TRANSIENT FIELD-CIRCUIT COUPLING BASED ON A TOPOLOGICAL APPROACH

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Introduction

The two-dimensional transient finite element method using a one-step time-stepping scheme deals with induced currents and saturation, and can be used to compute e.g. the behaviour of a three-phase induction machine. Moreover, when a moving-mesh model is used, i.e. using a moving band technique to move the rotor part of the finite element model relative to the stator, the effects of the rotation of the induction machine can be considered. The end-effects of the induction machine on the other hand, e.g. endwinding and ring impedance, have be taken into account by coupling an external electric lumped parameter model with the two-dimensional finite element model. Embedding both phenomena in the coupled system matrix is very attractive because of the linear nature of the coupling. As there are different possible coupling schemes [1], the question of a general and reliable coupling technique for transient finite element models arises. The choice of the electric circuit unknowns influences the matrix structure and system properties and is responsible for a significantly increasing complexity of the transient problem. Therefore the aim of this paper is to point out a framework for field-circuit coupling that is general, problem independent, effective and reliable. The method is illustrated by the calculation of an induction machine as end-effects play a rather important role in its analysis.

Transient Finite Element Analysis

In the two-dimensional transient finite element analysis, the magnetic vector potential is obtained by using a one-step time-stepping scheme [2]:

$$\left(\alpha\mathbf{K}+\frac{\mathbf{L}}{\Delta t}\right)\mathbf{A}_k+\left((1-\alpha)\mathbf{K}-\frac{\mathbf{L}}{\Delta t}\right)\mathbf{A}_{k-1} = \left(\alpha\mathbf{T}_k+(1-\alpha)\mathbf{T}_{k-1}\right). \tag{1}$$

$\mathbf{K}$ is the element coefficient matrix, $\mathbf{L}$ the stiffness matrix, $\mathbf{T}_{k-1}$ and $\mathbf{T}_k$ are the source vectors respectively at $t=t_{k-1}$ and $t=t_k=t_{k-1}+\Delta t$. Different difference schemes are obtained by changing the value of the parameter α in the recurrence relation between $\mathbf{A}_k$ and $\mathbf{A}_{k-1}$. $\alpha=1$ results in the backward difference Euler method. The Galerkin scheme is obtained for $\alpha=2/3$ and the Crank-Nicolson scheme for $\alpha=1/2$.

Solid and stranded conductors are distinguished to determine the source vector $\mathbf{T}_k$. The source vector $\mathbf{T}_{k-1}$ is easily determined as both current through and voltage drop over a solid or a stranded conductor are known at $t=t_{k-1}$.

$$\left(\alpha\mathbf{K}+\frac{\mathbf{L}}{\Delta t}\right)\mathbf{A}_k=-\left((1-\alpha)\mathbf{K}-\frac{\mathbf{L}}{\Delta t}\right)\mathbf{A}_{k-1}+\left(\alpha\begin{bmatrix}\mathbf{Q}_{sol} & \mathbf{P}_{str}\end{bmatrix}\begin{bmatrix}\mathbf{V}_{sol,k}\\ \mathbf{I}_{str,k}\end{bmatrix}+(1-\alpha)\mathbf{T}_{k-1}\right) \tag{2}$$

Solid Conductors

A solid conductor is a massive conductor and the voltage drop is constant along the section of the conductor in the two-dimensional model. The coupling with an external electric network requires the description of the solid conductor in terms of its total current $I_{sol,k}$ and $I_{sol,k-1}$, the voltage drop $V_{sol,k}$ and $V_{sol,k-1}$ and the magnetic vector potential $\mathbf{A}_k$ and $\mathbf{A}_{k-1}$ at respectively $t = t_k$ and $t = t_{k-1}$. The total current is

$$\alpha I_{sol,k} + (1-\alpha) I_{sol,k-1} = G_{sol} \left(\alpha V_{sol,k} + (1-\alpha) V_{sol,k-1} \right) - \sigma \frac{1}{\Delta t} \sum_{e=1}^{n_{e,sol}} \sum_{i=1}^{3} \left(A_{i,k} - A_{i,k-1} \right) \frac{\Delta_e}{3}, \tag{3}$$

$$\alpha I_{sol,k} + (1-\alpha) I_{sol,k-1} = G_{sol} \left(\alpha V_{sol,k} + (1-\alpha) V_{sol,k-1} \right) - \nu_0 \, \ell \frac{1}{\Delta t} \mathbf{Q}_{sol} \left(\mathbf{A}_k - \mathbf{A}_{k-1} \right), \tag{4}$$

with the conductance G_{sol} of the solid conductor, the electric conductivity σ, the active length ℓ of the magnetic model and the magnetic reluctivity ν.

