Function based approach in transient finite element analysis

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Abstract

Numerical simulation of electromagnetic devices using the finite element method is nowadays more than just a magnetostatic calculation. Induced currents, saturation and end-effects have to be taken into account. With the ever increasing computation power of state of the art computers, the transient method using a one-step time-stepping scheme is in common use in finite element analysis. Effects of higher harmonics caused by the non-linear behaviour of materials, motion, but also by non-sinusoidal excitation and faults, can be considered. Applying non-sinusoidal voltages and currents at the finite element model must be possible in an easy and comprehensible way. This is obtained by a function based description of the external circuits, i.e. the sources in the external circuits are described by a function and the actual value is calculated with a function evaluator in each time step of the transient analysis. This function based approach is not only extended to all circuit parameters, but also to the time-stepping and motional information.

1 Introduction

1.1 Transient Finite Element Analysis

In the transient finite element analysis, the magnetic vector potential is obtained by using a one-step time-stepping scheme [Preston[1], Arkkio[2]].

$$\left(\alpha \mathbf{K} + \frac{\mathbf{R}}{\Delta t}\right)\mathbf{A}_k + \left((1-\alpha)\mathbf{K} - \frac{\mathbf{R}}{\Delta t}\right)\mathbf{A}_{k-1} = \left(\alpha \mathbf{T}_k + (1-\alpha)\mathbf{T}_{k-1}\right) \tag{1}$$

K is the element coefficient matrix, **R** the stiffness matrix, $\mathbf{T}_{k-1}$ and $\mathbf{T}_k$ are the source vectors respectively at $t = t_{k-1}$ and $t = t_k = t_{k-1} + \Delta t$. Different difference schemes are obtained by changing the value of the parameter α in the recurrence relation between $\mathbf{A}_k$ and $\mathbf{A}_{k-1}$. The time-stepping scheme is started at $t = t_0$ and $\mathbf{A}_0$ is assumed to be zero or a time-harmonic solution is used [Arkkio[2]]. Saturation is considered by using a Newton-Raphson method, while the end-effects are considered by coupling an external lumped parameter model with the finite element model [Tsukerman[3]].

1.2 Equation of Motion

When a moving-mesh model is used [Williamson[4]], even the effects of the rotation of the machine can be considered. The differential equation of motion for a rotating machine in terms of the mechanical speed ω is given by

$$J \frac{\mathrm{d}\,\omega}{\mathrm{d}\,t} + C\,\omega = T_{em} - T_L \tag{2}$$

with the moment of inertia J of all parts reflected to the motor shaft, viscous friction coefficient C representing the mechanical losses, electromagnetic motor torque T_{em} and load torque T_L. Time integration results in a new value for the mechanical speed ω_k and position θ_k at the next time step.

$$\begin{aligned} \omega_k &= \frac{J - (1-\alpha) C\,\Delta t}{J + \alpha\, C\,\Delta t}\,\omega_{k-1} \\ &\quad + \frac{\alpha\,\Delta t}{J + \alpha\, C\,\Delta t}\left(T_{em,k} - T_{L,k}\right) + \frac{(1-\alpha)\Delta t}{J + \alpha\, C\,\Delta t}\left(T_{em,k-1} - T_{L,k-1}\right) \end{aligned} \tag{3}$$

$$\theta_k = \theta_{k-1} + \alpha\,\Delta t\,\omega_k + (1-\alpha)\Delta t\,\omega_{k-1} \tag{4}$$

As the electromagnetic torque T_{em} depends on the position θ, a simple approximation for $T_{em,k}$ is used to calculate the new position θ_k at which the finite element solution is computed. The value of the electromagnetic torque is updated afterwards.

$$T_{em,k} = T_{em,k-1} \tag{5}$$

2 Function Based Approach

A function based approach means that all variables are stored by their function descriptions instead of their particular value. The numerical values are calculated with a function evaluator at the beginning of each time step. Combined with the possibility of using parameters, changes with time are described in an easy and comprehensible way. Reserved parameters such as `time` and `speed` are set by the finite element program at the appropriate value.

