended with external circuit equations. Often a partial 2D model is sufficient. 3D models can be used for special coil arrangements, or to determine the concentrated circuit parameters of the external electric circuit.

The magnetic field  $B$  is solved, using the magnetic vector potential  $A$  [8]:

$$B = \nabla \times A \quad (3)$$

In 2D, equation (4) is obtained using this vector potential in the Maxwell equations in the time domain:

$$\nabla \cdot (\nu(A) \nabla A) - \sigma \frac{\partial A}{\partial t} = -J_s \quad (4)$$

with  $\sigma$  the electrical conductivity and  $\nu$  the saturable magnetic reluctivity. However, to study the steady state, it is more interesting to transform this equation to the frequency

domain by assuming that the solution can be written in the form (5). The resulting equation (6) has to be solved for every frequency  $\omega = 2\pi f$  of interest.

$$A(t) = A \cdot \exp(j\omega t) \quad (5)$$

$$\nabla \cdot (\nu(A) \nabla A) - j\omega\sigma A = -J_s \quad (6)$$

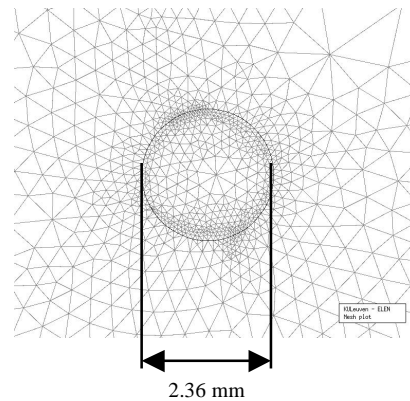
The aim of the simulation is to calculate the field in the short circuit test. The most easy way is to apply current sources  $J_s$  carrying the rated current. Because the field magnitudes are not large, saturation is relatively unimportant and the numerical solution is obtained relatively fast. The magnetic reluctivity  $\nu$  remains approximately constant.

##### B. Loss calculations

Saturation and iron losses can be neglected in the short circuit test. In this case, the set of equations of the type as found in (6), each having a different (harmonic) frequency, is linear and can be decoupled to calculate the joule losses. In the post-processing step, the losses are determined by adding the individual loss integrals  $q_e$  of the finite elements  $e$ . The principal losses in foil windings are due to joule losses associated with the source and the eddy currents:

$$P_{foil} = R_{AC}(f) I^2 = \sum_e q_e = \sum_e \int_{\Omega_e} \frac{(J_s + j\omega\sigma A)^2}{\sigma} d\Omega \quad (7)$$

The eddy current losses in wire windings may play an important role as well and have to be added separately, after a set of linear calculations. These losses do not follow directly out of the FEM result, since the conductors are not modelled individually – as in the case of foil conductors. For these conductors, the eddy current terms  $j\omega\sigma A$  are not included directly. These losses are generated by externally imposed leakage fluxes, but do not cause a significant change in these leakage fluxes as long as the size of the conductor is smaller than the local skin depth. So superposition of these eddy current and the other losses is allowed.



a) FEM mesh used for the eddy current loss calculation

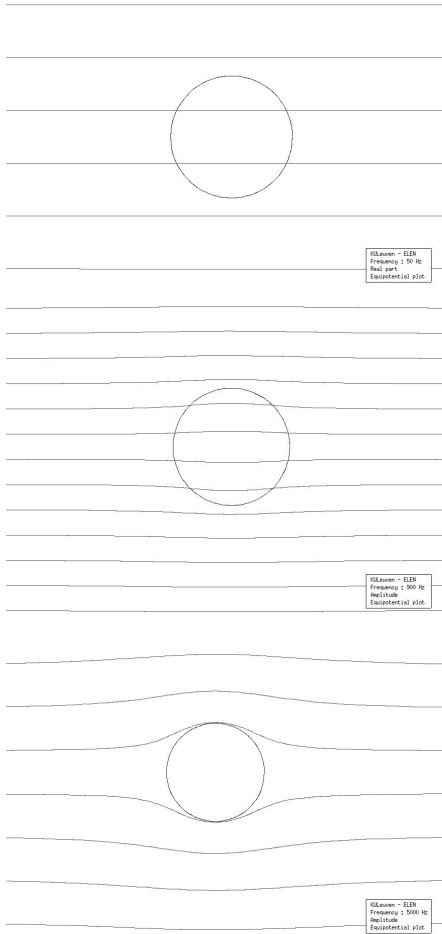


Fig. 1. Wire winding under externally imposed flux.

