

Study of Eddy Currents in Transformer Windings caused by a Non-Linear Rectifier Load

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Abstract — Transformers and chokes carrying currents originating from non-linear loads often contain harmonics causing additional heating of the device. A calculation scheme to model the eddy currents causing the hot spots by means of FEM-methods and post-processing is presented. It is developed to analyse transformers and chokes connected to a supply with a quasi-sinusoidal voltage feeding non-linear loads such as rectifiers. The method is illustrated at an application in which the current redistribution and its consequences inside a foil winding transformer carrying current harmonics is studied.

I. INTRODUCTION

In electrical power systems an increasing amount of power electronic systems is introduced. These systems exist in every size ranging from lighting, switched power supplies for office equipment, frequency converters for adjustable speed drives and huge rectifiers for electrothermal applications. These loads behave non-linearly towards the power net. Even with a sinusoidal voltage supply, their currents are non-sinusoidal, but still periodically in steady state. Hence they contain other spectral components, the current harmonics, frequencies which are a multiple of the fundamental supply frequency.

The harmonic components cause a current redistribution inside transformer or choke windings of the foil-type or parallel wires due to leakage fields and parasitic interwinding-couplings. These currents can cause internal hot-spots damaging the device [1,2]. Its therefore important to be able to model the current and related heat source distribution in these device by means of numerical field simulations, already in the design stage.

II. THE EFFECTS OF HARMONICS ON TRANSFORMERS

A. Transformer Loss Mechanisms

The heat sources in a transformer can be divided in two groups.

1. *The iron loss*: the losses in the core of the device due to the changing magnetic flux. This loss is approximately equal to the no-load loss if the resistance of the winding carrying the magnetising current can be neglected. In commercial transformers this loss amounts a few percent.

2. *The ohmic losses*: these losses are located in the current carrying windings. They only occur under loaded conditions.

The effect of power system harmonics with respect to the main loss mechanisms is discussed in the following sections.

B. The Effect of Voltage Harmonics

Voltage harmonics influence the flux in the transformer. This is explained by the law of Faraday-Lenz (1). This flux ϕ consists of the main part in the core ϕ_μ and a small leakage flux ϕ_σ .

$$\begin{aligned} N_1 \cdot \frac{d\phi}{dt} &= N_1 \cdot \frac{d(\phi_\mu + \phi_\sigma)}{dt} \cong N_1 \cdot \frac{d\phi_\mu}{dt} \\ &= \oint \vec{E} \cdot d\vec{l} = u_1(t) - R_1 \cdot i_1(t) \cong u_1(t) \end{aligned} \quad (1)$$

with N_1 the number of (primary) windings, R_1 the neglectable winding resistance, $i_1(t)$ the winding current and $u_1(t)$ the supplying voltage.

The transformation of (1) into the frequency domain shows the relationship of the flux components and the voltage harmonics (2).

$$N_1 \cdot \sum_h j(h\omega) \cdot \underline{\Phi}_\mu(h) \cong \sum_h \underline{U}(h) \quad (2)$$

This equation shows that the magnitude of the flux components is inversely proportional to the order of the harmonics. The effect of a higher order harmonic on the total flux is therefore small.

In realistic situations, the magnitude of the voltage harmonics remains small compared to the fundamental component. This is determined by the low internal impedance of most supply systems carrying current harmonics. The individual voltage harmonics rarely exceed the level of 2-3%. Therefore, only a slight error is made if the influence of the non-fundamental voltage components is neglected. This is even more justified by the inversely proportional relation at higher frequencies. Hence eq. (2) reduces to eq (3).

$$N_1 \cdot j\omega_1 \cdot \underline{\Phi}_{\mu,1} \cong \underline{U}_1 \quad (3)$$

The consequence of (3) is that the no-load losses are caused by the fundamental voltage component only. This is confirmed by measurements [1,2].

C. The Effect of Current Harmonics

For most power electronic systems, the amount of current harmonics is significant when compared to the fundamental component. The additional ohmic losses in the windings due to the harmonic currents can be substantial.