2.1 Circuit Information

The external circuit file contains for each branch of the circuit its type, start and end-node number, and other type-dependent parameters. The symbolic description of each parameter is stored instead of its actual value. Solid and stranded conductors, inductances and capacitances can be used in the same circuit part. As the coupling of the finite element method with an external circuit file is based on a topological method [De Gersem[5]], i.e. only the topological information and type of branch is used to perform the coupling, it is sufficient to evaluate the different function description before the matrix is assembled, e.g. at the beginning of each time step. However, no change in topology is allowed during the transient analysis. Due to numerical reasons it is not allowed to define resistances, inductances and capacitances with zero or infinite value. Therefore, switches have to be defined by using a step function in combination with an extra resistance that is already used to take the end-effects into account (figure 1).

```
Rextra+Ropen*step(tref1,time)
```

Figure 1: Symbolic description of a switch combined with an extra resistance.

2.2 Time Stepping and Motional Information

This function based approach is not only applied to all circuit parameters, i.e. the values of the end-effect parameters, but also to the time-stepping information. The weighting factor which determines the time-stepping scheme can be varied with time to avoid numerical instability caused by the additional circuit equations [Tsukerman[6]]. Starting with a value of 1, the value is slowly decreased (slope function) to a value of 0.6 (figure 2).

```
WEIGHTING FACTOR
1-0.4*slope(period/4,3*period/4,time)
```

Figure 2: Symbolic description of the weighting factor.

Additionally, the motional information can be function based to simulate e.g. a change in the load torque. In combination with the reserved parameter `time`, it is easy to define the load torque proportional to the square of the speed (figure 3).

```
PARA
Tload  Trated*speed^2/ratedspeed^2
```

Figure 3: Symbolic description of a load torque proportional to the square of the speed.

2.3 Special Functions

To improve the comprehensibility of the function descriptions, some special functions are implemented.

2.3.1 Step and Slope Function

The step function is used to simulate a sudden change in a parameter, while the slope function is used for a slow change. Both functions return a value between 0 and 1 depending on the reference value(s) (figure 4).

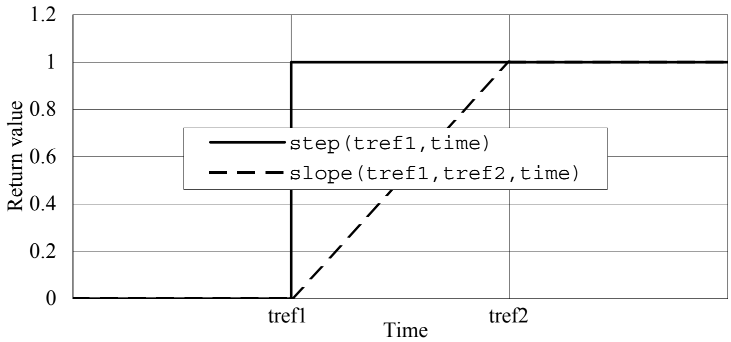

Figure 4: Return value of the step and slope function.

2.3.2 Pulse-Width Modulation Switching Scheme

In a simple pulse width modulation switching scheme, a triangular waveform at the switching frequency is compared with a control signal in order to generate the output signal. Figure 5 shows the function description and figure 6 shows the sinusoidal control signal and the output signal of the switching scheme with an amplitude modulation of 0.75 and a frequency modulation of 20.

```
pwm(amplitude/0.75*pwm(0.75*cos(omega*time+fase),fswitch,time)
```

Figure 5: Symbolic description of a simple pulse-width modulation switching scheme.

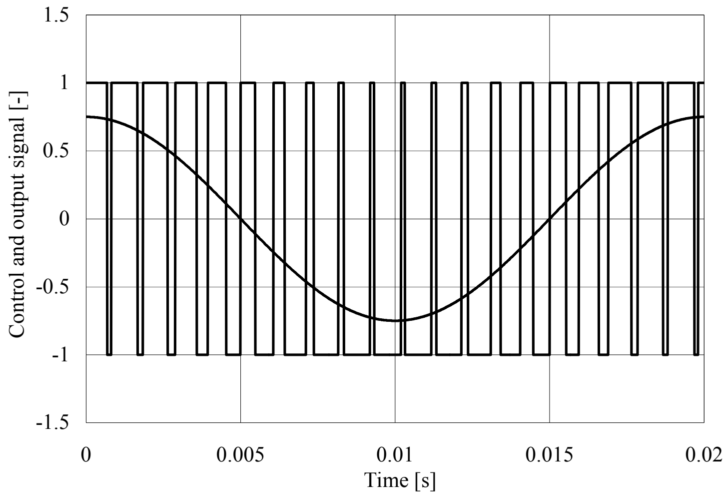

Figure 6: Sinusoidal control signal and output signal of a simple pulse-width modulation switching scheme.

