# Statistical Energy Analysis of Acoustic Noise and Vibration for Electric Motors: Transmission from Air gap Field to Motor Frame

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*Abstract*-A large number of vibration measurements are performed on the stator of a standard 5.2 kW electric motor. The rotor and the end-caps are removed. First, vibration measurements are performed on the stator without coils, i.e. consisting of the stator yoke (ferromagnetic iron) and the motor frame (cast iron) only. Second, vibration measurements are carried out on an identical stator with a standard coil system in the stator slots. Subsequently, a statistical energy analysis (SEA) is performed using these experimental data, in order to quantify the internal losses in the stator yoke and in the coils. The SEA also allows us to quantify the transmission of vibrations from stator yoke to motor frame (coupling loss factors), and to consider the influence of the presence of the coil system.

**Keywords:** statistical energy analysis, vibration analysis, acoustic noise control, electric motor design.

## I. INTRODUCTION

Not all aspects of acoustic noise and vibration analysis of electric machines can be captured by analytical or numerical techniques like Finite Element Analysis (FEA). Today's electric machine production still includes the assembly of a variety of components, using several different assembly techniques, e.g. shrink fitting the iron package into the stator frame. Several material parameters are not exactly known, especially for complex structures like the stator coil system, and for composite (anisotropic) materials like the iron package. Small deviations in dimension, even within production tolerance limits, can be important towards the acoustic behaviour of electrical machines [1].

Usually Modal Analysis is used to determine the eigenfrequencies and mode shapes (eigenvectors) of the stator structure. This can be done experimentally or using finite element models. Finite element models are confronted with the difficulties mentioned above, while a 2D model does not suffice to produce all mode shapes and eigenfrequencies that occur during operation [2]. While performing experimental modal analysis, the individual mode resonance peaks can usually be distinguished only up to 2500 Hz. For higher frequencies, the modal density becomes too large to fulfil the assumptions for modal analysis techniques. This high frequency range is however not unimportant, since power electronic switches will feed the electric machine with current harmonics up to 6000 Hz or more [2].

Statistical Energy Analysis (SEA) is designed to handle composed systems with a high modal density and a margin of uncertainty. Due to the constructional and material uncertainties of electric machines, and their high modal density (for high frequencies), SEA is a valuable tool in the acoustic noise research for electric machines. Measurement data enable us to quantify relevant phenomena like internal damping in the stator yoke, vibration transmission from yoke



Fig. 1. Main parts of electric machine stator involved in vibration transmission.

to frame, damping influence from the coils and sound power radiation from the frame [3][4]. Once the parameters of the SEA model of the stator are acquired through measurements, the model can be used to link the magnetic force excitation of the stator teeth due to the air gap field, to the stator surface vibration and noise radiation.

In general, SEA allows us to predict vibration amplitudes when the structure's parameters and power input are known, while the 'inverse' SEA enables us to determine the internal energy flow and system properties [5]. This kind of analysis is necessary for responsible mitigation of noise problems, e.g. using design optimisation with respect to component dimensions and damping localisation [6].

## II. VIBRATION TRANSMISSION AND ABSORPTION

Fig. 1 shows the main parts of the stator involved in the transmission of vibrations and the radiation of acoustic noise. Table 1 lists the main dimensions of the stator under consideration (Fig. 2). During operation, the stator teeth are excited by radial reluctance forces caused by the magnetic field in the air gap (Maxwell stress). The teeth transmit these forces to the yoke, leading to vibration. Since the longitudinal vibration of the individual teeth (2cm long) occurs at frequencies higher than 66 kHz [7], the vibration energy in the acoustic range can be considered to be transmitted without loss. The yoke transmits its vibrational energy to the machine's frame, which in turn is responsible for acoustic noise radiation off the frame surface. There is also vibration transmission through the mounting and the supporting structure, but this path is not considered here.