The frequency dependence of the winding resistance has to be considered. The skin depth δ in the winding material at a given frequency is given by (4). For a commonly used conductor material such as copper it approximates 1 cm at 50Hz.

$$\delta = \sqrt{\frac{2}{\mu_0 \omega \sigma}} = \frac{\delta_1}{\sqrt{h}} \quad (4)$$

Since the skin depth is inversely proportional to the square root of the harmonic order, the increased ac-resistance leads to a relatively higher ohmic loss for higher harmonics. Foil windings and large solid windings are connected in parallel, often have dimensions which are larger than the skin depth, so a substantial current redistribution may occur in the transformer windings due to internal eddy currents. Leakage fields and magnetic inter-winding couplings influence the current redistribution as well.

The eddy currents over the winding cross-sections cause an even worse unequal distribution of heat sources over the winding. This leads to local hot spots. Since the conductivity of the material is temperature dependent, the current density distribution is a function of the temperature field inside the winding, which itself is a function of the ohmic heat sources [3]. To obtain an accurate model of the effect of the current harmonics, the magnetic and thermal field have to be considered in a coupled way.

II. FINITE ELEMENT MODELLING ASPECTS

A.. Modelling the transformer

To model the magnetic field of a transformer, an eddy current problem has to be solved. This is accomplished by using the finite-element method on a vector potential formulation ($\mathbf{B} = \nabla \times \mathbf{A}$) of the problem. The current distribution in the transformer is simulated in a 2D-cross section. The equation describing the magnetic field is given by (5) [4], which is Ampère's law after the introduction of the vectorpotential. It is non-linear due to the saturation of the ferromagnetic core material.

$$\nabla(\nu(\mathbf{A})\nabla\mathbf{A}) - \sigma \frac{\partial \mathbf{A}}{\partial t} = \mathbf{J}_s \quad (5)$$

Strictly, the magnetic reluctivity is a periodic function in time. However, it is common to approximate it by a function which is constant in time: $\underline{\nu}(\mathbf{A})$. This can be done without severely affecting the field solution and loss calculation of realistic devices. The level of saturation is determined by the

field at fundamental frequency: $\underline{\nu}(\mathbf{A}_1)$. The transformation of (5) leads to a set of equations (6).

$$\nabla(\underline{\nu}(\mathbf{A}(h))\nabla\mathbf{A}(h)) + jh\omega\mathbf{A}(h) = \mathbf{J}_s(h) \quad (6)$$

with $h = 1, \dots, N$

This equation transformed into a functional and a discretisation with triangular finite elements is applied. The minimisation of the functional leads to a large system of complex algebraic equations. After the first equation is solved by a non-linear solver based on an adaptively damped successive substitution, combined with a (linear) equations solver (pre-conditioned iterative SSORCG solver), the linear equations for the higher harmonics are solved by the same linear equation solver.

The total rms current, necessary to obtain the ohmic loss is obtained by (7).

$$J_{rms} = \sqrt{\sum_h \|J_s(h) - jh\omega\sigma\mathbf{A}(h)\|^2} \quad (7)$$

B. Load and Supply Modelling

The finite element model is extended with electrical circuit equations modelling the load and the supply. The supply is modelled by a voltage source at fundamental frequency and an internal impedance. At harmonic frequencies, the voltage source is replaced by a short circuit. The load, for instance a bridge rectifier, is modelled by a set of current sources, one for each harmonic frequency. The magnitude and the phase of each current source are determined by the complex spectrum of the load current. This leads to a set of models to be solved, represented by figure 1.

