

# OPTIMIZING FERROMAGNETIC CHARACTERISTICS TO REDUCE NEWTON ITERATIONS

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**Abstract** - The standard Newton iteration scheme to solve a non-linear system of equations obtained from the finite element methods is based on the updating of the field dependent element reluctivity. Usually, the manufacturer of the ferromagnetic material provides a BH-characteristic as diagram (Fig. 1) or in form of a table of data samples. The influence of the material properties, in particular their accurate numerical representation, is significant for the rate of convergence during the Newton iterations. Here, a numerical optimization aiming at a technically smooth non-linear characteristic is performed to obtain a higher rate of convergence of the Newton iteration scheme.

## I. INTRODUCTION

The ferromagnetic properties of non-linear material are usually measured and plotted in diagrams (Fig. 1). The values are obtained by measurements on material samples. Available material characteristics are often erroneous, as they include measurement errors and the reading error when extracting numerical values from a diagram [1]. The engineer designing electromagnetic circuits has at his disposal usually the plotted characteristics only. He needs to construct a numerically useful diagram by choosing data samples.

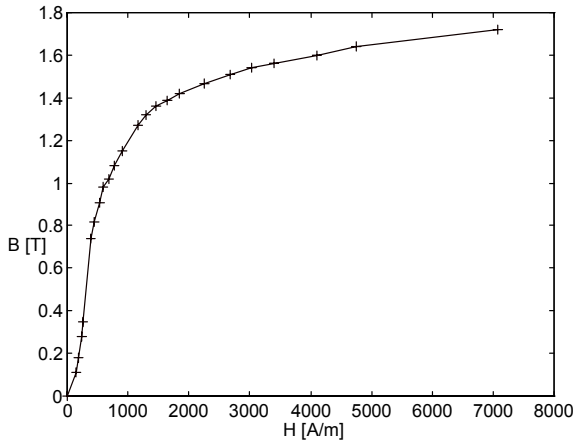


Fig. 1 Typical original BH-characteristic<sup>1</sup>

To approximate the field quantities between those given data, standard cubic spline interpolating polynomials are used. This

<sup>1</sup> These material data are based on data of an iron material used for the TEAM Workshop problem 25. The material has been slightly altered by the authors. By changing point (0.28 T, 215 A/m) to (0.28 T, 235 A/m) the numerical representation becomes unfavourable for the non-linear solution process, as will be shown in this paper. This change is optically invisible. However, the original data set includes other problematic sections as well.

approach inherits a defective representation of the non-linearity inside a finite element computation. The field computation algorithms require a representation of data that fulfil certain qualifications to ensure a fast rate of convergence and stable solutions [2].

Consider the differential equation for the magnetostatic case based on the vector potential  $\mathbf{A}$ :

$$\nabla(\nu \nabla \mathbf{A}) = -\mathbf{J} \quad (1)$$

These non-linear problems are typically solved using a Newton iteration:

$$\mathbf{A}^{k+1} = \mathbf{A}^k + \beta \delta \mathbf{A}^k \quad (2)$$

$$\mathbf{P} \delta \mathbf{A}^k = \mathbf{T} - \mathbf{K} \mathbf{A}^k \quad (3)$$

where  $\delta \mathbf{A}$  is the vector of residues,  $\mathbf{P}$  the Jacobian matrix and  $\beta$  a possible damping factor. Both the element matrix  $\mathbf{K}$  and the Jacobian matrix  $\mathbf{P}$  depend on the non-linear reluctivity  $\nu$  of the ferromagnetic material. Constructing the Jacobian  $\mathbf{P}$  furthermore depends on the derivative of the reluctivity:

$$P_{ij} = K_{ij} + 2 \int_{\Omega} \frac{d\nu}{d(B^2)} \sum_m \sum_n (\nabla \alpha_m \cdot \nabla \alpha_i) (\nabla \alpha_n \cdot \nabla \alpha_j) A_m A_n d\Omega \quad (4)$$

with  $i, j, m, n = 1(1)3$  denoting the nodal indices for the case of first order triangular elements, and  $\alpha$  the approximation or shape function. The reluctivity must be determined from the non-linear material characteristic based on the flux density inside each triangular element.  $B^2$  is computed by:

$$B^2 = \sum_m \sum_n (\nabla \alpha_m \cdot \nabla \alpha_n) A_m A_n \quad (5)$$

