FEM Modelling of Saturated Magnetic Devices under a Non-Linear Load

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Finite element methods, eddy currents, loss calculation, coupled problems, harmonics

ABSTRACT

Magnetic devices such as transformers and chokes often operate under a non-linear load, resulting in harmonic currents and voltages, causing additional heating of the device and internal hot spots. Calculation schemes to model the eddy currents at different frequencies, coupled to thermal calculations by means of FEM-methods are presented here. The method is discussed at an application example in which the current redistribution and its consequences inside a foil winding transformer carrying current harmonics is studied.

1. INTRODUCTION

In electrical power systems an increasing amount of power electronics is introduced. These systems exist in every size ranging from lighting, switched power supplies for office equipment, frequency converters for adjustable speed drives and huge rectifiers for electrothermal applications. These loads behave non-linearly towards the power supply system. Even with a sinusoidal voltage supply, their currents are non-sinusoidal, but still periodically in steady-state. Hence, they contain other spectral components, the current harmonics, frequencies which are a multiple of the fundamental supply frequency. Due to the non-zero internal impedance of the supply system, voltage harmonics arise.

The harmonic components cause a current redistribution inside transformer or choke windings of the foil-type or parallel wires due to leakage fields and parasitic inter-winding-couplings. These currents can cause internal hot-spots damaging the device [1,2]. It is therefore important to be able to model the current and related heat source distribution in these devices by means of numerical field simulations, already in the design stage.

2. NON-LINEARITIES

In general three sources of problem non-linearity are found in these types of problems.

- Many ferromagnetic materials show saturation in the BH-characteristic. This introduces solution-dependencies in the coefficients of the describing equations. The magnetising currents contain saturation harmonics as well.
- The power electronic loads behave non-linear and impose therefore non-sinusoidal currents. These currents also cause higher harmonic voltage components.
- Many material characteristics, such as electrical conductivities, are temperature dependent. This makes some coefficients dependent of a thermal solution. The entire non-linear problem is therefore coupled to a thermal equation.

3. FEM COUPLED MODELLING OF MAGNETIC DEVICES

3.1 Magnetic Time Domain Modelling

Magnetic fields can be calculated by means of the magnetic vector potential *A*.

$$B = \mathrm{curl}A \tag{1}$$

The introduction of this potential into Maxwell's equations leads to the next equation, or written as functional in the following equation [3].

$$\mathrm{curl}(\upsilon \cdot \mathrm{curl}A) - \sigma \frac{\partial A}{\partial t} = J_s \tag{2}$$

$$\left[\iint_\Omega \upsilon \left(\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y} \right) dxdy \right] A_i - \left[\iint_\Omega \upsilon N_i N_j \, dxdy \right] \frac{\partial A_i}{\partial t} = \iint_\Omega N_i J_s \, dxdy \tag{3}$$

This equation is discretised on a triangular finite element mesh and a time-stepping scheme is applied. The material parameters, such the reluctivity ν and the conductivity σ, are assumed to be constant per element. Non-linear loads can be represented in the time domain by means of time-dependent resistances or dependent voltage/current sources. In this way they can be introduced in the circuit equations coupled to the FEM-model. To account for the non-linearities, a successive substitution or a Newton-Raphson scheme can be derived.

However, to determine an accurate solution of this problem, many small time-steps have to be performed consecutively, leading to a time-consuming calculation, especially if interested in steady-state (coupled) solutions.

