Finite-Element/Equivalent-Circuit Two-Level Method for Magnetic Simulation

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INTRODUCTION

The arbitrary geometry of an electromagnetic device is discretised using non-structured and non-nested grids (Fig. 1). For technical devices as electric motors with winding slots, cooling channels and small air gaps, the coarsest finite element discretisation already requires a considerable number of elements. An exact solution of the corresponding finite element stiffness matrix will be very expensive. In this paper, a magnetic equivalent circuit is proposed as a small-sized and effective coarse representation of the magnetic field problem within a multilevel approach.

FINITE ELEMENT MODEL

The governing equation for magnetostatics is

$$-\frac{\partial}{\partial x}\left(\nu\frac{\partial A_z}{\partial x}\right)-\frac{\partial}{\partial y}\left(\nu\frac{\partial A_z}{\partial y}\right)=J_z\,, \qquad (1)$$

A_z and J_z are the z-components of the magnetic vector potential and the current density respectively. ν is the reluctivity of the ferromagnetic material. A finite element (FE) model is obtained by discretising (1) with linear triangular finite elements [1].

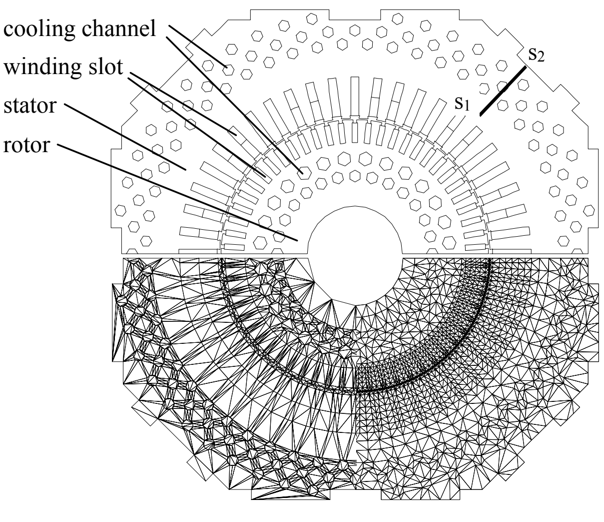

Fig. 1: Geometry, coarsest mesh and refined mesh of a four-pole, three-phase induction motor.

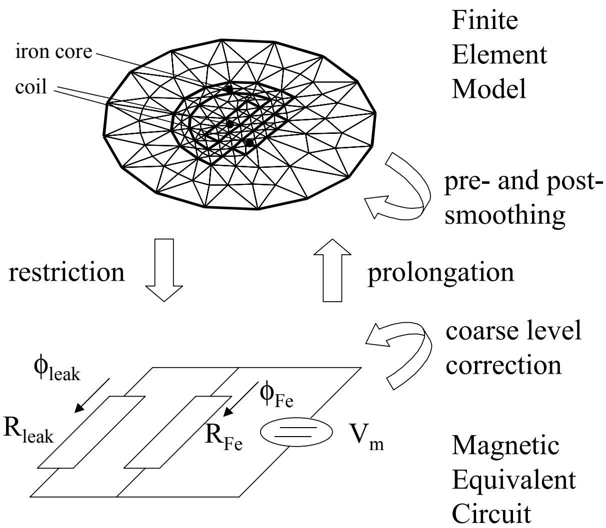

Fig. 2: FEM/MEC two-level approach applied to a benchmark inductor.

MAGNETIC EQUIVALENT CIRCUIT

The magnetic equivalent circuit (MEC) of a benchmark inductor model (Fig. 2) is built by distinguishing magnetic conductors, magnetic insulators and magnetomotive sources in the geometry. The reluctances of the magnetic paths are computed using the flux tube method [2]. An unknown loop flux is assigned to each independent loop in the circuit. To solve the circuit, Kirchhoff's voltage law and Hopkinson's law are expressed in terms of the unknown loop fluxes and the known magnetomotive sources.

FEM/MEC TWO-LEVEL METHOD

Both modelling techniques fit within a two-level hierarchy (Fig. 2). The magnetic vector potential distributions corresponding to the unknown loop fluxes, form the prolongation from the MEC to the FE model (Fig. 3). The restriction is defined as the adjoint of the prolongation [3]. As smoothers, damped Jacobi and Gauss-Seidel are applied. The MEC represents the jumps in the material properties and the far-field influences of the field. On the coarse level, an exact solution is performed.

APPLICATION

The two-level FEM/MEC method is applied to the benchmark model (Mod1) and the induction motor (Mod2) (Fig. 4). The sizes of the FE models and the corresponding MECs and the number of iteration steps of the novel approach compared to other stationary iterative solvers are collected in Table I. It can be noticed that the two-level FEM/MEC approach performs better compared to the pure relaxation schemes but is for this elliptic problem not competitive to an Algebraic Multigrid (AMG) technique [4]. The two-level method is also applied as a preconditioner for the Conjugate Gradient (CG) method.

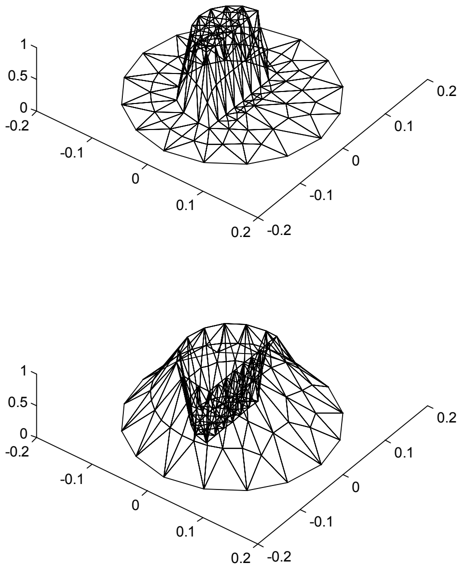

Fig. 3: Prolongations of ϕ_{Fe} and ϕ_{leak} .

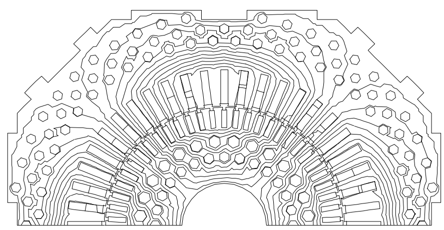

Fig. 4: Flux line plot of the induction motor.

Table I: Number of iteration steps: two-level FEM/MEC method compared to Jacobi (JAC), Symmetric Gauss-Seidel (SGS) and AMG.

	Mod 1	Mod 1 +CG	Mod 2 + CG
Size FE model	153	153	1951
Size MEC	2	2	53
JAC	768	27	2001
SGS	386	23	825
FEM/MEC + JAC	202	25	626
FEM/MEC + SGS	128	19	587
AMG			11

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