# Implementation of the Harmonic Balance FEM method for large-scale saturated electromagnetic devices

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## Abstract

The Harmonic Balance Finite Element Method (HBFEM) is an alternative for time-consuming transient analysis if only the steady-state solution is of interest. However, when used for realistic large-scale problems, such as simulation of the current redistribution in saturable transformers subject to a large spectrum of harmonic currents and voltages, practical implementation aspects have to be considered. An alternative derivation of the method in the complex frequency domain, leading to a block decomposition, is presented. Its implementation as a Gauss-Seidel iteration or a Jacobi-method, allowing a parallel algorithm, is discussed. A method to perform adaptive relaxation of the non-linear algorithm is presented. Effects associated with the representation of the material characteristic are studied. The computational effort in the outer non-linear iteration loop is minimised using FFT-algorithms, allowing to use adaptive relaxation methods to stabilise the overall procedure.

## 1 Introduction

The HBFEM method[1,2] presents an interesting method to obtain the steady-state solution of a non-linear electromagnetic field problem directly. Transient time-stepping computations can lead to steady-state solutions as well, but may be computationally expensive if start-up transients with large time-constants are studied. Often the describing ordinary differential equation has a stiff nature,

requiring small time steps and computationally expensive methods. The motion of bodies, inherent to the problem, can be modelled relatively easily when compared to the HBFEM[3]. However, if the number of field unknowns and/or the number of harmonics to calculate are large, the size of the system matrix of the complete HBFEM method may become too large to be handled by a single computer system. This is usually not the case if the same problems are solved by a transient approach. To allow larger problem sizes, a limited parallel computation model for the HBFEM has been suggested[4].

## 2 The HBFEM Method in the complex frequency domain

### 2.1 Derivation of the HBFEM

#### 2.1.1 Original HBFEM

Magnetic fields are generally described by a partial differential equation using the magnetic vector potential, being a time-dependent function, as $\upsilon$ is the reluctivity for saturable materials.

$$\nabla \cdot \left(\upsilon\left(A(t)\right)\nabla A(t)\right) - \sigma \frac{\partial A(t)}{\partial t} = J_s(t) \tag{1}$$

$$B(t) = \nabla \times A(t) \tag{2}$$

In the original HBFEM[1,2], the vector potential and the reluctivity are written as a truncated Fourier series with unknown coefficients for the sinus and cosinus terms, reflecting the periodicity of the steady state solution. Only one fundamental frequency is used, but DC-terms may occur. Evaluating this in the equation (1), numerous product terms arise in the absorption term of the PDE, leading finally to a large finite element equation.

#### 2.1.2 Complex HBFEM

An alternative derivation consists of applying the Fourier Transform to eqn (1). The product is replaced by a convolution operation on the complex spectra:

$$\nabla \cdot \left(\mathrm{F}\{\upsilon\} * \nabla \mathrm{F}\{A\}\right) - \sigma \frac{\partial \mathrm{F}\{A\}}{\partial t} = \mathrm{F}\{J_s\} \tag{3}$$

Since the functions are assumed periodically in time and real-valued, the spectra consist of sets of weighted Dirac functions, conjugate in the frequency domain. It is not necessary here to limit the number of fundamental frequencies to one. Several fundamental frequencies can be possible e.g. when dealing with PWM-modulated waveforms. This yields a set of coupled partial differential equations, the number of equations equals the number of occurring spectral components:

$$\nabla \cdot \left( \sum_{i=-N}^{N} \upsilon_{h-i} \nabla A_i \right) - j\omega_h \sigma A_h = J_{s,h}, \quad h = 0(1)N \tag{4}$$

If only saturation is assumed and hysteresis effects are neglected, the number of components of the material series is $2N$. It is possible to derive the complete coupled version of eqn (4), being a complex equivalent of the system obtained by the traditional HBFEM.

### 2.2 Decomposition of the Complex HBFEM

The set of partial differential equations (4) can be decomposed in a natural way into a decoupled system. In realistic devices, the DC-component of the material spectrum is several orders larger than the double frequency components. This is not necessarily the case for the magnetic spectrum. The convolution product term including $\upsilon_0$, multiplied by $A_h$, is therefore often dominant. The other terms can be assumed as locally constant, and moved to the right-handside, yielding $N$ decoupled equations. These off-diagonal convolution terms on the right-handside can be regarded as "fictitious current sources", forcing the flux to by-pass strongly saturated regions:

$$\nabla \cdot (\upsilon_0 \nabla A_h) - j\omega_h \sigma A_h = J_{s,h} - \nabla \cdot \left( \sum_{\substack{i=-N \\ i \neq h}}^{N} \upsilon_{h-i} \nabla A_i \right), \quad h = 0(1)N \tag{5}$$

The equations are discretised on triangular finite element meshes, yielding a set of matrix equations. If a Newton-Raphson method is applied, the matrices are Jacobians and the unknowns become corrections.

$$\boldsymbol{K}_0 A_h = \boldsymbol{F}_h - \sum_{\substack{i=-N \\ i \neq h}}^{N} \boldsymbol{K}_{h-i} A_i, \quad h = 0(1)N \tag{6}$$

Theoretically, the meshes may be different. For instance, local differences may be interesting when skin effects are present. The off-diagonal convolution terms are calculated after projection of the solution on the appropriate mesh. The same procedure is required if different types of elements are used.

