Coupled Thermo-Magnetic Simulation of a Foil-Winding Transformer connected to a Non-Linear Load

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***Abstract* — Foil-winding power transformers contain compact sets of flat conductors having a large aspect ratio. Modelling the eddy currents in the individual currents is therefore difficult and requires fully meshed conductors, especially when the influence of the thermal field on the local conductivity is considered. Coupled 2D magnetic and thermal models of a 30 kVA transfomer having 50 secondary foils are presented. In the thermal model, thin layer elements or equivalent anisotropic materials are used to model the insulation material between the conductors. The coupled problem is solved for the steady-state and transient case. A novel method will be introduced to solve the transient coupled problem using time steps, much larger than the magnetic subproblem's time constants.**

***Keywords* — Eddy currents, electro-thermal effects, finite element method, loss calculation, transformers**

I. Introduction

Power Quality problems such as power system harmonics caused by power electronic loads have a damaging impact on transformers. Voltage harmonics cause a limited rise in the iron losses, but the nowadays occurring substantial current harmonics may trigger dangerously high eddy currents. The necessary derating of such devices is performed using a '*K*-factor' [1], which can be accurately derived for wire winding transformers. Unfortunately this '*K*-factor' is difficult to determine for foil-winding transformers [2]. This paper reports on methods to perform this task using coupled thermal-magnetic transient and steady state finite element simulations.

III. Coupled Transient & Static FEM Techniques

A. Magnetic equation

The 2D transient magnetic equation written in terms of the magnetic vector potential is [3]:

$$\nabla \cdot (\upsilon \nabla (A)) - \sigma(T)\frac{\partial A}{\partial t} = -\sigma(T)V \tag{1}$$

with:
- A magnetic vector potential
- υ magnetic reluctivity tensor
- σ electrical conductivity
- V source voltage

The transient FEM simulation of the equation requires a time-stepping method with steps at least smaller than a half period. The time span to be simulated is very large, due to the thermal time constants. This will lead to simulations lasting a vast time. Thus a new method to simulate at time intervals of the size of the thermal time step needs to be developed.

It is possible to obtain a more efficient transient-type solver by assuming the solution can be written in the following complex form, with ω the field pulsation:

$$A(t) = \underline{A}(t)\cdot e^{j\omega t} \tag{2}$$

This is an extension of the assumption behind the time-harmonic method [3], but now we assume the solution part in the complex phasor form changes in time. Equation (2) splits the fast dynamics at the studied frequency (the exponential terms) and the slow dynamics in the phasor. The phasor can be interpreted as complex 'envelope' of the quickly oscillating harmonic function. Filled in, in (1), this leads to (assume the source is written in the form of (2) as well):

$$\nabla \cdot (\upsilon \nabla (\underline{A})) - \sigma(T)\left(j\omega \underline{A} + \frac{\partial \underline{A}}{\partial t}\right) = -\sigma(T)\underline{V} \tag{3}$$

This equation is transformed into FEM equations using the Galerkin method. The time derivative is replaced by a finite difference with the Δt of the thermal equation:

$$\begin{aligned} &\vartheta\left(K_A + j\omega H_A + \frac{R_A(T(t_n))}{\Delta t}\right)\underline{A}(t_n) = \\ &(1-\vartheta)\left(K_A + j\omega H_A - \frac{R_A(T(t_{n-1}))}{\Delta t}\right)\underline{A}(t_{n-1}) \\ &+ \vartheta \underline{F}_A(t_n) + (1-\vartheta)\underline{F}_A(t_{n-1}) \end{aligned} \tag{4}$$

with:
- K FEM matrix associated with diffusion term
- H FEM matrix associated with harmonic term
- R FEM matrix associated with transient term
- F FEM vector associated with source term

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ϑ time weight

An extension with circuit equations is made, but not given due to space limitations. The stationary magnetic equation in the time-harmonic domain is obtained by omitting the pure time derivative.