This eddy current loss value for wire windings of a given size can be calculated by a separate set of short FEM calculations in which a unity flux is imposed on a completely modelled individual wire (Fig. 1) of the exact size, followed by a loss calculation step. For flat wire conductors, small changes in the loss value might occur depending on their orientation towards the flux which requires additional calculations for different orientations towards the leakage flux.

This yields the graph of Fig. 2 in which the typical quadratic dependency can be recognised up to the point where the skin effect begins to occur. From this point on, a more linear characteristic is found. This graph describes the increased loss, compared to the loss due to external leakage fluxes at fundamental frequency.

b) 50 Hz unity flux, with wire diameter much smaller than skin depth.

c) 900 Hz unity flux: skin effect begins to occur.

d) 5000 Hz unity flux: full skin effect.

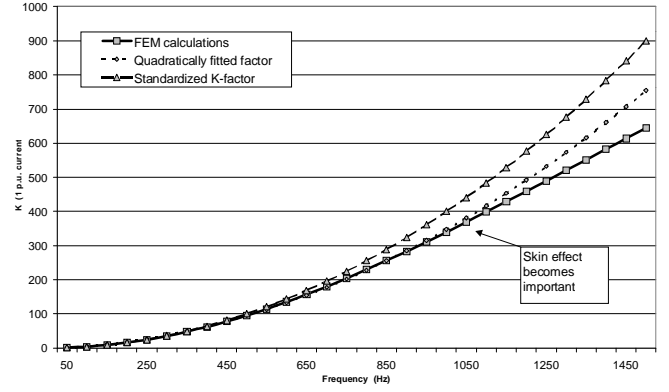


Fig. 2. Eddy current loss in the winding wire of Fig. 1 under unity flux as a function of frequency.

Together with the winding's filling factor, the function  $f_{ec}(f)$  plotted in Fig. 2 is used as a weighting function for the local eddy current loss in a wire winding element subject to the FEM-calculated leakage flux. This yields the formula (8) describing the loss in a wire winding, which is an adapted version of (7):

$$P_{wire} = R_{AC}(f)I^2 = \sum_e q_e = \sum_e \left( \frac{J^2}{\sigma} + f_{ec}(f)B^2 \right) \Omega_e \quad (8)$$

### C. Coupling to the thermal field

One has to be careful to take the correct electrical conductivity in (4), (6) and the loss formulae (7-8), since this material parameter depends on the local temperature:  $\sigma(T)$ . To determine this temperature, the thermal field associated with the local loss values is computed by (9), extended with appropriate thermal boundary conditions, such as convection.

$$\nabla \cdot (k \nabla T) - \rho c \frac{\partial T}{\partial t} = -q_e \quad (9)$$

with  $\lambda$  thermal conductivity  
 $\rho$  mass density  
 $c$  heat capacity

Particular attention has to be paid to the modelling of the insulation layers in the FEM models. Special types of elements are proposed for this purpose [9] and used in this approach.

Likewise, the magnetic equation is coupled to the thermal equation by the function describing the temperature dependency, expressed by factor  $\alpha$ , of the electrical conductivity:

$$\sigma(T) = \frac{\sigma_0}{1 + \alpha(T - T_{ref})} \quad (10)$$

The solution of this coupled problem set may be obtained by simply iterating over the non-linear set of steady-state equations (6) and (9) without the time derivative, using a block Gauss-Seidel algorithm [10]. Often the convergence of this approach is slow and difficult, especially when troublesome large conductive areas such as foil cross-sections are involved. In these regions, serious temperature gradients of up to a difference of 50°C between the hottest point and the average value can be identified. These gradients destabilise the numerical solution process. If convergence is problematic, other iteration methods may be required, such as:

- *Full coupled Newton-Raphson method* : In this case it is necessary to derive all partial derivatives in the Jacobian matrix. This can become a difficult approach when different meshes are used and advanced loss calculation techniques are required [9,10]. Moreover, since the thermal and magnetic material parameters differ several orders, the coefficients in the large Jacobian matrix show several orders of difference as well. Therefore, in practice the Jacobian equation usually is ill-conditioned and demands for a robust, but expensive solver such as GMRES. Fortunately this approach can be simplified by employing differences to obtain the (approximate) Jacobian.
- Calculate by means of a more stable *transient solution process* instead of iterating on the steady state equations: In the FEM equations, the presence of a transient term containing the time derivative, has a stabilising effect (strong diagonal dominant matrices). It can be observed that the transient calculation process converges better than the direct steady-state approach. However, it is not possible within a reasonable time to calculate the full sinusoidal waveforms based on equations (4) and (9). The only solution of interest here is in fact the slow dynamic change of the fields, induced by the slow time constants related to the temperature changes. This evolution can be extracted by solving a newly derived adapted formulation of (4) in which only the time evolution of the field amplitude and its phase are calculated. This novel formulation is obtained by replacing the constant complex phasor in (5) by a time dependent complex phasor in (11), yielding the time dependent complex equation (12):

$$A(t) = A(t) \cdot \exp(j\omega t) \quad (11)$$

$$\nabla \cdot (\nu(A) \nabla A) - \sigma \left( j\omega A + \frac{\partial A}{\partial t} \right) = -J_s \quad (12)$$

## V. MEASUREMENT APPLICATIONS

### A. Single-phase wire winding transformer

The additional loss factor of a commercial, single-phase dry-type transformer (1.6 kVA, 230/400 V) with a wire winding on the primary and secondary is determined by

measurements. During the short circuit tests, the transformer is supplied by a power amplifier. As predicted theoretically, the resulting K-factor follows the harmonic order quadratically (Fig. 3).

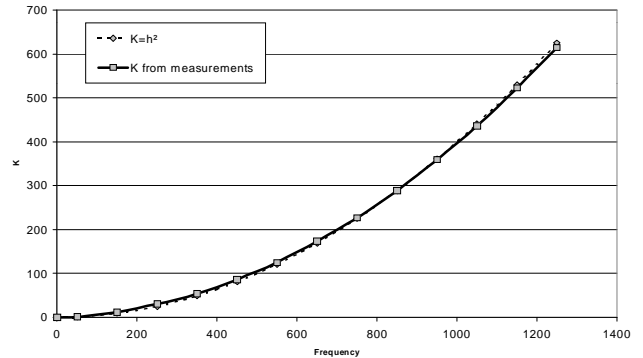


Fig. 3. Additional loss factor of a wire winding transformer.

### B. Three-phase transformer with low voltage foil winding and high voltage wire winding

A dry type, three-phase distribution transformer (30 kVA, 400 V/3 kV, connection Yy) was experimentally assessed as well. A high-speed synchronous generator was used as the three-phase supply. The low voltage winding of the transformer consists of a fifty layer foil winding in which a significant current redistribution was found. The outer high voltage winding is constructed as a conventional wire winding.

Due to the presence of the foils, the frequency dependency of the eddy current and stray field losses is not quadratic (Fig. 4), as is found in the shape of the additional loss factor curve. When fitted, the exponent of the curve is 0.74, so a quadratic 'K-factor' relation is not appropriate for this type of transformer. The numerical values of the factor are generally lower than the values obtained with corresponding wire wound transformers.

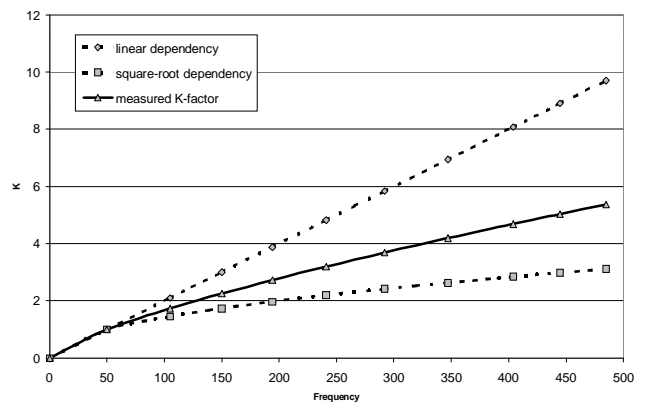


Fig. 4. Additional loss factor of a three phase foil winding transformer.