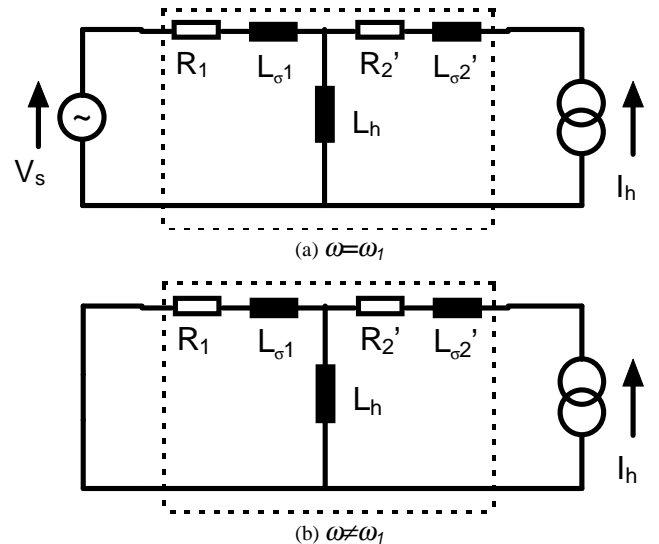


Fig. 1. Simulation models of the transformer, load and supply; the part inside the dotted line is modelled by the FEM equations.

III. APPLICATION

The described method is applied to a design of a 4 kVA single-phase transformer. The core is O-shaped and the winding is distributed symmetrically over both legs of the

core. The inner low-voltage winding consists of a foil winding made of a copper sheet with a thickness of 0,5 mm. The height of the winding is 147 mm, so a current redistribution is likely to appear. The outer, primary winding is stranded and therefore the current density can be assumed uniformly distributed over the conductors' cross-sections. Because of the symmetry in the design, only a quarter of the device is modelled. The geometry of the quarter model is shown in figure 2. To model the air around the transformer, an open boundary technique is used: from a certain distance, a $1/r$ -transformation is applied. The air beyond that boundary, carrying a very low flux, is contained in the small region on the right.

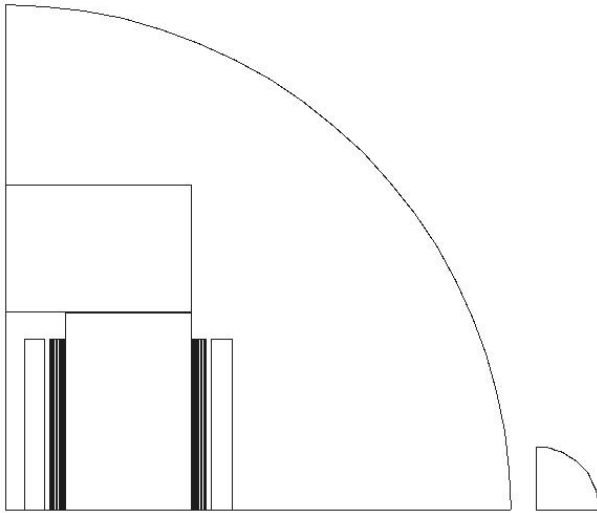


Fig.2. Geometry of the transformer design under study (1/4 model).

The mesh used to calculate the magnetic field is constructed gradually by applying h-adaptation. The error estimators used to select the elements to be refined are chosen in function of the desired result. For the core region the error in the magnetic induction is estimated, whereas in the foil conductors the current density or the ohmic loss density is used. This leads to the mesh shown in figure 3. The size of the largest element in the top of the foil conductors is about 0,2 mm. Eq. 4 is used to estimate the maximum frequency of the current that can be modelled by these elements; this shows that up to the 20th harmonic could be modelled if the fundamental frequency is 50 Hz.

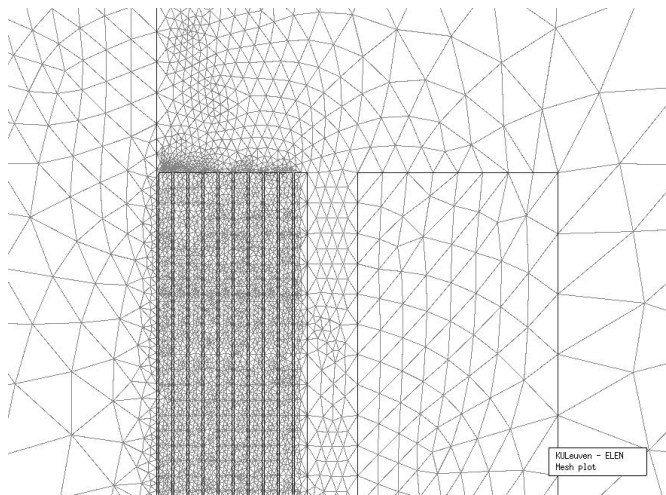


Fig.3. Detail of the mesh used to calculate the magnetic field.