3 Examples

This function based approach enables various possibilities of electromagnetic analysis. To show the versatility and comprehensibility of the approach and for reasons of simplicity, only one rough finite element model (5524 elements) of a 4-pole induction machine is used (figure 7). The results of the different simulations are shown in per-unit values based on rated current, rated torque and synchronous speed.

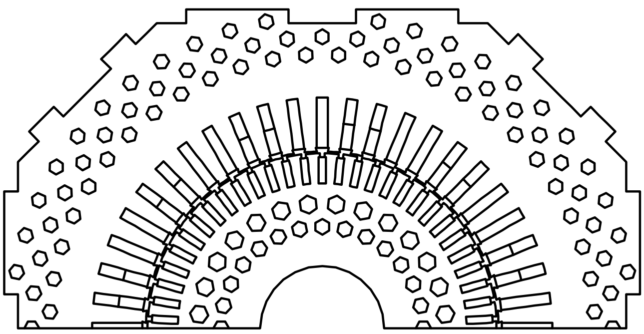

Figure 7: Half-symmetry model of a 4-pole induction machine.

3.1 Line Start

Figure 8 and 9 show the simulation results of a line start of the induction machine. The load torque is proportional to the square of the speed.

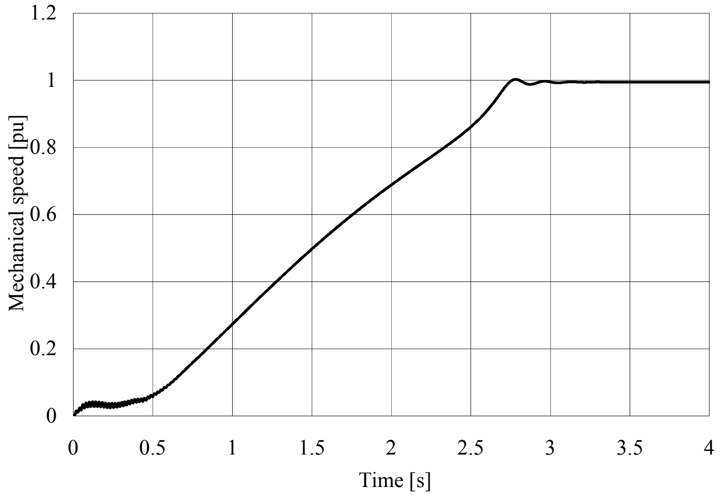

Figure 8: Mechanical speed versus time during line start.

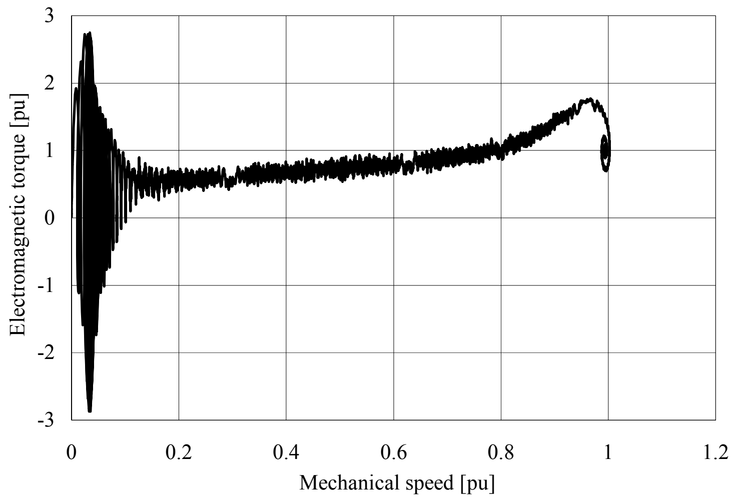

Figure 9: Dynamic torque-speed characteristic during line start.