The teeth and the yoke are in close contact with the coil

TABLE 1. MAIN STATOR PROPERTIES	
stator length	420 mm
iron package length	155 mm
inner diameter cast iron frame	210 mm
inner diameter iron yoke	150 mm
stator teeth length	20 mm
iron yoke thickness	10 mm
number of stator slots	54 slots



Fig. 2 Standard 5.2 kW induction machine stator being measured

system, so it can be expected that a certain amount of energy is transmitted to (and absorbed by) the coil system. Vibration energy is absorbed due to structural damping (in the contact surfaces between coils and yoke and between yoke and frame) and material damping (iron package, coil system) [8].

## III. SEA MODEL OF THE STATOR

SEA deals with structures consisting of several interacting



Fig. 3. SEA model with three subsystems: coupling loss  $\omega \eta_{ij} E_i$ , internal loss  $\omega \eta_{ij} E_i$ , power input  $P_{in}$  and noise radiation  $W_{rad}$ .



Fig. 4. SEA model for measuring the stator: a) structure composed of three subsystems (rings), b) view inside the stator; cast iron frame ring sections on both sides of the iron yoke.

subsystems. Fig. 3 shows an SEA model consisting of three subsystems at vibration energy level  $E_i$ , the averaged total energy stored in subsystem *i* [3]. Averaging is performed in space, in time and over frequency bands. The parameters  $\eta_{ij}$ and  $\eta_{ii}$  are the *coupling loss factors* and *internal loss factors* respectively. The term  $\omega \eta_{12}E_1$  represents the energy transfer from subsystem 1 to subsystem 2 due to their coupling, while the term  $\omega \eta_{11}E_1$  represents the energy dissipated in subsystem 1 due to damping (for clarity, only a few terms are indicated in Fig. 3). The loss factors are a function of angular frequency  $\omega$  (third-octave band resolution). The resulting vibration energy transfer from subsystem 1 to subsystem 2 is given by the difference  $\omega(\eta_{12}E_1-\eta_{21}E_2)$ .

The Power Injection Method (PIM) [3] measures the power input into the subsystem under excitation and records the resulting vibrational energy level of all subsystems (both directly and indirectly excited). The subsystems are covered randomly with *N* accelerometers to probe the vibration energy level (Fig. 4b). The subsystem under excitation is excited at *N* randomly distributed locations using hammer or shaker. All subsystems are excited in turn, giving a set of  $S^2 N$  by *N* frequency response matrices, where *S* is the number of



Fig. 5. Frequency response function (FRF) of two representative points on the stator, indicating the modal density of the stator.

subsystems. The excitation and accelerometer positions are systematically adjusted in order to determine all relevant loss factors  $\eta_{ij}$ . The energy measure  $E_i$  is a spatial as well as a temporal mean of the surface vibration, yielding one parameter value per third-octave band.

In this case, the stator yoke, the machine frame and the coil system are the most obvious physical subsystems. When the stator yoke is taken as subsystem 1, the magnetic force excitation  $P_{in}$  is acting on system 1 alone. The other subsystems are excited indirectly through the coupling. When the frame is taken as subsystem 3, the energy radiated as acoustic noise  $W_{rad}$  can be incorporated in the model.

During vibration measurements, it is not possible to individually excite the three physical subsystems (yoke, frame and coils), since the coil system can only be reached in both end-winding regions and cannot be sufficiently excited on the parts embedded in the stator slots. Therefore, stator yoke and coil system are considered as one subsystem for the measurements.

All excitation (using shaker or hammer) is applied at the inner surface of the frame or on the teeth of the yoke. The inner surface of the frame cannot be reached in the region where the yoke iron is connected to the frame. Therefore, the frame is separated into two rings, one on each side of the yoke iron. This results in three subsystems (Fig. 4):

- ring 1: frame on left side of yoke,
- ring 2: iron yoke (and coil system),
- ring 3: frame on right side of yoke.

For the measurements on the stator without coils, subsystem 2 consists of the iron yoke only. These three subsystems are determined by the measurement conditions and are not to be confused with the three physical subsystems mentioned before. The general model shown in Fig. 3 can still be used. When all measurement data are acquired, subsystems 1 and 3 are reassembled to represent the cast iron frame as a whole. Here the analysis concentrates on quantifying the internal loss

factors as well as the coupling loss factors between the three ring subsystems (inverse SEA).