For the assembly of the system of equations that defines one solution step in the Newton process, it is therefore advantageous to represent the ferromagnetic properties by the reluctivity versus the flux density squared  $\nu = f(B^2)$ . To avoid oscillations of the Newton iteration scheme, the original material data samples must be corrected to obtain a technically and numerically smooth characteristic [2]. Considering the error sources and magnitude of the original

material data, such a correction does not introduce a larger error bound.

## II. TECHNICALLY SMOOTH MATERIAL CHARACTERISTICS

As shown by eqns. (1) to (5) it is unfavourable to store the BH-data directly, as the reluctivity and its derivative are required for the assembly and update of the Jacobian  $\mathbf{P}$  and the element matrix  $\mathbf{K}$ . Therefore, it is common practice to store a  $vB^2$ -characteristic instead (Fig. 2).

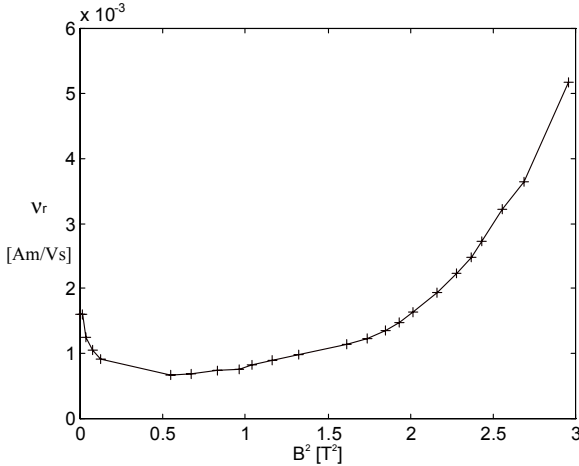


Fig. 2  $vB^2$ -representation of the ferromagnetic material (linear interpolated data points).

Due to the required derivative of the reluctivity, a linear interpolation of the data points as shown in Fig. 2 is not sufficient. It is useful to represent the material characteristic with cubic splines in each interval. Natural cubic splines pass through the data points, having continuous first and second derivative at each data point. The advantage of using cubic splines is the continuity of the slope of the material characteristic. Ferromagnetic material characteristics are smooth by nature. This continuity and smoothness is desirable, as it allows the Newton process to converge fast, without missteps due to rapid changes of the Jacobian. It can be shown however, that an erroneous material data set can cause a spline representation unfavourable for the Newton process. Even though continuity of the characteristics is guaranteed using splines, it can be non-smooth and its derivative may not be monotonically increasing (Figures 3 and 4).

The transformation of the BH-pairs into the  $vB^2$ -pairs already inherits the first problem: it is impossible to determine the reluctivity at the origin. The reluctivity in the origin is usually taken equal to the reluctivity determined at the first non-zero set of data. This results in a spline representation of this part of the characteristic, of which the derivative is not monotonically increasing. This is not related to the possibly intended modelling of the Rayleigh-part of the material characteristic, which similarly results in non-monotonically increasing derivatives.

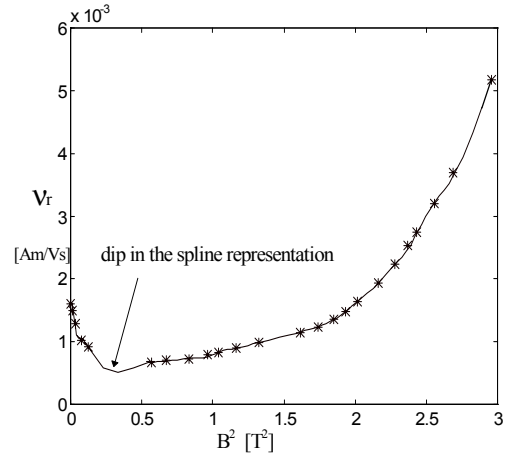


Fig. 3 Cubic spline representation of the  $vB^2$ -characteristic based on the  $v$ -values of the original data.

