Combined Time-Harmonic - Transient Approach to Calculate the Steady-State Behaviour of Induction Machines

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Abstract: Due to less computational expenses, a time-harmonic approach is often preferred over a transient approach to calculate the steady-state behaviour of induction machines. However the computation time of the transient approach with a one-step time-stepping scheme is significantly reduced by using a combined time-harmonic - transient approach and by varying the parameter which determines the difference scheme with time.

Keywords: induction machines, electromagnetic analysis, transient analysis, finite element methods.

I. INTRODUCTION

Although three-dimensional finite element calculations of induction machines belong to the present state of possibilities, two-dimensional calculations are still preferred to obtain an acceptable solution in a reasonable time. As the calculation method has to consider induced currents, saturation and end-effects, two approaches are widely used: the time-harmonic and the transient approach.

A. Time-Harmonic Approach

The magnetic vector potential A is assumed to vary sinusoidal with time at the frequency f or angula frequency ω and is represented by its complex phasor notation. The correct induced rotor currents are obtained by applying the net frequency f_n and multiplying the conductivity in the rotor by the slip s [1, 2].

$$\text{Stator:}\quad \nabla\cdot\left(\nu\nabla A\right)-\mathrm{j}\,2\pi f_n\,\sigma\,A=-J_s \tag{1}$$

$$\text{Rotor:}\quad \nabla\cdot\left(\nu\nabla A\right)-\mathrm{j}\,2\pi f_n\left(s\sigma\right)A=-J_s \tag{2}$$

ν is the reluctivity, σ the conductivity and J_s the source current density. A complex Newton-Raphson method using effective reluctivity curves considers the saturation [1, 3], while the end-effects are taken into account by coupling an external lumped parameter model with the finite element model [2, 4, 5].

B. Transient Approach

The magnetic vector potential is obtained by using a one-step time-stepping scheme [6, 7].

$$\left(\alpha\,\mathbf{K}+\frac{\mathbf{R}}{\Delta t}\right)\mathbf{A}_k+\left(\left(1-\alpha\right)\mathbf{K}-\frac{\mathbf{R}}{\Delta t}\right)\mathbf{A}_{k-1}=\left(\alpha\,\mathbf{T}_k+\left(1-\alpha\right)\mathbf{T}_{k-1}\right) \tag{3}$$

$\mathbf{K}$ is the element coefficient matrix, $\mathbf{R}$ the stiffness matrix, $\mathbf{T}_{k-1}$ and $\mathbf{T}_k$ are the source vectors respectively at $t=t_{k-1}$ and $t=t_k=t_{k-1}+\Delta t$. Different difference schemes are obtained by changing the value of the parameter α in the recurrence relation between $\mathbf{A}_k$ and $\mathbf{A}_{k-1}$. The time-stepping scheme is started at $t=t_0$ and $\mathbf{A}_0$ is assumed to be zero. Saturation and end-effects are taken into account by the same techniques as in the time-harmonic approach.

Fig. 1 shows a one pole pitch finite element model with 9178 elements of a 400 kW squirrel cage induction machine with a two-layer winding. Four periods (128 time steps per period) are simulated using a time-stepping scheme with $\alpha=2/3$ (Galerkin method). A voltage-driven locked rotor test is taken as example because no motion effects are involved. Fig. 2 shows the stator currents through the four parts of the stator windings that appear in the finite element model of Fig. 1. The number of periods and thus the computation time of the transient approach is significantly reduced by using a combined time-harmonic - transient approach.

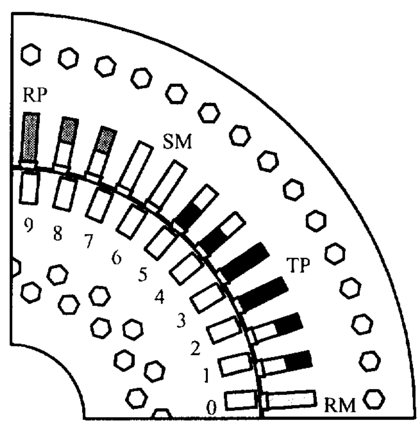

Fig. 1. Outline and material labels of a one pole pitch finite element model of a squirrel cage induction machine.