## VI. DESIGN APPLICATION USING FEM SIMULATIONS

The transformer is designed as an alternative for the single-phase transformer in V.A. It contains with a low voltage foil winding and high voltage wire winding. The method is applied to a FEM model of this small single-phase transformer. The foils have to be fully discretised by triangular finite elements in order to be able to model the internal eddy current distribution with the necessary accuracy [10]. The strands in the high voltage wire winding are not modelled individually. The stray and eddy currents losses occurring there are computed by different FEM models with individual strands in an external (leakage) field. These values are superposed in a post-processing step. A detail of the model is shown in Fig. 5.

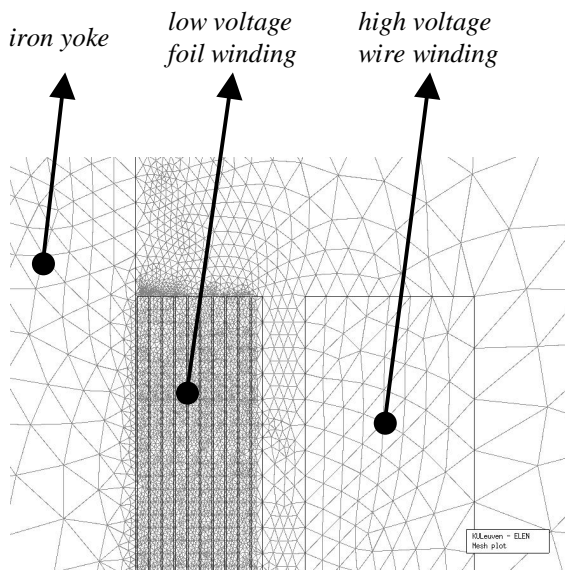


Fig. 5. Detail of the finite element mesh, used to compute the load losses at harmonic frequencies.

The FEM model allows determining the load losses for the different foil layers. Fig. 6 illustrates that layers close to the core show an almost linear frequency dependency, whereas the outer layers rather show a square-root like characteristic. This behaviour can be explained by the shielding effects, leading to stronger internal eddy currents, of the inner foil layers towards the enforced leakage flux. A graph such as Fig. 4 is obtained by appropriately adding and averaging the curves in Fig. 6.

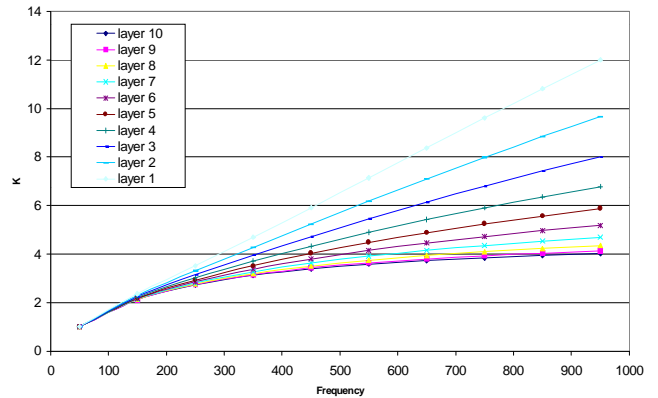


Fig. 6. Additional loss factor obtained from FEM simulations for the different foil layers; the lower curves originate from the outer layers.

## VII. CONCLUSIONS

A practical method to determine the frequency-dependent additional loss factor for transformers, subject to a harmonically distorted load current, is presented. Only short circuit tests at harmonic frequencies have to be performed experimentally or simulated.

The approach is applied to a transformer with a conventional wire winding, showing a quadratic frequency dependency. The application to a transformer with the low voltage winding manufactured as a foil winding showed that the K-factor can not be calculated with quadratic, but with rational exponents. With this approach in the design stage, using field simulations based on the finite element method, the losses in the different foil layers in a low voltage winding show different frequency dependencies, ranging from linear to square-root characteristics. These losses are calculated with great detail to allow an accurate modelling of the eddy current losses in both foil as well as wire windings.