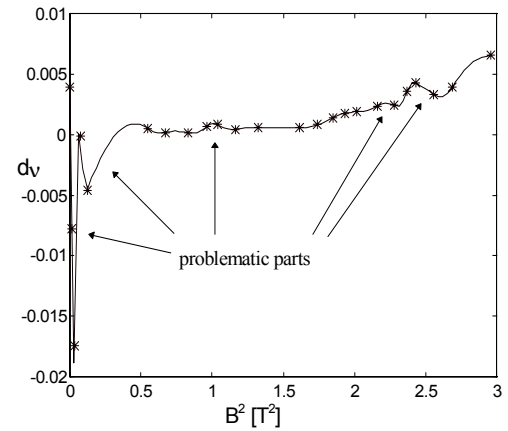


Fig. 4 Derivative of the cubic spline representation in Fig. 3.

To avoid slow convergence of the Newton iteration, two different solutions are possible:

1. Apply a different numerical representation of the original material properties (e.g. approximation techniques instead of interpolations).
2. Correct the erroneous original data set in such a way that the cubic spline representation is technically smooth.

A technically smooth material characteristic is defined by:

- the corrected data set differs only marginally from the original one,
- the reluctivity is continuous, locally smooth, two times differentiable and having one minimum only (if modelling the Rayleigh-region),
- the first derivative must be smooth and monotonically increasing.

The first approach is not applicable if the user of the system has no access to the source code of the FE solver module. Furthermore, it can be computationally expensive, if the material representation is constructed at the start-up-phase of the processing module for each non-linear material present in the model. The second approach has its advantage as it needs to be applied only once, outside the solver module. It is therefore not time-critical with respect to the solution time of

the problem. This approach can be adopted to produce good results for existing finite element packages if the internally applied interpolation method is known to the user.

In *OLYMPUS*, our in-house finite element package, the  $vB^2$ -characteristic for each non-linear material is constructed at the start-up phase of the solver module, when reading the problem data. This allows the maintenance of a material library in ASCII-format. A standard natural cubic spline interpolation algorithm is applied.

### III. OPTIMIZATION OF THE MATERIAL DATA

To find such a technically smooth characteristic, a numerical optimization algorithm is applied subject to a new set of reluctivities  $\mathbf{v}$  that minimizes:

$$\min f(\mathbf{v}) = \sum_{i=2}^N i \frac{(v_i - v_{o,i})^2}{v_{o,i}^2} + p(S(\mathbf{B}^2)) + p(S'(\mathbf{B}^2)) \quad (6)$$

with  $N$  the number of original data sets,  $v_{o,i}$  the original reluctivities and  $S(\mathbf{B}^2)$ ,  $S'(\mathbf{B}^2)$  the spline representation of  $vB^2$  and its derivative respectively. The weighting of each squared difference with its index guarantees smaller deviations at data sets with larger field strength. This accounts for the fact that relative errors of the original set (e.g. due to reading errors) are always smaller at higher field values than at lower ones. The problem of identical reluctivities for the first two points is solved by allowing the second reluctivity to change while the first is fixed. The penalty terms are formulated by:

$$p(S(\mathbf{B}^2)) = \begin{cases} 0 & \forall (S(B_j^2) > S(B_{j-1}^2)), \\ \frac{S(B_{j-1}^2) - S(B_j^2)}{S(B_j^2)} & \forall (S(B_j^2) < S(B_{j-1}^2)), \end{cases} \quad (7)$$

with  $j = 2(1)4N$ ,  $0 < B_j^2 < B_N^2$  and

$$p(S'(\mathbf{B}^2)) = \begin{cases} 0 & \forall (S'(B_k^2) > S'(B_{k-1}^2)), \\ 10 \frac{S'(B_{k-1}^2) - S'(B_k^2)}{S'(B_k^2)} & \forall (S'(B_k^2) < S'(B_{k-1}^2)), \end{cases} \quad (8)$$

with  $k = 2(1)4$  per segment. It should be noticed that the penalty (8) for the derivative is computed at four points equally spaced over each segment of the original data set, whereas (7) uses four times the number of points over the whole range of the curve. The second penalty term (8) is only applied after an initial minimum in the reluctivities is passed. This allows a correct treatment of a possible Rayleigh-region. The higher weighting factor for the derivative is applied to enforce this penalty against the other.