B. Thermal equation

The transient thermal equation is [4]:

$$\nabla \cdot \left(k \nabla (T)\right) - \rho c \frac{\partial T}{\partial t} = -q(A, T) \tag{5}$$

with: K thermal conductivity tensor
ρ mass density
C specific heat
q loss term

This equation is extended with convection boundary equations and translated into standard FEM equations. The time derivative is replaced by a finite difference of the same type as in the transient magnetic equation:

$$\vartheta \left(K_T + \frac{R_T}{\Delta t} \right) T(t_n) = (1-\vartheta) \left(K_T - \frac{R_T}{\Delta t} \right) T(t_{n-1}) + \vartheta F_T(t_n) + (1-\vartheta) F_T(t_{n-1}) \tag{6}$$

C. Coupling

Equation (3) and (5) are coupled by to relations:

- *Material temperature dependence :*

$$\sigma(T) = \frac{1}{\sigma_{ref}\left(1 + \alpha_\sigma \left(T - T_{ref}\right)\right)} \tag{7}$$

- *Loss calculation* (mainly joule and eddy current losses) :

$$q(A, T) = \frac{1}{\Omega_e} \iint_{\Omega_e} \sigma (V - j\omega A)^2 \, d\Omega \tag{8}$$

D. Iteration algorithm

The non-linear coupled transient equations (4)/(6) and the coupling equations (7)/(8) are calculated using a predictor-corrector-like algorithm in every time step *n*:

PREDICTOR:
Calculate σ^1, *using (7)*
Calculate $\underline{A}^1(t_n)$, *using (4),* (ϑ *may be 0)*
Calculate q^1, *using (8)*
Calculate $T^1(t_n)$, *using (6),* (ϑ *may be 0)*

CORRECTOR: (until convergence)
Calculate σ^{i+1}, *using (7)*
Calculate $\underline{A}^{i+1}(t_n)$, *using (4),* ($\vartheta \neq 0$)
Calculate q^{i+1}, *using (8)*
Calculate $T^{i+1}(t_n)$, *using (6),* ($\vartheta \neq 0$)

Hence the algorithm will consist of several loops: the time loop, the non-linear loop in the corrector and usually iteration loops to solve the linear subsystem. By rearranging the sequence of the calculation steps, parallelism can be brought into the algorithm.

III. SIMULATION MODELS

A. 3D Magnetic models (2D validation)

The leakage fields and three-limp core basically require a 3D model. However, the shape of the cross section of the core is almost round, yielding an almost axially symmetrical leakage field, which can be modelled with a 2D model.

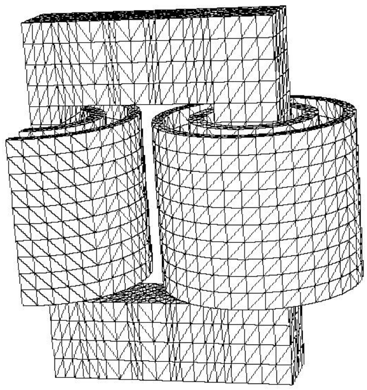

Fig. 1: 3D model used to determine the circuit parameters; the foil conductors are modelled as a solid block.

To check this approach a 3D model was constructed (Fig.1). The foil conductor packs are replaced with a solid block, since it is only meant to study the symmetry. Several axial slices through the coils were made and compared. The leakage field and internal eddy current distribution seemed to occur in two states: the part of the conductor within the core frame and the conductor section the core frame. The two field solutions are also found in the magnetic field calculated in the central plane through the core. The conductors in the 2D model with circuit equations are therefore assumed to have lengths proportional to the two distinguished coil sections.

B. 2D Magnetic analysis

2D FEM models allow the calculation of joule and eddy current losses. To model the eddy current distribution in the foils, a full discretisation of every foil is required (Fig. 2). Special attention has to be given to the surrounding air, since the leakage inductances are an important part of the short circuit impedance. A Kelvin transformation is used to model the air region.

A complete model is used. This is required since asymmetry will arise in the temperature dependent material properties due to the vertical thermal gradient arising in the device.

C. 2D Thermal FEM Modelling

The thermal behaviour of transformers is heavily dependent on the insulation, more in particular the insulation between the windings. These are very thin layers, compared to the thickness of the conductors. To model them, and to model other thermal contact resistances $\Re_c$ (for instance core air gaps), special thin layer elements can be used [5]. To be able to apply these element relations, the mesh has to be split at the edge on top of the thin layer. The element equations for the elements with common boundary Γ_{ef} are derived for different types of elements such as hierarchical elements, using a Galerkin method :

$$
\begin{aligned}
&-\iint_{\Omega_e} \nabla w_j \lambda_e \nabla T_i d\Omega - \iint_{\Omega_e} w_j q_e d\Omega + \int_{\Gamma_{ef}} w_j \frac{T_i}{\Re_c} d\Gamma = 0 \\
&-\iint_{\Omega_f} \nabla w_j \lambda_f \nabla T_i d\Omega - \iint_{\Omega_f} w_j q_f d\Omega + \int_{\Gamma_{fe}} w_j \frac{T_i}{\Re_c} d\Gamma = 0
\end{aligned}
\qquad (10)
$$

An alternative that does not introduce extra unknowns is to use equivalent anisotropic materials. The thermal conductance in the radial direction of the foil is almost identical to the insulation conductance. In the axial direction, the copper or aluminium value is dominant. The equivalent thermal capacitances and mass densities are calculated using the relative volume ratios.

