Optimization of Radial Active Magnetic Bearings Using the Finite Element Technique and the Differential Evolution Algorithm

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Abstract—An optimization of radial active magnetic bearings is presented in the paper. The radial bearing is numerically optimized using Differential Evolution – a stochastic direct search algorithm. The nonlinear solution of the magnetic vector potential is determined using the 2D finite element method. The force is calculated by Maxwell's stress tensor method. The parameters of the optimized and non optimized bearing are compared. The force, the current gain, and the position stiffness are given as functions of the control current and rotor displacement.

Introduction

The design of Active Magnetic Bearings (AMBs) [1] is expected to satisfy the static and dynamic requirements in the best possible way. It can be found either by experience and trials or by applying numerical optimization methods. AMBs are nonlinear systems. The dependency of the objective function and its gradients from the design parameters is unknown. For the optimization of such constrained, nonlinear electro–mechanical problems, the use of stochastic search methods in combination with the Finite Element (FE) analysis is recommendable [2].

In this paper the numerical optimization of radial AMB using Differential Evolution (DE) [3] is presented. It is the aim to achieve maximum force at a minimum mass of the entire construction. The objective function is evaluated by FE–based 2D calculations. The optimization has been performed in a special environment tuned for FE–based numerical optimizations [4]. The linearized equations are applied to compare the performance of designs prior and after the optimization.

Radial active magnetic bearing

The voltage balance in the coil of an electromagnet is described by (1)

$$u = Ri + L\frac{di}{dt} + k_u\frac{dx}{dt} \qquad (1)$$

where u is the voltage, i the current, R the Ohmic resistance, L the inductance, k_u the coefficient of induced voltage, and $\frac{dx}{dt}$ the derivative of the rotor displacement in the axis x. The resultant force of two electromagnets located at the opposite sides of the rotor in the axis x (legs 3, 4 and legs 7, 8 in Fig. 2), linearized about operating point x_0, i_{p0}, is given by (2).

$$F(x, i_p) = F(x_0, i_{p0}) + k_i(i_p - i_{p0}) + k_x(x - x_0) \qquad (2)$$

k_i is the current gain, k_x the position stiffness, i_p the control current, and $F(x_0, i_{p0})$ the force in the operating point x_0, i_{p0}. The same bias current i_b is supplied into the coils of both opposing electromagnets. Force control is done by adding the control current i_p into the coil of one electromagnet and subtracting it in the coil of other one. The motion of the mass point with the mass m between two electromagnets is described by (3).

$$F = m\frac{d^2x}{dt^2} \qquad (3)$$

One axis of the radial AMB is mathematically described by the pair of equations (1), by (2) and (3).

Optimization

The optimization of the radial magnetic bearing is briefly described in the following six steps:

- **Step 1:** The geometry of the bearing is described parametrically and the initial parameter values are determined by a first analytical design (Fig. 2).
- **Step 2:** The new parameter values are determined by DE [3]. The electromagnets in the y axis are supplied by the current i_b (legs 1, 2 and legs 5, 6 in Fig. 2), while the electromagnets in the x axis are supplied by the currents $i_b + i_p$ (legs 7, 8) and $i_b - i_p$ (legs 3, 4) at $i_p = i_b$.
- **Step 3:** The bearing geometry, the material, the current densities, and the boundary conditions are defined. The procedure continues with **Step 2** if the parameters of the bearing are outside the geometrical constraints.
- **Step 4:** First the mesh is generated. Then the nonlinear solution of the magnetic vector potential is determined using the conjugate gradient algorithm and Newton-Raphson's algorithm.
- **Step 5:** The force is calculated by Maxwell's stress tensor method. A contour along the center of the air gap is used as integration path
- **Step 6:** The objective function and the penalties are found empirically and described by (4). Their values are determined from **Step 3** through **Step 5**.

$$q = \frac{mF_0}{Fm_0} + p_1 + p_2; \qquad \begin{array}{lll} p_1 = \frac{F_0}{F} & \text{if} & F < F_0 \\ p_2 = \frac{m}{m_0} & \text{if} & m > m_0 \end{array} \qquad (4)$$

m_0 and F_0 are the initial mass and the initial force of the bearing. m and F are the mass and the force at instantaneous parameter values. p_1 and p_2 denote the penalties.