This optimization problem is solved using the Hooke and Jeeves algorithm. The high number of free parameters ( $N-1$ ) leads to a high number of function evaluations, each requiring the construction of the spline interpolation. However, as this material correction is not time critical, such computational expense is acceptable. Typical data sets consist of 15-40 BH-pairs. The present set has 23 points, and the optimization problem is solved with 10363 function evaluations (overall execution time is 4s on a HP C160). The resulting material

characteristics are shown in figures 5 to 7. The change in the BH-characteristic is almost invisible. The enhancements become visible in figures 6 and 7. The dip in the characteristic between 0.1 and 0.5 T<sup>2</sup> is removed.

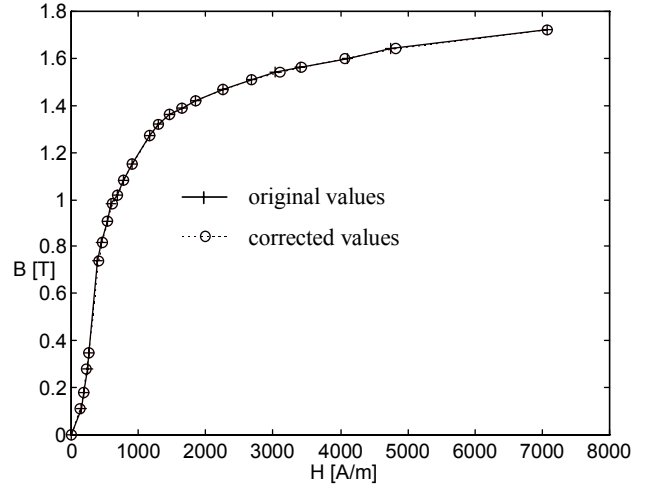


Fig. 5 Original and corrected BH-curve.

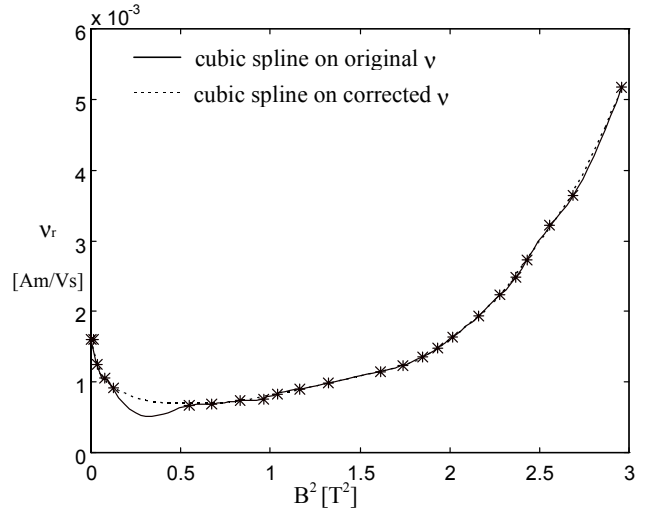


Fig. 6 Spline interpolation of the original and the corrected reluctivities.

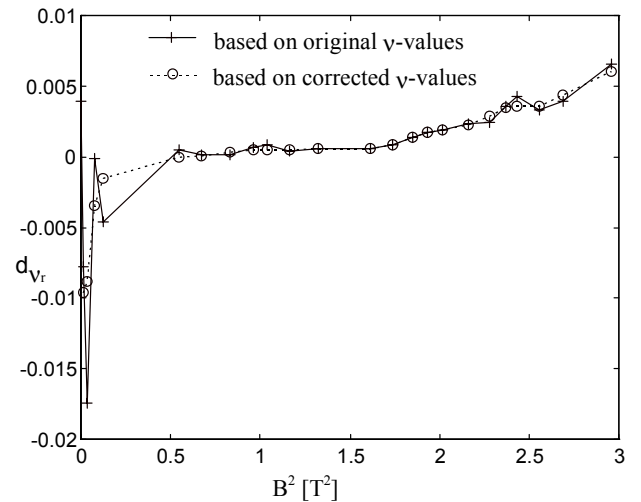


Fig. 7 Derivative of the spline interpolated reluctivities in the points of the original curve. The derivative is now monotonically increasing.