The accuracy of the convection boundary condition has a large influence on the obtained thermal field. The convection parameter for the side walls has to be a function of the height to account for the rising hot air. This will cause a higher temperature in the upper part.

IV. Application results: 30 kVA transformer

A 30 kVA transformer with 50 foil conductors in its secondary winding (closest to the core) is modelled in the described way. The magnetic mesh (detail in Fig. 2) is made using *h*-adaptive refinement after domain based error estimation. The real component of the magnetic solution of the simulated short-circuit test is shown in Fig. 3. Fig. 4 shows a detail of the leakage field.

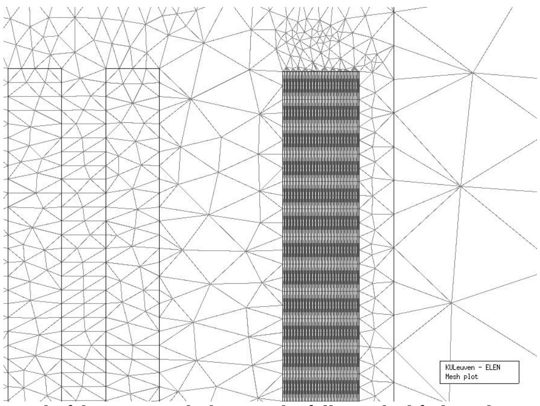

Fig. 2: Detail of the FEM mesh showing the fully meshed foil conductors.

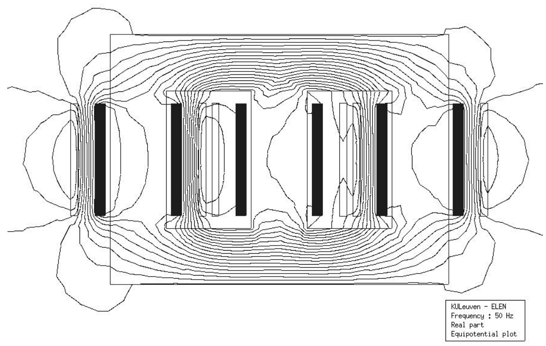

Fig. 3: Real part of the final magnetic field solution of a simulated short circuit test.

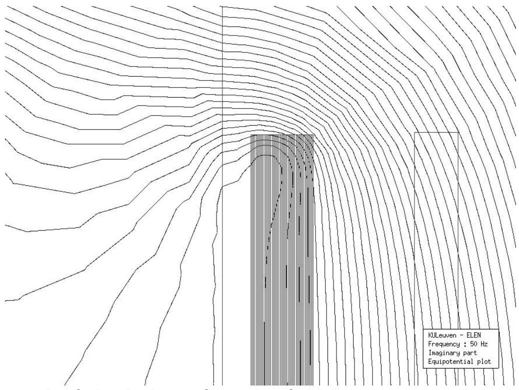

Fig. 4: Detail of the leakage flux; the field lines passing through the foil conductors in the top the coil are associated with additional eddy currents and losses at that location.

The short-circuit test simulation is used to determine the losses in the windings at rated conditions. Due to the low core flux, saturation effects are almost inexistent, simplifying the magnetic model. Clearly the foils closest to the core produce more eddy currents (cut by more field lines) and shield the outer foils. This effect can be seen as well in the current density plot of Fig. 5. The apparent discontinuities in the graphs are caused by the FEM discretisation.

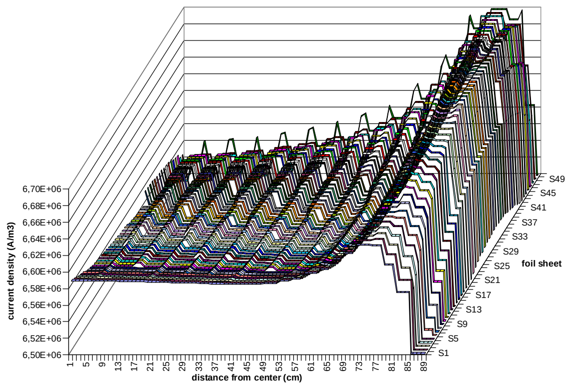

Fig. 5: Current density distribution in the 50 secondary foils, outside the core frame. Though there is a slight difference between the upper part and lower part of the coil, only the density in the (hotter) upper part is plotted; the foils closest to the core, having more eddy currents, are plotted in the background.