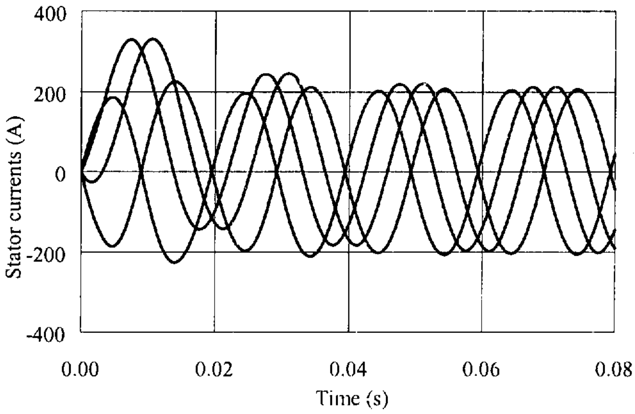

Fig. 2. Transient simulation of the stator currents in the finite element model for four periods (Galerkin method).

II. COMBINED TIME-HARMONIC - TRANSIENT APPROACH

A. Start Solution

Instead of assuming the start solution $\mathbf{A}_0$ to be zero, a time-harmonic solution is used [5]. This method can also be used for the circuit unknowns $\mathbf{X}$ [4].

$$\begin{bmatrix} \mathbf{A}_0 \\ \mathbf{X}_0 \end{bmatrix} = \sqrt{2}\ \mathrm{Re}\left(\begin{bmatrix} \mathbf{A}_{TH} \\ \mathbf{X}_{TH} \end{bmatrix}\right) \tag{4}$$

$\mathbf{A}_{TH}$ and $\mathbf{X}_{TH}$ are the solutions of an equivalent time-harmonic problem. If the meshes of the time-harmonic and the transient problem differ, efficient mesh projection methods can be used [8]. Fig. 3 shows again the stator currents and Fig. 4 the stator voltages. Only two periods are simulated with the Galerkin method ($\alpha = 2/3$) to obtain the steady-state behaviour of the induction machine. As can be seen in these figures, it is sometimes sufficient to simulate less than two periods and the simulation can be terminated based on the periodicity of the circuit unknowns.

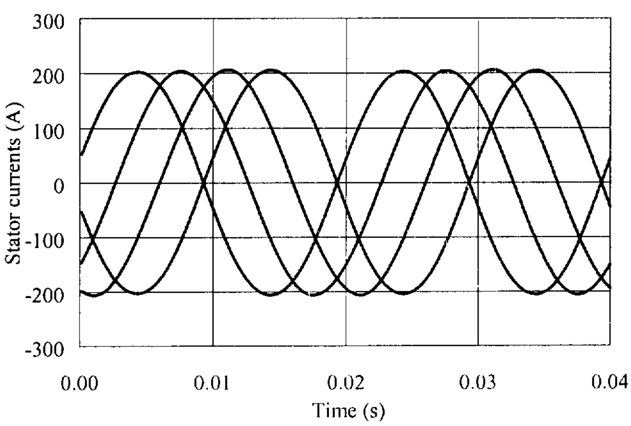

Fig. 3. Combined time-harmonic - transient simulation of the stator currents for two periods (Galerkin method).

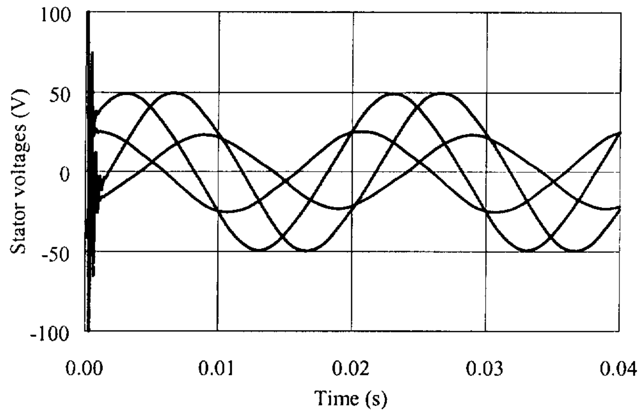

Fig. 4. Combined time-harmonic - transient simulation of the stator voltages for two periods (Galerkin method).

B. Stability and Accuracy

The choice of the value of the parameter α in (3) influences the stability and the accuracy of the numerical results. The recurrence scheme is stable for $\alpha \geq 1/2$. Oscillations are not prevented but they do not grow out of control. The Crank-Nicolson method ($\alpha = 1/2$) gives on top a second order accuracy with time and is therefore a good choice. Unfortunately, in combination with field-circuit coupling peculiar effects may occur. Fig. 5 shows the stator voltages over the four parts of the stator windings in the finite element model. In this context, the term 'stability paradox' is used [9] and is advised to choose the value of the parameter α closer to 1 than to $1/2$.