3.2 Change in Load Torque

The step function can be used to define a sudden change in the load torque at $t = t_{event}$ (figure 10). Figure 11 shows the corresponding variation in the mechanical speed due to this sudden decrease in load torque.

```
PARA
Tload  Trated*(1-0.5*step(tevent,time))
```

Figure 10: Symbolic description for a sudden decrease in load torque.

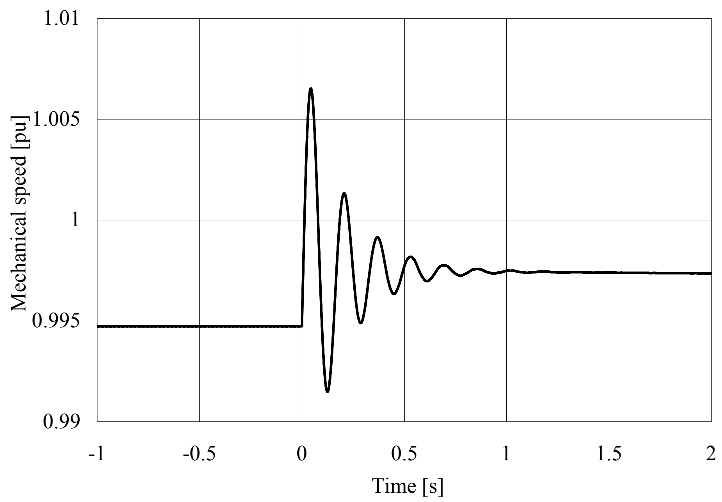

Figure 11: Variation of the mechanical speed due to a sudden decrease in the load torque.

3.3 Shortcircuit of One Phase of a Three-Phase System

Shortcircuiting one phase of the induction machine and opening that phase of the power supply can be simulated by setting the voltage equal to zero at $t = t_{event}$ (figure 12). Figure 13 shows the current through the 'unhealthy' phase, i.e. the phase that is shortcircuited.

```
PARA
voltage1  amplitude*cos(omega*time+phase)*(1-step(tevent,time))
```

Figure 12: Symbolic description for the shortcircuit of the first phase of a three-phase system.

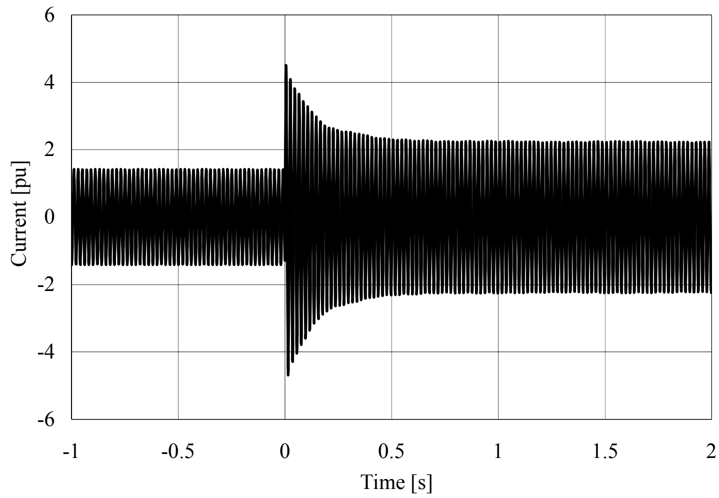

Figure 13: Current through the 'unhealthy' phase after shortcircuiting.

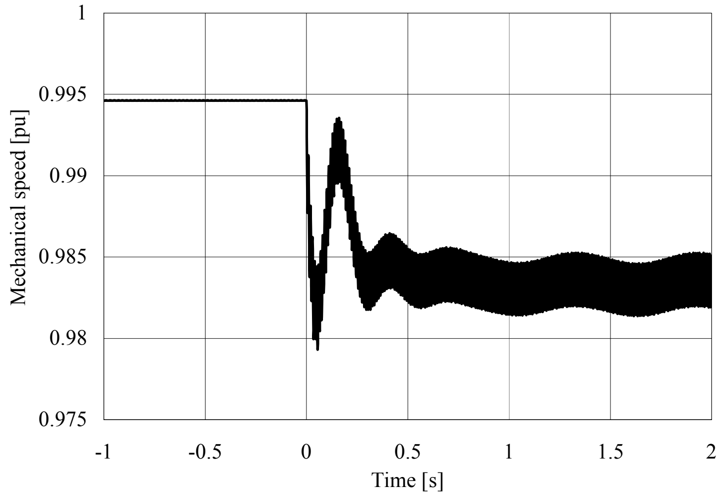

Figure 14: Resulting speed fluctuations after shortcircuiting one phase.

