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Adaptive Coupling of Differential Evolution and Multiquadrics Approximation for the Tuning of the Optimization Process

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Abstract **– Recently, the combination of global convergent stochastic search methods with approximation schemes based on radial basis functions has been introduced.[1,2] This paper presents a new approach: instead of a procedural sequencing of approximation algorithm and optimization algorithm, the presented optimization scheme is characterized by a direct and adaptive coupling of both algorithms. An approximation of the feasible space is constructed and updated during the progress of the evolutionary search. If the approximation fulfils certain accuracy-criteria, the evolutionary search algorithm starts sampling the approximation (indirect search) instead of directly sampling the objective function. This can lead to a significant reduction of function calls, which is desirable if the function evaluation is computational expensive (e.g. involving finite element analysis steps).**

INTRODUCTION

In direct search methods, the optimization algorithm samples the objective function directly, in indirect methods the optimization algorithm is applied to an approximation of the N-dimensional feasible surface. A trial on a fitted surface is computationally comparatively cheap if it replaces, e.g. a finite element analysis [4]. In the classical Response Surface Methodology (RSM) only one global polynomial is fitted. It becomes an optimization method if the following successive steps are performed until an accuracy criteria is met: sampling the feasibility space, constructing an approximating, finding the optimum on the approximation, construct a new approximation closer to the optimum found. The Generalized RSM (GRSM) applies the same methodology as the RSM, except of using radial basis functions to construct the approximations. The presented new scheme does not have a well defined sampling grid, it rather uses the sampling points of the evolutionary search to construct and update an approximation. Instead of constructing a highly accurate approximation in each successive step, the approximation is gradually improved during the course of the evolutionary search.

MULTIQUADRIC APPROXIMATION

The new feature of the GRSM is the usage of multiquadrics to achieve a response surface including multiple minima [1,2]. The GRSM uses approximations of the objective function at any point $\mathbf{x}_i$ of the form:

$$f(\mathbf{x}_i) = \sum_{j=1}^{M} c_j h\left(\left\|\mathbf{x}_i - \mathbf{x}_j\right\|\right) \qquad (1)$$

with c_j the approximation coefficients, M the number of experiments and a possible radial basis function $h(\|\mathbf{x}-\mathbf{x}_j\|)$ chosen to be:

$$h\left(\left\|\mathbf{x} - \mathbf{x}_j\right\|\right) = \sqrt{\left\|\mathbf{x} - \mathbf{x}_j\right\|^2 + s} \qquad (2)$$

The advantage of the multiquadrics approximation (on a multiminima objective function) arises with higher factorial designs (Fig. 1) or by accumulating the sample points in successive zooming steps. More detail of the original curve is present in the approximation, however at the expense of significantly more sample points when compared to single polynomial approximations (Fig. 1).

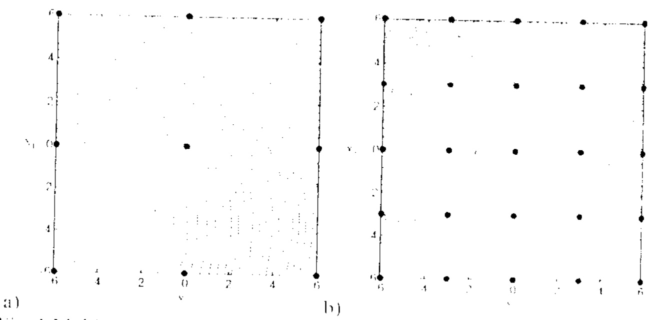

Fig. 1 Multiquadric approximation based on a) full 3^N and b) full 5^N factorial (25 samples) of an analytical test function given in [2]

DIFFERENTIAL EVOLUTION STRATEGY

Differential evolution (DE) [3,4], is a rather recent approach for the treatment multiobjective optimization problems. As is typical for stochastic search algorithms, differential evolution does not require any prior knowledge of the variable space, nor of the derivatives of the objective functions towards the design variables. DE generates new parameter vectors by adding the weighted difference between a defined number of randomly selected members of the previous population to another member. More detail on Differential Evolution will be presented in the final paper.