The value of the objective function is minimized in the optimization procedure. The optimization proceeds with

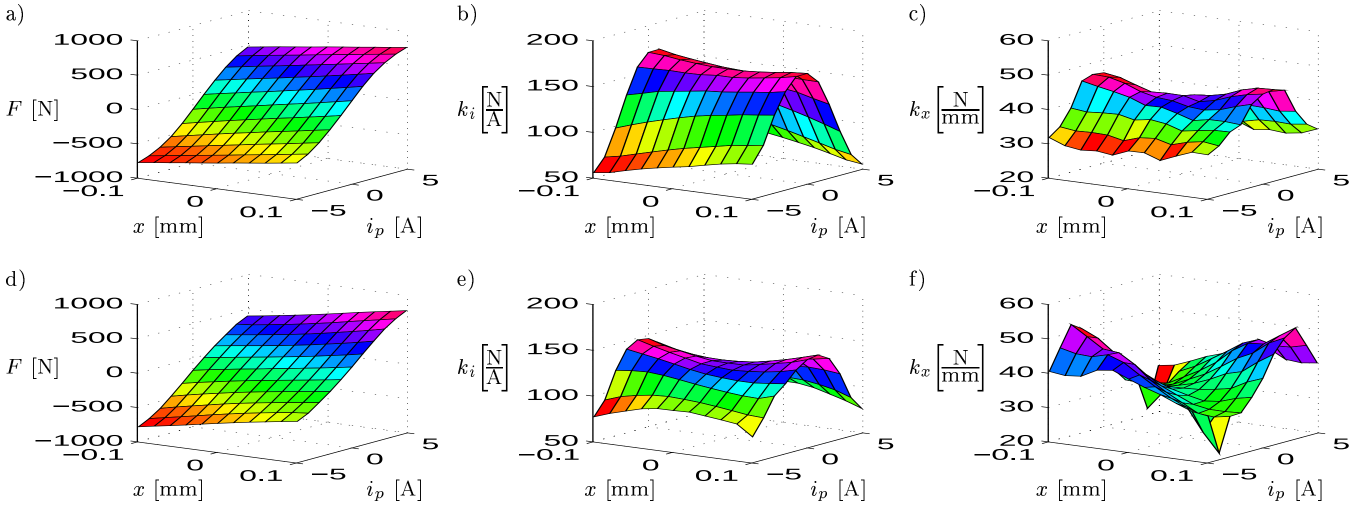


Fig. 1. Force F, current gain k_i and position stiffness k_x calculated in different operating points $i_p = i_{p0}$, $x = x_0$; The optimized bearing: a) force F, b) current gain k_i, c) position stiffness k_x; The non–optimized bearing: d) force F, e) current gain k_i, f) position stiffness k_x;

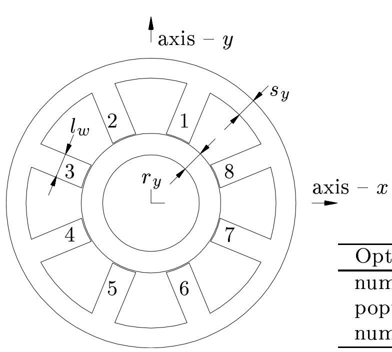
Step 2 until a pre-set minimum parameter variation or a maximum number of evolutionary iterations are reached.

Results

The optimization has been performed in the operating point $i_p = i_b = 5$ A, $x = 0$ mm and $y = 0$ mm. The data of the non–optimized and of the optimized bearing are given in Table I. The bearing geometry and the optimization data are given in Fig. 2.

TABLE I. Data of the initial and of the optimized design

data	parameter	initial	optimized
stator yoke	s_y [mm]	8.5	7.2
rotor yoke	r_y [mm]	9.0	7.8
leg width	l_w [mm]	10.0	9.0
bearing length	l [mm]	53.0	56.3
bearing mass	m [kg]	2.691	2.688
force	F [N]	580.01	629.74
objective function	q	1	0.92



Optimization data	
number of parameters	4
population size	20
number of iterations	60

Fig. 2. The bearing geometry and the optimization data

The values of the force F, the current gain k_i and the position stiffness k_x (equation (2)) calculated for the optimized and for the non–optimized bearing by **Step 3** through **Step 5** are given for different values of the control current $i_p = i_{p0}$ and rotor displacement $x = x_0$ in Fig. 1.

Conclusion

The paper describes the optimization of a radial AMB and the determination of the bearing model parameters linearized about various operating points. It has been shown that the use of optimization methods in combination with the FE calculations can increase the maximum bearing force at an unchanged mass and a negligible increase of magnetic nonlinearities. The values of the force, position stiffness and current gain in different operating points have been determined using FE analysis tools. These results enable the evaluation of the robustness of the control algorithm. Moreover, they can be approximated by a continuous function, which is further used for the linearization in the entire operating range, and altogether applied in the synthesis of the nonlinear bearing control. The presented results have been partially verified by measurements performed on a prototype.

References

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