## 3. Linear and Non-linear Iterative Solution Algorithms

The solution of the set of equations (5), contains three nested iterative loops. At first, the individual linear systems have to be solved. Next, the iteration loop at block level has to be considered and finally, the non-linear iteration loop is to

converge. If error estimation and mesh refinement is used, a fourth iteration loop can be added.

### 3.1 Linear system solving

The matrix equations to be solved are complex, therefore requiring an efficient complex iterative solver. The numerical properties of symmetric complex systems indicate to be more favourable with respect to iterative solvers than their double-sized real-valued counterparts[5]. The decomposition into multiple linear systems with off-diagonal eddy current terms in one single frequency is advantageous from the numerical point of view. The higher the frequency $\omega_h$ becomes, the more unfavourable the numerical properties of the system become. If different frequencies were mixed as in the complete HBFEM, a preconditioner would never be as effective as for the decomposed equations.

### 3.2 Block iteration loop

#### 3.2.1 Sequential Block Gauss-Seidel

An obvious choice would be the sequential solution of the individual systems in eqn (5). The obtained solutions are immediately used in the consecutive equations as formulated in eqn (6). Obviously, the block algorithm is sequential. The old solution can be immediately overwritten with the newly obtained one.

$$\boldsymbol{K}_0 A_h^{(j+1)} = \boldsymbol{F}_h - \sum_{|i|<h} \boldsymbol{K}_{h-i} A_i^{(j+1)} - \sum_{h<|i|\le N} \boldsymbol{K}_{h-i} A_i^{(j)}, \quad h = 0(1)N \tag{7}$$

#### 3.2.2 Parallelisable Block Jacobi

The updates are made after solving all systems. Theoretically, this method converges slower than the Gauss-Seidel method, but is more stable. However it must be noted that this has never appeared to be a problem in the studied models. The iteration process can be formulated:

$$\boldsymbol{K}_0 A_h^{(j+1)} = \boldsymbol{F}_h - \sum_{\substack{i=-N \\ i\neq h}}^{N} \boldsymbol{K}_{h-i} A_i^{(j)}, \quad h = 0(1)N \tag{8}$$

This method can be parallelised. The simultaneous solution procedures of the individual matrix systems can be performed on different computational units. A comparable parallelisation for the traditional HBFEM is made in reference [4], where it is suggested to set the number of linear iteration steps to a fixed number. Here, however, it is suggested to let the linear iterative solvers converge to an equal relative norm of their residuals. For higher frequencies, the convergence is generally slower. Meanwhile, the already converged sub-process can transfer its

solutions to the master process, proving to minimise the overall time spent for communication and synchronisation. By using multi-casting, if available, the time necessary for transmitting the new set of solutions to the slave processes is limited. The norm of the difference between two consecutive solutions is used as a stopping criterion for the block iteration loop.

### 3.3 Non-linear Iteration loop including Adaptive relaxation

#### 3.3.1 Non-linear iteration

In the non-linear iteration loop, the material parameters of the finite elements covering the sub-domains with saturable materials have to updated – whether a Newton-Raphson or a substitution based non-linear algorithm is applied. These material updates have to be performed in the time-domain. Therefore an Inverse Fourier Transform is required. The most efficient implementation to perform this, is the (I)FFT on the double-sided spectrum. After the update, another FFT per saturable element is necessary to obtain the material coefficient series. The number of points in the FFT should be a multiple of two and depends on the spreading of the spectral components in the series. Suppose one fundamental frequency exists in $N$ harmonics, and only odd harmonics are non-zero. The (I)FFT's are then $4N$-points. Then the number of floating point operations for $M$ elements approximately is:

$$flops \approx 2 \cdot M \frac{4N}{2} \log(4N) \tag{9}$$

In principle, the non-linear and linear iteration loops could be exchanged[6], but in that case, the number of calculations given by eqn (9) would increase the calculation time.