ADAPTIVE COUPLING

The basic steps of this new scheme are summarized in Fig. 2. Three levels of adaptivity determine the algorithm:

1. The contraction or zooming of the approximated region is adaptive to the progress of the optimization by considering a search space with a maximum radius of $k\delta^{(i)}$. The factor k is empirically chosen as:

$$k = k_1(\alpha) \cdot 10.0 \qquad (3)$$

with α the step length factor of the evolution strategy.

2. The acceptance of the approximation check is determined based on the variance of the objective function value of the iteration underlying the active approximation:

$$s(\mathbf{x}_i) < 0.1 \cdot \frac{1}{(m-1)} \sum_{k=1}^{m} \left(f(\mathbf{x}_k) - \frac{1}{l} \sum_{l=1}^{m} f(\mathbf{x}_l) \right)^2 \qquad (4)$$

with $m = 1(1)\lambda$, λ the population size of the evolutionary search.

3. The number of indirect search iterations only depends on the acceptance ratio achieved with the active approximation. A higher acceptance ratio allows a larger number of iterations on $f'(\mathbf{x})$. Tests have let to the following determination of the number of indirect search iterations n_h:

$$n_h = \left(\frac{\frac{n_a}{\lambda} - 0.5}{0.5} \right)^2 \cdot 10 + 1 \qquad (5)$$

with n_a the number of accepted trials per iteration.

The performance of the scheme is demonstrated on a test example (Fig. 1). Using the same DE-strategy settings as for the direct approach, only 43% of the direct objective function evaluations are required (Fig. 3, 4).

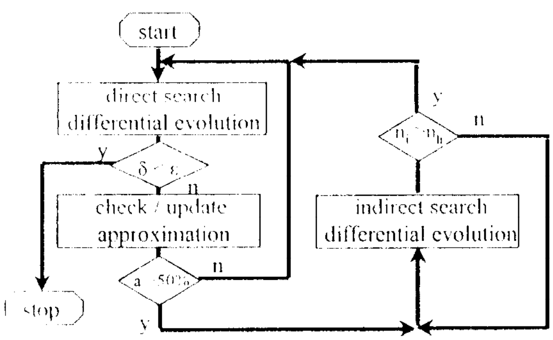

Fig. 2: Flowchart of the basic steps of the new method.

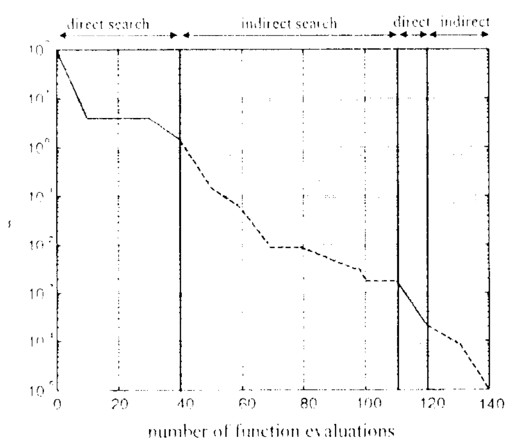

Fig. 3: Typical convergence of the error using the proposed method

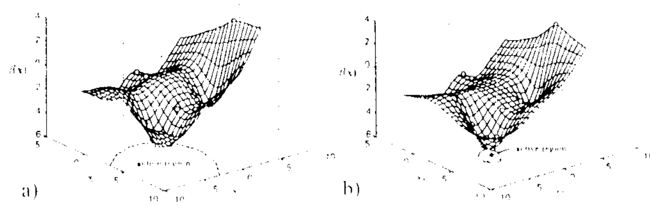

Fig. 4: Updated approximations after a) 2, b) 4 iterations of the differential evolution algorithm ($\lambda = 10$) with the active search region indicated.

A remarkable feature of this proposed algorithm is that the efficiency depends on the curvature of the objective function. Tests with simple second order feasible surfaces have shown reductions of objective function calls of up to 80%.

Further studies are necessary to increase the robustness of the algorithm by finding better criteria for the adaptivity of the algorithm. Some possible treatments of constraints have been outlined for the GRSM by Ebner [5]. So far, the new method performs with good results only for low dimensional optimization problems. This has been reported for the GRSM as well and remains an active research topic [5].

Conclusions

A new optimization scheme has been introduced, featuring an adaptive coupling of the differential evolution strategy and multiquadric function approximation. A remarkable reduction of supposed computationally expensive objective function calls is the result. The three levels of adaptivity provide control over progress dependent accuracy and computational expense of the method.

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