3.2 Magnetic Frequency Domain Modelling

If only steady-state solutions are of interest, the problem can also be calculated in the frequency domain. To derive the necessary equations, a Fourier Transform is applied on the time domain equation.

$$\mathrm{curl}\left(\mathrm{F}(\upsilon) * \mathrm{curl}\,\mathrm{F}(\mathrm{A})\right) - \sigma \mathrm{j} \omega \mathrm{F}(\mathrm{A}) = \mathrm{F}(\mathrm{J_s}) \qquad (4)$$

If the solution is periodic in time, the Fourier spectra in this equation become infinitely long discrete complex series. For practical reasons, the solution series has to be limited to *N* components. It can be proven that due to the convolution, the series of the material reluctivity must have length *2N.*

$$\mathrm{curl}\left(\{\upsilon_h\}_{h=-2N(1)2N} * \mathrm{curl}\{A_h\}_{h=-N(1)N}\right) - \sigma \mathrm{j} \omega \{A_h\}_{h=-N(1)N} = \{J_h\}_{h=-N(1)N} \qquad (5)$$

When this equation is completely written, a large system of complex equations remains. For small problems in the sense of a small mesh or a small *N*, it is possible to solve the entire system [4]. However, for large problems it is necessary to decompose the problem into a set of *N* coupled equations that can be solved separately. This is accomplished in the equation concerning harmonic *h* by moving the convolution terms with the other harmonics to the righthand-side. They can be interpreted as fictitious currents. The solution in the negative side of the spectrum can be calculated from the positive side of the spectrum since the solution is real valued in time.

$$\mathrm{curl}\left(\upsilon_0 \cdot \mathrm{curl} A_h\right) - \sigma \mathrm{j} \omega A_h = J_h - \mathrm{curl}\left(\sum_{\substack{i=-N \\ i \neq h}}^{N} \upsilon_{h-i} \cdot \mathrm{curl} A_i\right), h = 1(1)N \qquad (6)$$

It is possible to derive a Newton-Raphson scheme to solve the non-linear material parameters. If successive substitution is used, and *N* is a multiple of two, fast FFT algorithms can be used to calculate the new ν_r-series.

In practice, the terms moved to the righthand-side are relatively small. At first, the saturation, causing the non-DC terms in the material reluctivity series, is not extreme under standard operating conditions for which most devices are designed. This makes the harmonic terms in ν_r-series small, when compared to ν_{r0}. The magnitude of the higher harmonic solution terms can be relatively small as well, when there is no significant driving force at that frequency, e.g. a significant voltage source harmonic. Hence, often only a relatively small error is made when the convolution terms are omitted. This approximated solution is obtained much faster since the equations become physically much more weakly coupled.

3.3 Thermal Modelling

The equation coefficients containing the conductivity are temperature dependent. In some models, it is necessary to take this into account for every element, since small local changes in conductivity already influence the current redistribution in the windings. The possibility to calculate the location of the hot spots is a driving factor as well.

Hence, a thermal model has to be solved. The thermal source terms are the total joule and iron losses, calculated for every element. The boundary conditions are of the convection type and are usually applied at different locations in the model. Hence, a different mesh is generated and mesh projection is necessary to transfer the solutions.

3.4 Calculation Scheme

The algorithm used to calculate the total problem is shown in figure 1 [5]. The magnetic problem is solved in the frequency domain and decomposed per harmonic component. The error estimation and the mesh refinement step are introduced to enhance the total solution.

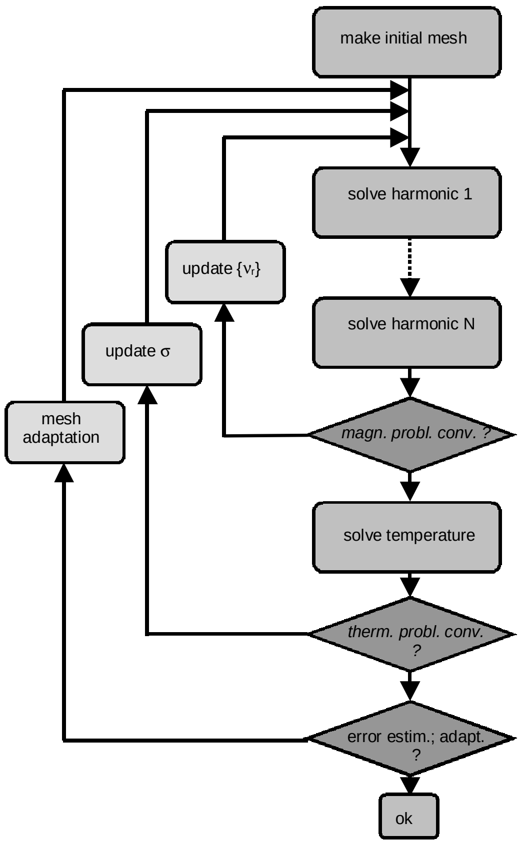

Figure 1: Calculation scheme.