## VIII. ACKNOWLEDGEMENT

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## X. BIOGRAPHIES



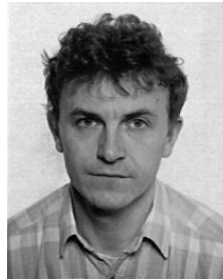
**Johan Driesen** (M'97) graduated in 1996 as Electrotechnical Engineer from the Katholieke Universiteit Leuven (KULeuven). Since 1996 he is a research assistant of the Fund for Scientific Research of Flanders (F.W.O.-VI.). He received the 1996 R&D-award of the Belgian Royal Society of Electrotechnical Engineers (KBVE) for his Master Thesis on power quality problems. He is currently working towards the Ph.D. degree in Electrical Engineering at KULeuven. His research topics are the finite element solution

of coupled thermal-electromagnetic problems and related applications in electrical machines and drives, microsystems and power quality issues.

J. Driesen is member of the Koninklijke Vlaamse Ingenieursvereniging (KVIV) and the IEEE.



**Bert Brouwers** (M '98) received his M.S in electrical engineering in 1998 from the Katholieke Universiteit Leuven (KULeuven) He is currently working as a research assistant at the KULeuven. His interests include automated measurement systems, and power quality related problems.



**Thierry Van Craenenbroeck** (SM '99) graduated in 1989 as electrotechnical engineer from the Katholieke Universiteit Leuven (KULeuven). From 1990 to 1992 he worked as lecturer at the Anton-de-Kom University of Surinam. In 1998, he obtained the Ph.D. degree in Electrical Engineering at KULeuven with a thesis on three-phase ferroresonance in distribution networks. He is currently a post-doctoral research fellow. His research interests include nonlinear and transient phenomena in power systems, and power

quality related problems.



**Kay Hameyer** (SM' 1995) received the M.S. degree in electrical engineering in 1986 from University of Hannover, Germany. He received the Ph.D. degree from University of Technology Berlin, Germany, 1992.

From 1986 to 1988 he worked with the Robert Bosch GmbH in Stuttgart, Germany, as a design engineer for permanent magnet servo motors. In 1988 he became a member of the staff at the University of Technology Berlin, Germany. From November to December 1992 he was a visiting professor at the COPPE Universidade Federal do Rio de Janeiro, Brazil, teaching electrical machine design. In the frame of a collaboration with the TU Berlin, he was in June 1993 a visiting professor at the Université de Batna, Algeria. Beginning in 1993 he was a scientific consultant working on several industrial projects. From 1993 to March 1994, he held a HCM-CEAM fellowship financed by the European Community at the Katholieke Universiteit Leuven, Belgium. Currently he is professor for numerical field computations and electrical machines with the K.U.Leuven and a senior researcher with the F.W.O.-V. in Belgium, teaching CAE in electrical engineering and electrical machines. His research interests are numerical field computation, the design of electrical machines, in particular permanent magnet excited machines, induction machines and numerical optimization strategies.

Dr. Hameyer is member of the Koninklijke Vlaamse Ingenieursvereniging (KVIV), the International Compumag Society and the IEEE.



**Ronnie J.M.Belmans** (S'77-M'84-SM'89) received the M.S. degree in electrical engineering in 1979 and the Ph.D. in 1984, both from the Katholieke Universiteit Leuven, Belgium, the special Doctorate in 1989 and the Habilitation in 1993, both from the RWTH Aachen, Germany.

Currently, he is a full professor with the K.U. Leuven, teaching electrical machines and CAD in magnetics. His research interests include electrical machine design (permanent magnet and induction machines), computer aided engineering and vibrations and audible noises in electrical machines. He was the director of the NATO Advanced Research Workshop on Vibrations and Audible Noise in Alternating Current Machines (August 1986). He was with the Laboratory for Electrical Machines of the RWTH Aachen, Germany, as a Von Humboldt Fellow (October 1988-September 1989). From October 1989 to September 1990, he was visiting professor at the McMaster University, Hamilton, ON., Canada. He obtained the chair of the Anglo-Belgian Society at the London University for the year 1995-1996.

Dr. Belmans is a member of the IEE (U.K.), the International Compumag Society and the Koninklijke Vlaamse Ingenieursvereniging (KVIV).