#### 3.3.2 Efficient Adaptive Relaxation

With strongly non-linear materials, as e.g. laminated iron, immediate use of the newly obtained solution leads to divergence if large sets of harmonics are calculated and far extrapolation of the material characteristic would be necessary. Therefore, (under)relaxation is required. The relaxation factor has to change in the different non-linear iteration loops, using adaptive correction. In order to estimate the locally optimal damping factor, it can be assumed that it should minimise the non-linear residual over the total solution vector, the norm being $\|\bullet\|_1$, $\|\bullet\|_2$ or $\|\bullet\|_\infty$. This residual can be used as a stopping criterion.

$$\min \left\| \left[ \sum_{i=-N}^{N} \mathbf{K}_{h-i}\left( A_{h-i}^{(j+1)'} \right) A_i^{(j+1)'} - \mathbf{F}_h\left( A_h^{(j+1)'} \right) \right] \right\| \tag{10}$$

$$A_h^{(j+1)'} = \alpha^{(j)} \cdot A_h^{(j+1)} + \left(1 - \alpha^{(j)}\right) \cdot A_h^{(j)} \tag{11}$$

Different strategies can be used to find the minimum of the residual. It is possible to find a global minimum over the global solution vector (all harmonics considered), but since the different solution components may vary some orders of magnitude, this can be basically reduced to the search of the minimum for the dominant component. Using a relative norm with different denominators for each partial solution may help, but it is difficult to find a good denominator. It is possible to use the solution itself, though it is not situated in the same vector space as the residual vector(s). The right-handside lies in that vector space, but is not interesting in realistic problems, where usually many entries are filled in with zeros or small numbers. This yields very small norms in the denominator.

It appears to be better to look for the minimum of every partial residual (every harmonic) separately. These minima are usually not found for the same $\alpha^{(j)}$ and thus certain components may be slightly overrelaxed in some iteration steps. However, this proves to be a robust divergence-free method for strongly non-linear materials.

### 3.4 Mesh Adaptation

The final iterative loop consists of the error estimation and the adaptation (*h*-, *p*- or *hp*-refinement) of the mesh or possible multiple meshes for the sub-problems. The error can be estimated, based on the total solution or on the partial solutions. This is still subject of further research. The projection of the previous solution on the altered mesh, proves to be an excellent starting solution.

## 4. Implementation

### 4.1 Implementation aspects

#### 4.1.1 Linear loops

The described complex HBFEM method has been implemented in the in-house FEM software environment 'Olympos'[7]. There, first order triangular finite elements are applied. The possibility to extend the models with circuit equations has been implemented. Fast mesh adaptation and projection algorithms are available.

The numerical algorithm used for the iterative solution of the linear systems (blocks) is QMR, preconditioned by SSOR. This algorithm proves[5] to be very fast and robust for the occurring complex symmetric systems. In the beginning of the non-linear process, often almost homogeneous systems have to be solved e.g. when the right-handsides contain only contributions of saturation harmonics, qnd the equations are still very damped. For these types of systems, (SSOR)QMR converges fast when compared to other iterative solvers such as the complex (SSOR)CG.

### 4.1.2 Non-linear loops

The determination of the relaxation factor in the non-linear loop can be implemented in an effective way. The considered relaxation factors are calculated by $\alpha_j=\beta^\kappa$ with $\beta$=0.5 and $\kappa$ a natural number. Every calculation of a residual requires a non-linear material update, the composition of the matrix system and a matrix-vector product. To minimise the amount of time necessary for searching the relaxation factor with the minimal residual, the number of residual calculations can be limited to three: $\beta*\alpha_j$, $\alpha_j$ and $\alpha_j/\beta$ with $\alpha_j$ the relaxation factor of the previous step. The minimal residual is usually to be found in the covered interval, since the non-linear process converges smoothly, when appropriate damping is applied.

### 4.1.3 Round-off errors

The HBFEM proves to be sensitive to round-off of errors, since the interacting solution components may vary several orders of magnitude. If these round-off errors become too large, the smoothness of the convergence at block iteration or at non-linear level gets lost. However, this can be detected by comparing with an estimated convergence criterion. In this case, it is the best to keep the previous solution and stop the iteration.

### 4.2 Parallelisation aspects

If the block-level iteration uses the Jacobi algorithm, parallelisation of the entire approach is possible. For implementation, the PVM[8]-library ("parallel virtual machine") is used. The parallel machine consists of a cluster of computation units (CU's), with distributed memory, e.g. a group of workstations connected over a LAN. A master process controls the calculation, assigning a spectral component to each CU. When each CU finished its task, it transmits its solution vector to the master. The master then checks the convergence criterion and if needed, broadcasts the total solution vector to the slave processes on the CU's. Due to the slight differences in computation time, for reason of numerical properties, there is an overlap in the communication of the partial solution to the master and the computations of the slower converging processes and not much time is lost. The time lost in the broadcast phase is larger, but efficient implementations exist to perform multi-casting over a LAN, instead of peer-to-peer communication.