4. APPLICATION

The described method is applied to a design of a 4 kVA single-phase transformer. The core is O-shaped and the winding is distributed symmetrically over both legs of the core. The inner located low-voltage winding consists of a foil winding made of a copper sheet with a thickness of 0,5 mm. The height of the winding is 147 mm, therefore current redistribution caused by eddy currents appears. The outer, primary winding is stranded and therefore the current density can be assumed to be uniformely distributed over the conductors' cross-sections. For reasons of symmetry, only a quarter of the device is modelled. The geometry of the quarter model is shown in figure 2. To model the air around the transformer, an open boundary technique is used: from a certain distance, a 1/r-transformation is applied. The air beyond that boundary, carrying a very low flux, is contained in the small region on the right side of figure 2.

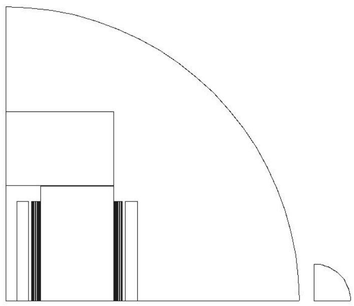

Figure 2: Geometry of the transformer design under study (¼ model).

The mesh used to compute the magnetic field is constructed gradually by applying h-adaptation. The error estimators used to select the elements for refinement, are chosen with respect to the desired result. For the core region the error in the magnetic induction is estimated, whereas in the foil conductors the current density is used in order to obtain an accurate joule loss distribution. This yields the mesh shown in figure 3.

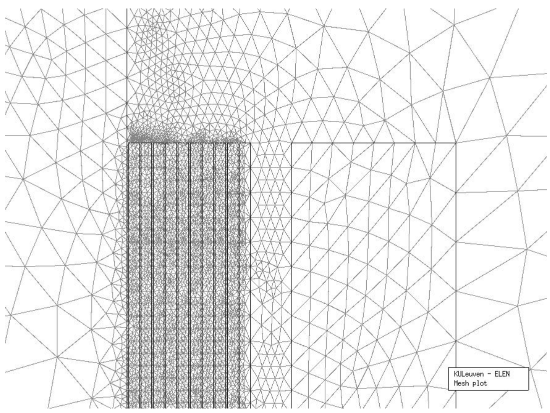

Figure 3: Detail of the mesh used to compute the magnetic field.

The applied load current is assumed to be generated by a single phase diode rectifier forming the net-side of a frequency converter driving an AC induction machine. Two periods of the current and its magnitude spectrum are shown in figure 4. This current was measured when the rectifier was operated by a quasi-sinusoidal voltage with less than 2% of voltage harmonics (mostly 5^{th} and 7^{th} harmonics).

The finite element model is extended with electrical circuit equations modelling the load and supply. The supply is modelled by a voltage source with the fundamental frequency and an internal impedance. At harmonic frequencies, the voltage source is replaced by a short circuit (figure 5). The load, e.g. a bridge rectifier, is modelled by a set of current sources, one for each harmonic frequency. The magnitude and the phase of each current source are determined by the complex spectrum of the load current.