The surrounding air and the air between the winding packs is replaced by appropriate convection boundary conditions in the thermal mesh with thin layer elements or anisotropic maerials between the foils. The model is shown in Fig. 6. To transfer the loss data from the magnetic solution to the thermal mesh and to feed back the thermal data, fast projection algorithms are used, as well as to transfer the material updates. A detail of the solution is shown in Fig. 7, clearly indicating the location of the hot spot in the top of the foil winding coil.

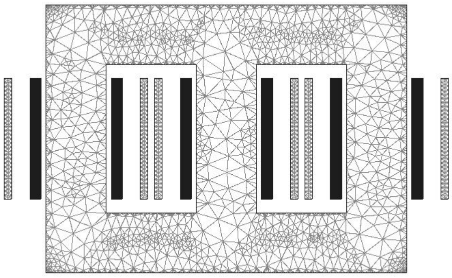

Fig. 6: Mesh used for thermal model; thin layer elements are put between the individual foils or anistropic material is used.

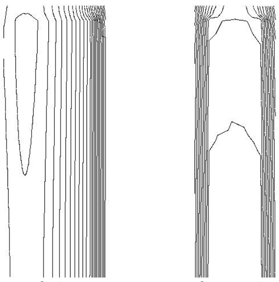

Fig. 7: Isothermal lines of the upper part of a coil set; on the left the foil conductor; on the right the wire coil. The hot spot is visible in the top of the foil pack.

Simulations for different harmonic frequencies followed by post-processing of the losses allows to determine the additional losses due to power system harmonics. With this information, the *K*-factor can be determined [6]. This is accomplished by simulating the coupled problem solution of a short-circuit test involving a set of harmonics, thus a set of equations such as (4). The loss integral (6) is extended, allowing the determination of the relative amount of additional losses due to current harmonics. To determine these losses, the steady state version of the coupled iteration algorithm is used.

The transient coupled algorithm is used to study the heating effects under temporarily higher harmonics loads: in this way it can be studied quickly the transformer will be damaged. To validate the transient method, measurements were made and compared with simulations. Fig. 8 demonstrates that there was a good correspondence. The difference between the steady state temperature in the measurements and the simulation is explained by the difficult to model natural convection cooling, especially for the coil parts within the yoke. The time constant of the simulation is a bit lower, which can be explained by the simple convection model as well.

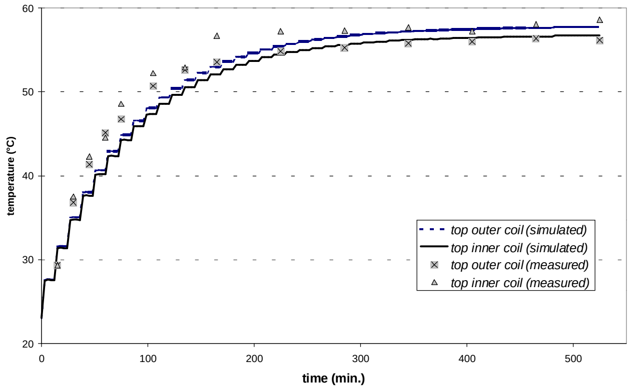

Fig. 8:.Comparison of measured and calculated heating of the transformer in a short circuit situation. The temperature near the tops on the outer side is measured optically.

The measured and simulated currents were close as well. The current magnitude dropped 1.5% for reason of higher resistance when hot, but the current phase angle changed 3% from the cold to the hot state because of the larger losses.

CONCLUSION

Practical static and transient coupled magnetic-thermal simulations of transformers having foil-type windings are presented. To be able to compute the magnetic field evolution, a novel transient method is derived, simulation the change of the 'solution envelope' with time stepping at the pace of the thermal problem.

2D magnetic FEM analysis with fully meshed foil layers is coupled to 2D thermal FEM models with thin layer elements or anisotropic materials to model the inter-foil insulation layers. The results can be used to determine the transformer's *K*-rating.

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