### 4.3 Source Modelling

The voltage or current source driving the device has to be modelled in the frequency domain. Two options exist to model the complex source values, appearing on the right-handside.

#### 4.3.1 Truncated Fourier Series

If the periodic waveform can be represented by a known Fourier Series, this series may be truncated. The equivalent of this operation is the application of a low-pass filter to the waveform, leading to the Gibbs phenomenon with overshoots. Such an overshoot may lead to severe errors since it can drive the model into a non-existing state of deep saturation, hereby causing large errors in the saturation harmonics.

#### 4.3.2 Data Sampling

An alternative not suffering from overshoots is based on data sampling in the time domain. On these samples (or measurement samples), an FFT has to be performed to obtain the spectral components. In this case, it cannot be avoided that aliasing occurs, thereby increasing the source components at the highest frequencies, when compared to the physical reality. In general, this does not influence the algorithm as is the case with overshoots. The use of anti-aliasing filters re-introduces moderate overshoots.

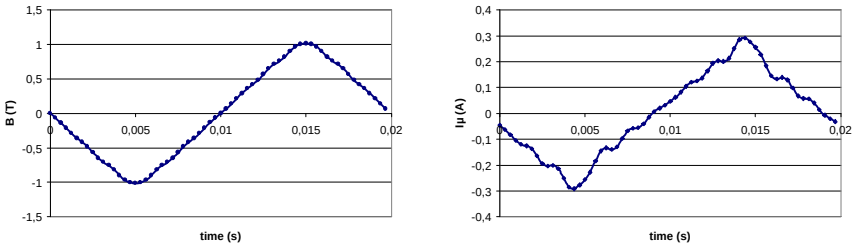
### 4.4 Material Characteristic Representation

The calculation process is also influenced by the representation of the non-linear magnetic saturation characteristics. Spline interpolated reluctance curves, based on manufacturer supplied data, sometimes lack local smoothness, especially in the linear region, giving rise to non-existing higher saturation harmonics in the calculated fields. These occur even in the beginning of the non-linear calculation and require stronger relaxation, leading to longer computational times. Analytically interpolated expressions such as polynomials are smoother and perform better.

## 5. Application

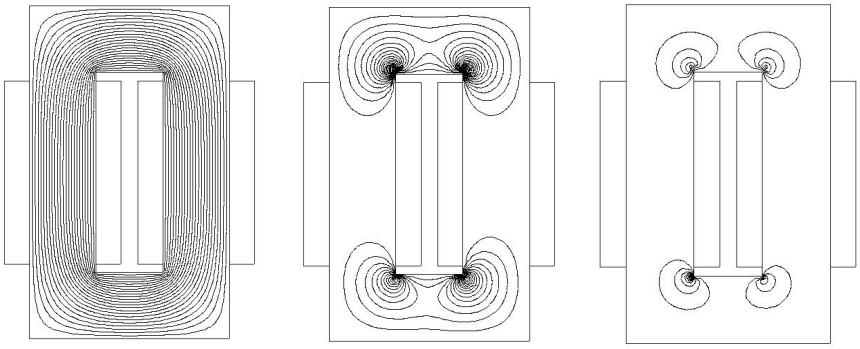
The complex HBFEM is applied to a single-phase transformer, subject to a squarewave voltage. The voltage harmonics are determined using an FFT. The flux is triangular shaped, with the top of the triangle reaching into the saturation region of the material characteristic, causing the magnetising current to be triangular shaped and having locally high peak values. This model is calculated up to the $15^{th}$ harmonic, considering only the odd harmonics. Figure 1a contains graphs of the flux magnitude near a saturated corner in which the round-off effect due to saturation is seen. The saturation harmonics up to the $15^{th}$ order in the current are plotted in Figure 1b. Figure 2 shows the first three component fields of the solution.

The time required to compute this model is roughly distributed in 40% for the non-linear algorithm (residual calculations) and 60% for the (block) linear solving algorithm in case in case of sequential implementation. This can be reduced to an 80% / 20% distribution if the parallel implementation is used.




a) flux magnitude b) magnetisation current

Figure 1. Time evolution of $B$ and $I_\mu$.



a) fundamental b) 3rd harmonic c) 5th harmonic

Figure 2. Set of solutions obtained from the application of the complex HBFEM on a single-phase transformer model.

## 6. Conclusions

The implementation of a complex version of the HBFEM method for large-scale FEM-simulation of non-linear electromagnetic devices is discussed and compared with the complete HBFEM method. A parallel implementation is derived. A four-level nested iteration process is explained, including block iteration and the adaptively relaxed non-linear loop. Practical implementation aspects such as the determination of the relaxation factor, the effect of round-off errors, the choice of the linear solver and the use of PVM are studied. Finally, the application of the method to a transformer is shown to demonstrate the feasability of this approach at an example out of the engineering practice.

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