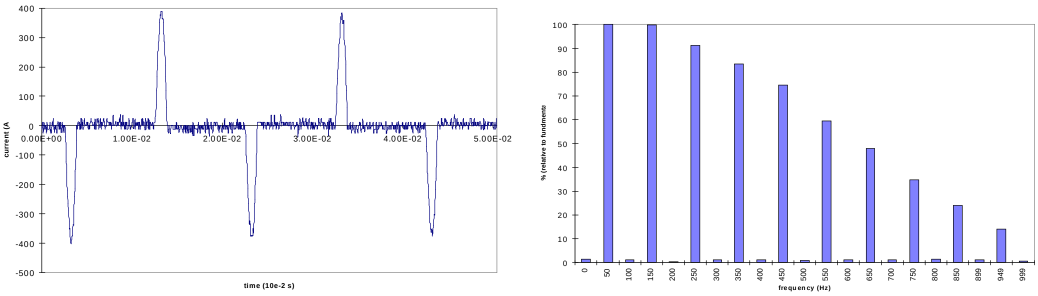

Figure 4: Load current and its amplitude spectrum.

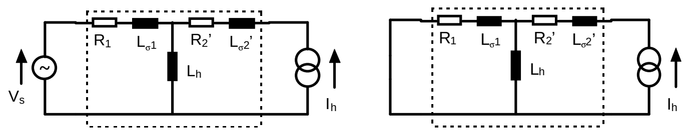

Figure 5: Simulation models of the transformer, load and supply; the part inside the dotted line is modelled by the FEM equations.

The real and imaginary component of the complex solution at fundamental frequency (first equation of (6)) is shown in figure 6. Since the source voltage is assumed to be the phase reference, the main part of the magnetic field is found in the imaginary solution. The real part of the field is associated with the stray field and the internal eddy currents.

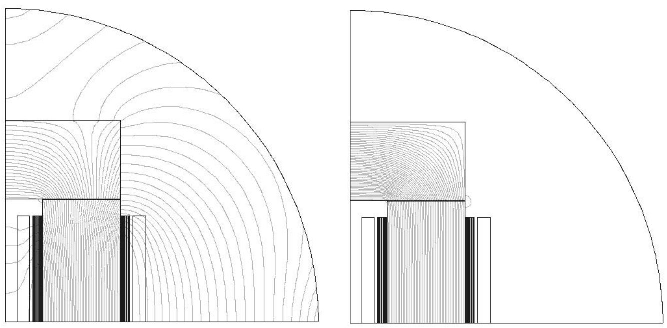

Figure 6: Field lines of the magnetic field solution (real and imaginary component) at fundamental frequency.

The current density distribution is obtained as a post-processing result of each calculated magnetic field. Figure 7 shows the current density distributions of the fundamental component and a higher harmonic. Two sets of curves are shown: the conductors on the right side (outside the core) and on the left side (inside the core). As it can be seen, the distributions differ significantly. The oscillating current causes a higher current density at the tops of the conductors. The difference in the leakage fields inside and outside the core causes the profiles to be different. The distance to the core is of importance since there are local differences in leakage flux.

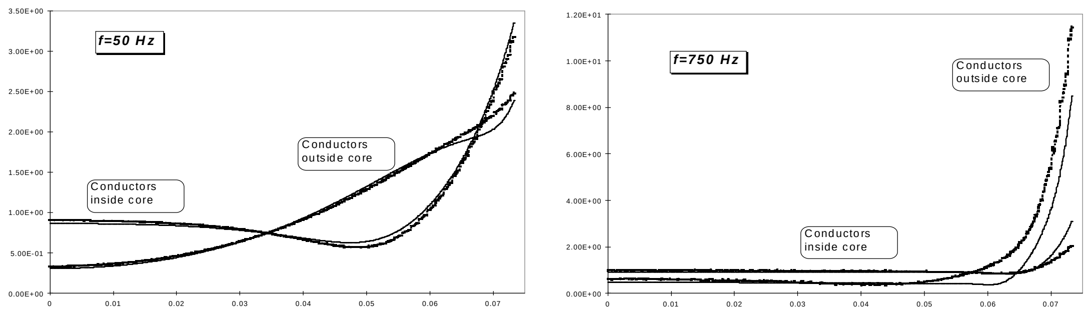

Figure 7: Profile of the current density distribution (in A/mm²) inside the foil conductors at the fundamental frequency and the 15th harmonic. The x-axis (m) starts at the middle of the conductor and runs upwards. The solid line represents the 1st conductor (close to the core); the dashed line represents the 10th conductor.