### IV. INFLUENCE ON THE NEWTON ITERATION

To illustrate the impact of a technical smooth ferromagnetic material characteristic on the convergence of the Newton process, the non-linear actuator problem in Fig. 8 is taken as an example. The stopping criterion for the Newton process is set to 0.01 % change. Interesting is the effect of reducing the number of Newton iterations in combination with adaptive mesh refinement. Less Newton iterations at the final refinement step reduce the overall solution time considerably. The elapsed solution time of the test model Fig. 8 is reduced by approximately 40%.

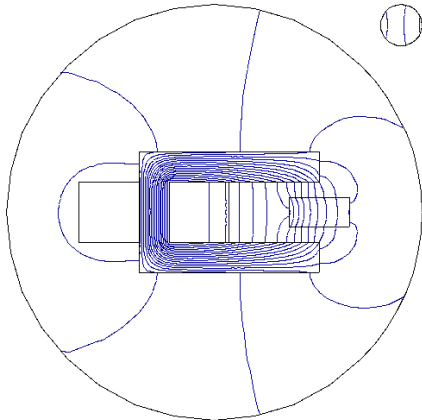


Fig. 8 Equi-potential plot of the C-core actuator model.

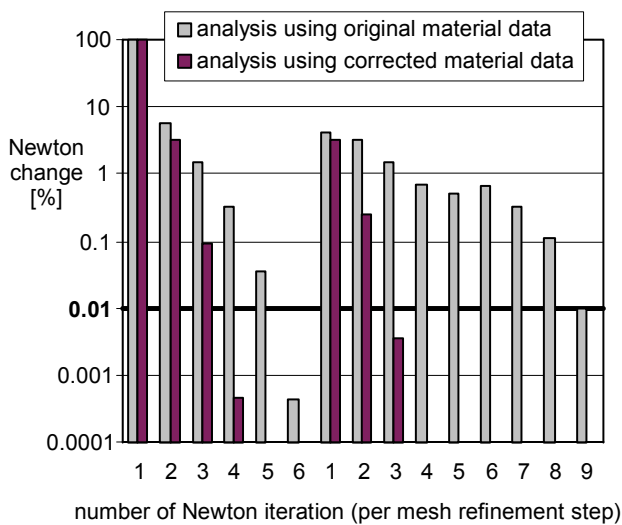


Fig. 9 Reduction of Newton steps due to the optimised material characteristic.

## V. CONCLUSION

An approach of correcting the material data to obtain an optimized and favourable numerical representation for solving non-linear problems including ferromagnetic material is introduced. It improves the convergence of the Newton algorithm in case of a unfavourable numerical representation of the erroneous original material data. The difference of the corrected data set compared to the original set is only marginal, justified by the error margins experienced for the original data sets.

This approach has effect on ferromagnetic characteristics that do not fulfil the technical smoothness criteria. Correcting the original data set is not time-critical, as it has to be applied

only once to each non-linear iron material before it is included into the material library of the finite element package used.

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## REFERENCES

- [1] R.Rosenberger, K.-D.Dettmann, K.Heuck, R.Kegel, "Ausgleichsverfahren zur Bestimmung technisch glatter Magnetisierungskennlinien als Grundlage zur Simulation elektrischer Netzwerke mit nichtlinearen Induktivitäten", *etzArchiv*, vol. 6, no. 3, 1984, pp. 115-120.
- [2] P.P.Silvester and R.L.Ferrari, *Finite elements for electrical engineers*, Third edition, Cambridge University Press, New York, 1996.
- [3] H.-P.Schwefel, *Numerical Optimization of Computer Models*, John Wiley & Sons, Chichester, New York, 1981.