3.4 Wye-Delta Start

Figure 15 shows the current in the first phase during a wye-delta start of an induction machine. The motor is connected in wye untill $t = t_{event} = 5\,\text{s}$ and reconnected in delta at $t = t_{event} + t_{open} = 5\,\text{s} + 50\,\text{ms}$. Figure 16 shows the mechanical speed of the machine during the wye-delta start.

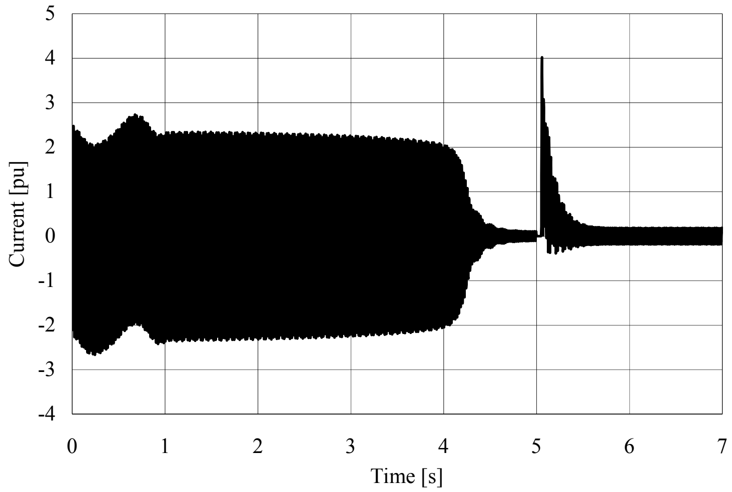

Figure 15: Current in the first phase versus time during a wye-delta start of an induction machine.

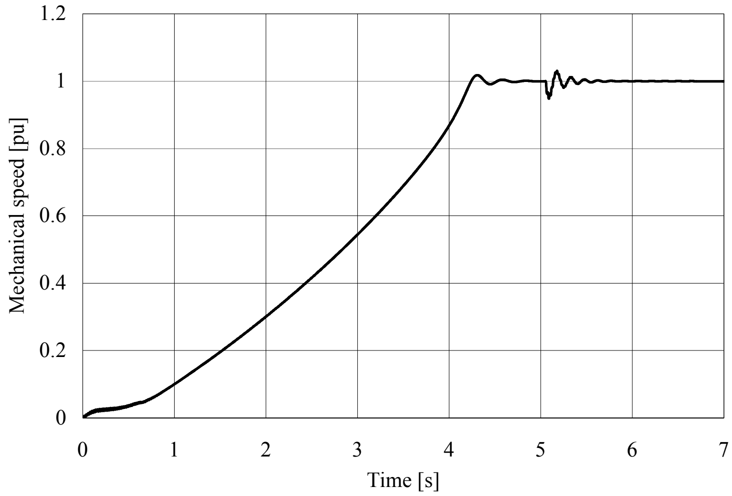

Figure 16: Mechanical speed versus time during a wye-delta start of an induction motor.

4 Conclusion

Function based transient finite element analysis allows various possibilities of electromagnetic analysis in an easy and comprehensible way. The parameters describing the external electric circuits, the time stepping and motional information are stored with their function description instead of their particular value. The numerical value is simply calculated with a function evaluator at the beginning of each time step. Special defined function even improve the comprehensibility. The Matlab-like syntax and the possibility of using user-defined and reserved parameters in the expressions allow almost any kind of excitation or events in time. There is no need to indicate in advance, e.g. by means of a special flag, what kind of excitation the finite element program can expect. The versatility of the approach is shown by various examples.

Acknowledgement

The authors are grateful to Dr. J.-Ch. Mercier of Alstom, Nancy, France for providing the necessary data of the studied induction machine, to the Belgian "Fonds voor Wetenschappelijk Onderzoek Vlaanderen" for its financial support of this work and the Belgian Ministry of Scientific Research for granting the IUAP No. P4/20 on Coupled Problems in Electromagnetic Systems. The Research Council of the K.U.Leuven supports the basic numerical research.

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