Stranded Conductors

A stranded conductor is the approximation for a winding in which skin effect is negligible, i.e. **L** is omitted in (1). Instead of modelling all wires separately and connecting them in series, a constant current density is assumed across the section of the stranded conductor in the two-dimensional model. The voltage drop across the winding is computed as

$$\alpha V_{str,k} + (1-\alpha) V_{str,k-1} = R_{str} \left(\alpha I_{str,k} + (1-\alpha) I_{str,k-1} \right) + \frac{N_{str}\, \ell}{S_{str}} \frac{1}{\Delta t} \sum_{e=1}^{n_{e,str}} \sum_{i=1}^{3} \left(A_{i,k} - A_{i,k-1} \right) \frac{\Delta_e}{3}, \tag{5}$$

$$\alpha V_{str,k} + (1-\alpha) V_{str,k-1} = R_{str} \left(\alpha I_{str,k} + (1-\alpha) I_{str,k-1} \right) + \nu_0 \, \ell \frac{1}{\Delta t} \mathbf{P}_{str} \left(\mathbf{A}_k - \mathbf{A}_{k-1} \right), \tag{6}$$

with the resistance R_{str} of the stranded conductor, the number of turns N_{str} and the total cross-section S_{str}.

Inductors and Capacitors

Analogue to solid and stranded conductors, inductors and capacitors are distinguished. Equations (7) and (8) give respectively the voltage-current relation of an inductor and a capacitor.

$$\alpha V_{ind,k} + (1-\alpha) V_{ind,k-1} = L \frac{I_{ind,k} - I_{ind,k-1}}{\Delta t} \tag{7}$$

$$\alpha I_{cap,k} + (1-\alpha) I_{cap,k-1} = C \frac{V_{cap,k} - V_{cap,k-1}}{\Delta t} \tag{8}$$

Topological Description of the External Circuit

The nature of the electromagnetic coupled system is studied by tracing a tree through the circuit using a strict priority scheme (Table 1) [3, 4]. A distinction is made between *voltage driven* branches (priority > 0) and *current driven* branches (priority < 0). The tree will mainly be formed by voltage sources, solid conductors and capacitors whereas the most of the current sources, stranded conductors and inductors will belong to the cotree. Resistors have a neutral nature as they can equivalently be described in terms of resistances or conductances.

Table 1. Tree branch priorities.

Branch	Priority	Branch	Priority
Voltage source	+3	Inductor	-1
Solid conductor	+2	Stranded conductor	-2
Capacitor	+1	Current source	-3
Resistor	0		

The topological treatise is concluded by representing the fundamental loops and cutsets associated respectively with the links and tree branches in appropriate incidence matrices. The *fundamental loop matrix* $\mathbf{B}$ represents the incidences of the circuit branches to the fundamental loops [5]. The *fundamental cutset matrix* $\mathbf{D}$ represents the incidences of the circuit branches to the fundamental cutsets.

$$\mathbf{B} = \left[\mathbf{B}_T \;\vdots\; \mathbf{1}\right] \tag{9}$$

$$\mathbf{D} = \left[\mathbf{1} \;\vdots\; \mathbf{D}_L\right] \tag{10}$$

Hybrid Circuit Description

At this stage of the coupling process the unknowns of the electric network are tree branch voltages $\mathbf{v}_T$, tree branch currents $\mathbf{i}_T$, link currents $\mathbf{i}_L$ and link voltages $\mathbf{v}_L$. The behaviour of the network is fully described by the Kirchhoff voltage law relations for the fundamental loops, the Kirchhoff current law relations for the fundamental cutsets and the voltage-current relations of links and tree branches:

$$\mathbf{B}_T \mathbf{v}_T + \mathbf{v}_L = 0 , \tag{11}$$

$$\mathbf{D}_L \mathbf{i}_L + \mathbf{i}_T = 0 , \tag{12}$$

$$f_L\left(\mathbf{i}_L, \mathbf{v}_L\right) = 0 , \tag{13}$$

$$f_T\left(\mathbf{v}_T, \mathbf{i}_T\right) = 0 . \tag{14}$$

This is the matrix representation of the tableau analysis method for electric circuits. A hybrid circuit description is developed by replacing $\mathbf{v}_L$ in (11) and $\mathbf{i}_T$ in (12) by their corresponding branch voltage-current relations. This is not always possible (e.g. a stranded conductor as tree branch or a solid conductor as link), but the priority based tree construction enables the application of some partial transformations to overcome this [3, 4]. Multiplying the circuit loop equations with χ and the circuit cutset equations with $-\chi$ leads to a symmetric coupled field-circuit matrix.

$$\chi = \frac{\alpha \Delta t}{\ell \nu_0} \tag{15}$$

The cutset equations preserve the positive definiteness of the finite element matrix part. The loop equations are responsible for a negative definite diagonal block. As a result, the system is not suited for solving with the Conjugate Gradient (CG) algorithm. Instead, an algorithm for symmetric indefinite systems like Minimal Residual (MINRES), Generalised Minimal Residual (GMRES) or symmetric Quasi Minimal Residual (QMR) is used. It is possible to eliminate all loop equations, giving a positive definite system matrix. The fill-in in the finite element matrix part [6] causes the matrix-vector product, used by the CG algorithm, to be computationally expensive so that this approach has to be avoided.

Example: Calculation of an Induction Machine

Figure 1 shows the field plot of a 45 kW induction machine at rated load. Skewing is considered by putting the different slices in series by means of the external electric circuit model. Table 2 gives the results of the topological description of the external electric circuit model, resulting in 60 circuit equations in which solid and stranded conductors coexist.

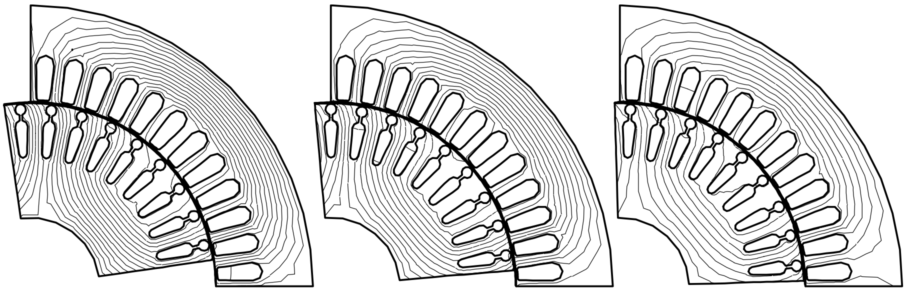

Fig. 1. Field plot of an induction machine at rated load (skewing is considered).

Table 2. Topological description of the external electric circuit model.

Number of branches	84	Number of tree branches	72
Number of stranded conductors	12	Number of links	12
Number of solid conductors	27	Number of circuit equations	60
Number of connected circuit parts	2		

Conclusion

The hybrid field-circuit coupling method offers a general and robust way to couple an electric lumped parameter model to a two-dimensional finite element model. The method adds a minimal amount of extra equations to the system matrix. Both stranded and solid conductors, arbitrarily connected in the network, fit in the description and matrix symmetry is retained. This coupling method is easily extended to non-linear circuits and circuits with dependent sources. However, the additional circuit equations may cause numerical instabilities in combination with the Crank-Nicolson scheme [7].

Acknowledgement

The authors are grateful to Dr. J. Harger of Schorch GmbH, Mönchengladbach, Germany for providing the necessary data of the studied induction machine, to the Belgian "Fonds voor Wetenschappelijk Onderzoek Vlaanderen" for its financial support of this work and the Belgian Ministry of Scientific Research for granting the IUAP No. P4/20 on Coupled Problems in Electromagnetic Systems. The Research Council of the K.U.Leuven supports the basic numerical research.

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