The load current applied is assumed to be produced by a single phase diode rectifier forming the net-side of a frequency converter driving an ac induction machine. One period of the current and its amplitude spectrum are shown in figure 4. This current was measured when the rectifier was fed by a quasi-sinusoidal voltage with less than 2% of voltage harmonics (mostly 5th and 7th harmonics).

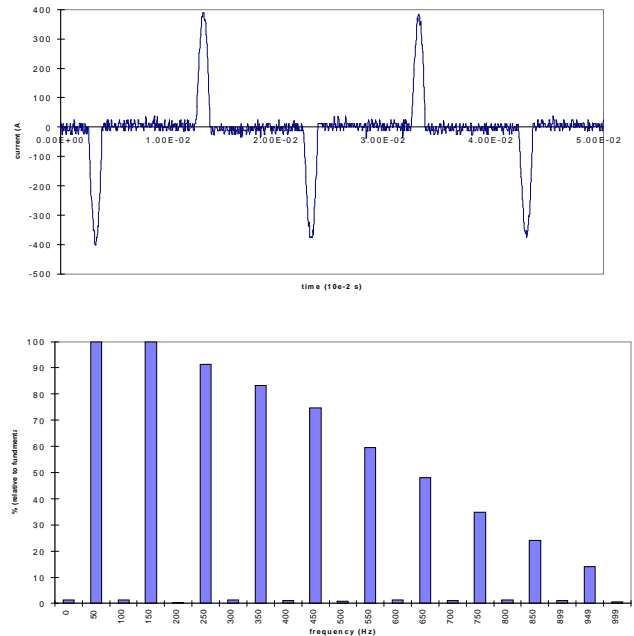


Fig.4. Load current and its amplitude spectrum.

The magnetic FEM calculation results in a set of complex fields. The curl of each of these fields gives the local \mathbf{B} -field. The real and imaginary part of the first equation of (6) after the curl operation is performed is shown in figure 5. Since the source voltage is assumed to be the phase reference, the main part of the magnetic field is found in the imaginary solution due to the integration of eq. (3). The real part of the field is associated with the stray field and the internal eddy currents.

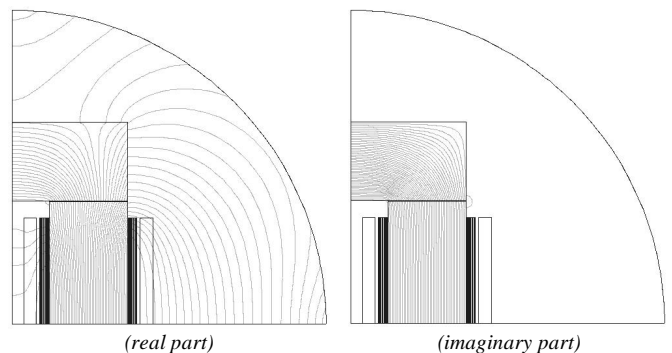


Fig.4. Field lines of the magnetic field solution at fundamental frequency.

The current density distribution is obtained as a post-processing result of each calculated magnetic field. Figure 5 shows the density distributions of the fundamental component and a higher harmonic. Two sets of curves are shown: the conductors on the right side (outside the core) and on the left side (inside the core).

As can be seen, that current density distributions differ significantly. The oscillating current causes a higher density at the tops of the conductors. The difference in the leakage fields inside and outside the core causes the profiles to be different. The distance to the core is of importance since there are local differences in leakage flux.