## **IV. MEASUREMENT SET-UP**

The stator is suspended in free-free condition. The vibration measurements on the stator without coils are performed in a frequency range from 10 Hz to 10 kHz. The measurements on the stator with coils reach from 10 Hz to 20 kHz. Due to the averaging of the results over third-octave frequency bands, the highest frequency where a parameter value is obtained is 6.3 kHz for the stator without coils and 12.5 kHz for the stator with coils.

The SEA analysis requires a sufficiently high modal density. From a previous modal analysis on the same stator, it was found that the modal density becomes sufficiently large for frequencies higher than 2.5 kHz. Fig. 5 shows a representative frequency response function (FRF) for two points on the stator, indicating the modal response of the structure. For frequencies below 2.5 kHz, the mode resonance peaks can be distinguished individually and modal analysis techniques are better suited. For frequencies higher than 4 kHz, the modal density has become too high to use modal analysis; this is the domain of SEA.

At first, hammer excitation proved to be inadequate to excite the structure sufficiently in order to measure the power input. This is due to the fact that the stator has a high mass density and a high thickness in comparison to the relatively thin-walled mechanical structures that are usually investigated. Therefore, shaker excitation needed to be applied, where 1 kg shakers were glued firmly onto the structure. Since the stator with coils could not be damaged, no glueing was allowed and hammer excitation had to be used. Several hammer-accelerometer configurations where tried out to simulate the use of an impedance head; the best result was obtained by hammering very close to the accelerometer position. This approximation of the PIM is usually not allowed, but proved adequate for this kind of structure.

The full set of measurements necessary to obtain the internal and coupling loss factors of both stators (with and without coils) requires 80 man-hours. The measurements were performed using a 10-channel acquisition system with commercially available SEA software.

## V. MEASUREMENT RESULTS: INTERNAL LOSS FACTORS

Each subsystem is excited while accelerometers are applied to the directly excited subsystem to determine internal loss, as well as the indirectly excited subsystems to determine coupling loss. Fig. 6 shows the internal loss factors of all three subsystems as a function of frequency, for both cases; with and without coils. The following conclusions can be drawn:

1. For frequencies below 2 kHz, the internal loss factors behave relatively jumpy due to the low modal density; this is the area of Modal Analysis rather than SEA.

2. The internal loss factor of the cast iron frame (dotted and double line) remains fairly constant as a function of frequency.

3. The internal loss factor of the yoke (solid and dashed line) grows larger for increasing frequency in the range 3 kHz to 8 kHz; the difference between low and high frequency is about 3 dB. The internal loss factor reaches a constant value for frequencies over 8 kHz.

4. Damping in the iron yoke is larger than in the cast iron frame. This difference grows up to 4 dB when coils are added.

5. The damping in the yoke with coils (solid line) is larger than the damping in the yoke without coils (dashed line); the difference is about 5 dB.

6. The frame and the yoke cannot be separated entirely; the frame's internal damping also rises when coils are added,



Fig. 6. Internal loss factors as a function of frequency (third-octave bands):

- iron yoke with coils (ring 2), solid line
- cast iron frame, stator with coils (ring 1 and 3), dotted line
- iron yoke, no coils (ring 2), dashed line
- cast iron frame, stator without coils (ring 1 and 3), double line



Fig. 7. Confidence interval on internal loss factor of iron yoke with coils (90% confidence).

although the coils are not physically in contact with the frame. This is caused by the choice of the three ring subsystems for the measurements: the middle part of the frame is constrained by the iron yoke, while the outer rings are free.

Since the SEA is a statistical method, all quantities obtained have to be interpreted as mean values for the thirdoctave band under consideration. The standard deviation on these mean values can be expressed using confidence intervals. Fig. 7 shows the internal loss factor of the yoke and coils subsystem and the corresponding limiting curves indicating the upper and lower bounds of the 90% confidence interval. Up to 8 kHz, the uncertainty on the internal loss factor is limited to 3 dB. For higher frequencies, the results are less precise since the high frequencies are more difficult to excite than the low frequencies, especially using hammer excitation.