The coupled thermal calculation is performed on a mesh covering only a part of the magnetic model. The computed thermal field lines are shown in figure 8. Some temperature profiles found in the foil conductors are shown in figure 9. In average, the conductors inside the core are hotter due to the moderate convective heat transfer there. The conductors close to the core are less heated because of the resistive thermal path through the core, compared to the heat path to the environment as seen from the outer conductor.

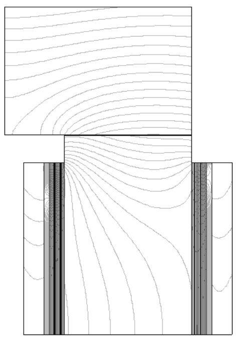

Figure 8: Thermal field lines of the transformer model.

These results can be used to estimate the so-called *'K-factor'* that can be used as a derating factor or a design variable for transformers operating non-linear loads.

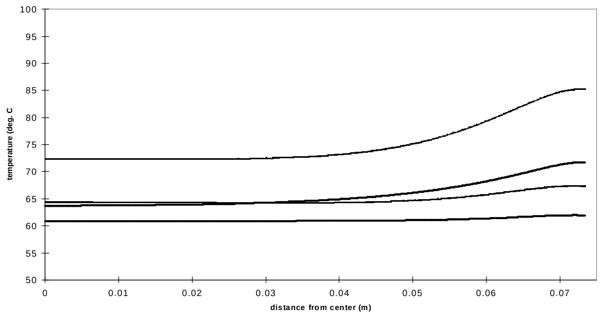

Figure 9: Thermal profile of the foil conductors. The thinner lines are the conductors inside the core; the thicker lines are the conductors outside the core.

5. CONCLUSION

A method to model the effect of non-linear loads on transformers and chokes is presented. Fields at harmonic frequencies originating from saturation or source harmonics are modelled. Load-generated current harmonics are considered. Based on the obtained set of magnetic solutions, an estimation of the joule and iron losses is performed, serving as the input for a thermal model. The results of this thermal model allow to adjust the electrical conductivities locally in order to obtain a more accurate current distribution by a coupled problem iteration.

This approach is illustrated by an example of a transformer design with foil conductors carrying a rectifier load. Computations are performed in a multiple iteration loop over magnetic fields decomposed in different frequencies, the thermal field and mesh enhancement steps. Internal current redistributions occur in the solid conductors due to thermal effects.

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REFERENCES

[1] M. Bishop, J. Baranowki, D. Heath, S. Benna, “Evaluating harmonic-induced transformer heating,” *IEEE Trans. on Power Delivery*, Jan. 1996, vol. 11, no. 1, pp. 305-311.

[2] A. Girgis, E. Makram, J. Nims, “Evaluation of temperature rise of distribution transformers in the presence of harmonics and distortion,” *Electric Power Systems Research*, Jan. 1990, vol. 20, no. 1, pp. 15-22.

[3] K. Binns, P. Lawrenson, C. Trowbridge, *The Analytical and Numerical Solution of Electric and Magnetic Fields*, John Wiley & Sons, Chichester, reprint 1994.

[4] S. Yamada, K. Bessho, J. Lu, “Harmonic Balance Finite Element Method Applied to Nonlinear AC Magnetic Analysis,” *IEEE Trans. on Magnetics*, July 1989, vol. 25, no. 4, pp. 2971-2973.

[5] J.Driesen, J.Fransen, R.Belmans, K.Hameyer, ”A general-purpose environment for the calculation of coupled thermo-electromagnetic problems” *Proc. 4th International workshop on Electric and Magnetic fields*, Marseille, France, 12-15 May 1998, pp. 541-546.

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