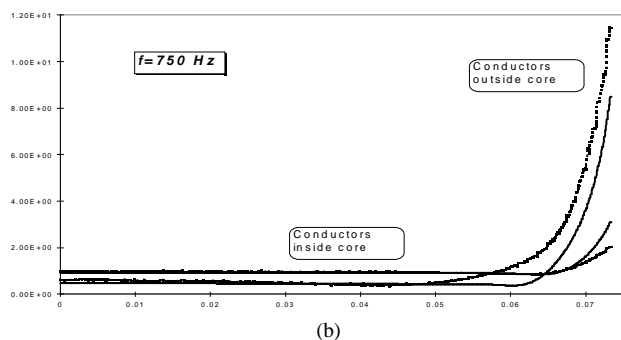
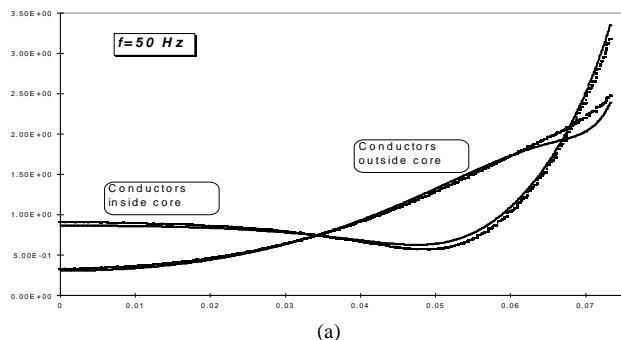


Fig.5. Profile of the current density distribution inside the foil conductors at the fundamental and the 15th harmonic;. The x-axis starts at the middle of the conductor and runs upwards. The solid line represents the 1st conductor (close to the core); the dashed line represents the 10th conductor.

The coupled thermal calculation is performed on a mesh covering only a part of the magnetic model. The computed thermal field lines are shown in figure 6. Some temperature profiles found in the foil conductors are shown in figure 7. In average, the conductors inside the core are hotter due to the moderate convective heat transfer there. The conductors close to the core are more cool because of the resistive thermal path through the core, compared to heat path to the environment as seen from the outer conductor.

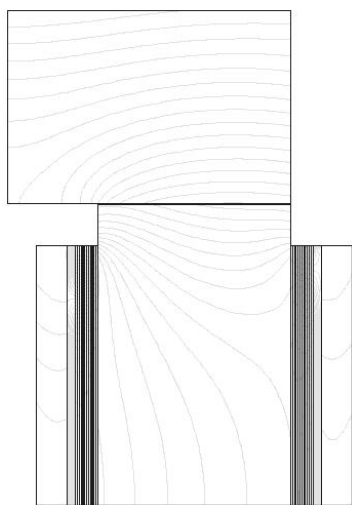


Fig. 6. Thermal field lines of the transformer model.

These results can be used to estimate the so-called ‘K-factor’ that can be used as a derating factor or a design variable for transformers feeding non-linear loads.

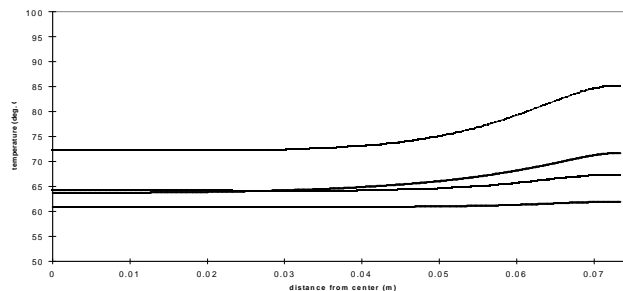


Fig.7. Thermal profile of the foil conductors. The dashed lines are the conductors inside the core; the solid lines are the conductors outside the core.

V.CONCLUSION

A method to model the effect of current harmonics on transformers and chokes under a quasi-sinusoidal voltage is presented. These currents are produced by non-linear loads fed by the transformer. The saturation level in the core and the related iron losses can be determined from a voltage-driven non-linear, FEM-simulation of the magnetic field component at fundamental frequency. The ohmic losses for the fundamental current are obtained as well. The harmonic components cause additional ohmic losses that are obtained from a current-driven, linear magnetic field simulation. The combination of these post-processing results forms the input for a FEM thermal field model that is used to locate the hot-spots in the design. The results of this thermal model allows to adjust the electrical conductivities locally in order to obtain a more accurately current distribution after a coupled problem iteration.

This approach is discussed at an example of a transformer design with foil conductors carrying a rectifier load.

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