### VI. MEASUREMENT RESULTS: COUPLING LOSS FACTORS

Fig. 8 shows the coupling loss factors between the two

subsystems (iron yoke and cast iron frame) as a function of frequency, for both cases (with and without coils). The following conclusions can be drawn:

1. For frequencies below 2 kHz, the coupling loss factors behave relatively jumpy due to the low modal density; this is the area of Modal Analysis rather than SEA.

2. The coupling loss factor from yoke to frame for the stator with coils (solid line) shows a minimum for 5 kHz. From 5 kHz to 12.5 kHz the coupling increases remarkably with 12 dB. This is an important argument against the tendency to neglect this frequency range in noise and vibration analysis for electric machines.

3. The coupling loss factor from yoke to frame for the stator without coils (double line) shows the same behaviour as the corresponding loss factor for the stator with coils, but the minimum occurs at a lower frequency, around 3 kHz.

4. The coupling from frame to yoke for the stator with coils (dotted line) roughly coincides with the curve for the inverse direction (from yoke to frame, solid line) from 3 kHz to 6.3 kHz. This means that both systems exchange vibrational energy equally in both directions. For higher frequencies, the coupling from frame to yoke decreases while the coupling from yoke to frame increases; now the energy transmission occurs mainly in the direction from yoke to frame.

5. The coupling loss from the frame to the yoke for the stator without coils (dashed line) shows the same tendency as the corresponding coupling loss for the stator with coils (dotted line). Again, the presence of the coil system causes a shift of 3 kHz before the curve starts to decrease.

All loss factors found are positive. The validity of the SEA measurements can further be estimated using the triple-product (Fig. 9). This check enables the quality of each coupling loss factor to be verified and compared. The factors



Fig. 8. Coupling loss factors for the three subsystems (third-octave bands):

- from iron yoke to cast iron frame, stator with coils, solid line
- from cast iron frame to iron yoke, stator with coils, dotted line
- from iron yoke to cast iron frame, stator without coils, *double line*
- from cast iron frame to iron yoke, stator without coils, dashed line



Fig. 9. Triple-product bar chart for the three subsystems.

$$\frac{\eta_{ij} \eta_{jk} \eta_{ki}}{\eta_{ii} \eta_{kj} \eta_{ik}}$$
<sup>(1)</sup>

are averaged for all possible combinations i, j, k and assembled into one matrix. The inverse of (1) has to be used when this ratio is larger than one. The closer the off-diagnal terms to unity, the better the quality of the coupling loss factor. The diagonal terms should be zero [9][10]. Fig. 9 shows that these requirements are met.

### **VII.** CONCLUSIONS

A standard electric machine stator is subjected to an extensive set of vibration measurements in order to quantify the internal and the coupling loss of vibrational energy absorbed in and transferred between the subsystems. This analysis is performed in the high frequency and high modal density range, where the Statistical Energy Analysis (SEA) is a useful and common tool in vibration analysis. The influence of the coils is quantified by performing the measurements both with and without a coil system in the stator slots. The parameters in the SEA model are determined, so that the resulting SEA model is complete and ready to be used to link the magnetic force excitation from the air gap to the acoustic noise radiated off the stator surface.

The internal loss factor of the cast iron frame is found to remain fairly constant as a function of frequency, while the internal loss factor of the iron yoke increases 3 dB in the range 3 kHz to 8 kHz. Vibration damping in the iron yoke is larger than in the cast iron frame (up to 4 dB in the presence of coils). Damping in the yoke with coils is about 5 dB larger than in the yoke without coils.

The coupling loss factor from iron yoke (with coils) to cast iron frame increases by 12 dB with increasing frequency from 5 kHz to 12.5 kHz. For 3 kHz to 6.3 kHz, the yoke and the frame exchange vibrational energy equally in both directions, but for higher frequencies, the energy transmission occurs mainly in the direction from yoke to frame.

### ACKNOWLEDGMENT

The authors are grateful to the Belgian "Fonds voor Wetenschappelijk Onderzoek Vlaanderen" for its financial support; Koen Delaere has a FWO-V scholarship. The authors thank the Belgian Ministry of Scientific Research for granting the IUAP No.P4/20 on Coupled Problems in Electromagnetic Systems. The research Council of the K.U.Leuven supports the basic numerical research.

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