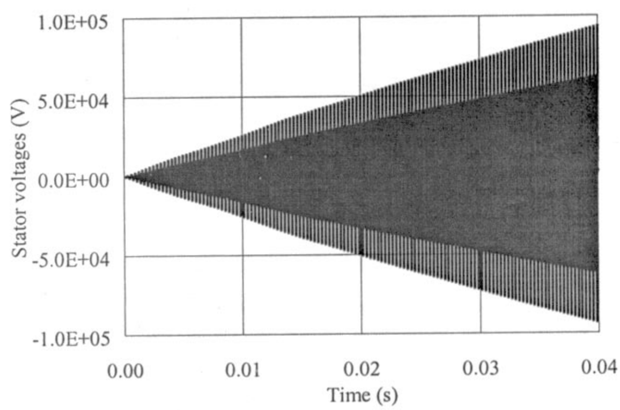

Fig. 5. Combined time-harmonic - transient simulation of the stator voltages (Crank-Nicolson method).

As the transient method is function based, i.e. all parameters are described by a function description and the actual value is calculated with a function evaluator before assembling the coefficient matrix, it is possible to vary the value of the parameter α with time (Fig. 6). Starting with a value of 1, the value is slowly decreased (*slope*-function) to a value of 0.6. This corresponds to an error reduction in the stator voltages by $1/3$ in each step. It is now possible to use a one-step time-stepping scheme with a value of the parameter α closer to $1/2$ than to 1. The steady-state behaviour of the the induction machine is obtained as fast and accurate as possible. Fig. 7 shows again the computed stator voltages.

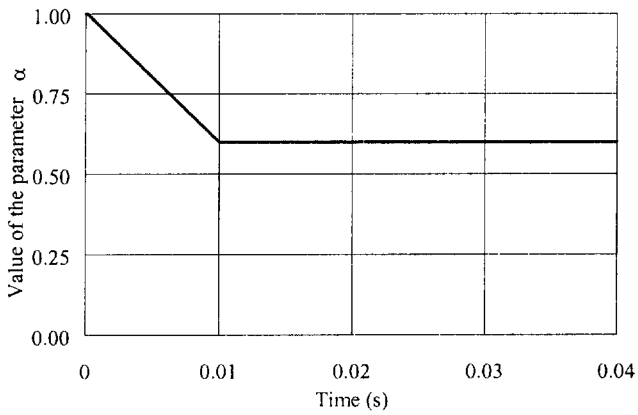

Fig. 6. Variation of the value of the parameter α in the one-step time-stepping scheme with time.

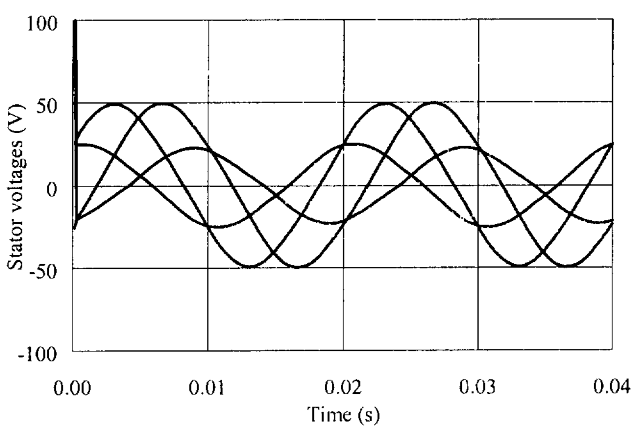

Fig. 7. Combined time-harmonic - transient simulation of the stator voltages (parameter α varies with time).

III. CONCLUSION

Although the ever increasing computation power of computers, a transient approach using a one-step time-stepping scheme is still a time consuming task. The total computation time is significantly reduced by using a combined time-harmonic - transient approach. Less than two periods have to be simulated to obtain the steady-state behaviour of an induction machine. The simulation can be terminated based on the periodicity of the circuit unknowns. The time needed for the time-harmonic calculation is negligible and efficient mesh projection can be used if the meshes of the two approaches differ. Due to the function based approach, the parameter α in the one-step time-stepping scheme is varied with time. Starting with a value of 1 (high error reduction at the beginning), the value is slowly decreased to a value closer to $1/2$ to obtain a steady-state solution as fast and accurate as possible.

IV. ACKNOWLEDGMENTS

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V. REFERENCES

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