MANUFACTURING TOLERANCES AS A CAUSE FOR AUDIBLE NOISE OF INDUCTION MOTORS

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1. INTRODUCTION

In designing and manufacturing induction motors, it is necessary to allow dimensional tolerances on every part, e.g. on stator and rotor stampings, stator frame, rotor assembly, bearings and endshields. Generally, the larger the tolerance, the lower the manufacturing cost [1]. On the other hand, the tolerances are often related to performance of the machine, and increases intolerance cause a deterioration in the performance; e.g. lower efficiency, torsional vibrations, radial oscillations, shorter lifetime due to increased overall temperature or hot spots. In several papers (e.g. [2,3,4]) it has been reported that audible noise data vary considerably between nominably identical small machines, and this variation is most likely to be caused by the variations of the dimensions of parts between the tolerance limits in mass production. For electric machines with small airgaps, such slight variations in the dimensions of the stator, rotor, end shields and bearings may lead to a considerable variation of the airgap at different angular positions between stator and rotor. This nonuniform airgap gives rise in operation to unbalanced magnetic pull. The u.m.p. due to a static eccentricity (incorrect positioning of the rotor with respect to the stator bore) is a static force, in most cases not causing any problem. In two-pole motor, and to a minor extend also in more pole machines, vibrating components at double supply frequency are present that may cause detrimental effects [5]. In a dynamically eccentric machine, i.e. a machine with a bended shaft due to mechanical unbalance, the unbalanced magnetic pull leads to a decrease of the critical speed of the machine [6]. Again a two pole machine gives a special forces component, having double slip frequency.

2. ECCENTRICITY FIELDS-UNDAMPED

When the rotor is eccentrically positioned with respect to the stator bore, the airgap length is a function of time and space:

$$\delta(\alpha,t) = \delta''\left\{1-\varepsilon\cos\left(\alpha-\omega_\varepsilon t-\varphi_\varepsilon\right)\right\} \tag{1}$$

$$\omega_\varepsilon = \begin{cases} 0 & \textit{for a static eccentricity} \\ (1-s)\,\dfrac{\omega}{p} & \textit{for a dynamic eccentricity} \end{cases} \tag{2}$$

(p: number of polepairs; s:slip; ω: supply pulsation). δ'' is the airgap length, increased for the slotting and saturation. The relative eccentricity ε is (e: eccentricity)

$$\varepsilon = \frac{e}{\delta''} \tag{3}$$

The airgap permeance is the inverse of the airgap length:

$$\lambda(\alpha,t) = \frac{1}{\delta(\alpha,t)} = \sum_{\lambda=0}^{\infty}\Lambda_\lambda \cos\left[\lambda\left(\alpha-\omega_\varepsilon t-\varphi_\varepsilon\right)\right] \tag{4}$$

$$\Lambda_\lambda = \begin{cases} \dfrac{1}{\delta''}\dfrac{1}{\sqrt{1-\varepsilon^2}} & (\lambda = 0) \\ \dfrac{2}{\delta''}\dfrac{1}{\sqrt{1-\varepsilon^2}}\left(\dfrac{1-\sqrt{1-\varepsilon^2}}{\varepsilon}\right)^{\lambda} & (\lambda > 0) \end{cases} \tag{5}$$

For a small relative eccentricity:

$$\lambda(\alpha, t) \approx \frac{\mu_0}{\delta''}\left\{1 + \varepsilon\cos\left(\alpha - \omega_\varepsilon t - \varphi\right)\right\} \tag{6}$$

During transients, insufficient time is available to generate the large vibration amplitudes required for increased and sustained audible noise. Therefore, with regard to audible noise, the steady state condition is important and therefore, the slip is small $\left(0 \le s \le s_{rated}\right)$. If damping by rotor currents is neglected, the resulting airgap flux density is

$$\begin{aligned} b(\alpha, t) &= B_p\cos\left(p\alpha - \omega t - \varphi_m\right) \\ &\quad + \frac{\varepsilon}{2}B_p\Big\{\cos\left[(p+1)\alpha - (\omega + \omega_\varepsilon)t - (\varphi_m + \varphi_\varepsilon)\right] \\ &\qquad + \cos\left[(p-1)\alpha - (\omega - \omega_\varepsilon)t - (\varphi_m - \varphi_\varepsilon)\right]\Big\} \end{aligned} \tag{7}$$

with B_p the almost constant, undamped amplitude of the fundamental field in the machine. This situation is definitevely true when the machine is supplied directly from the grid, where a constant voltage is applied. When the induction motor is supplied from a frequency inverter, field weakening is applied in the high speed range (above base speed given by the motor rated voltage and frequency and the inverter dc link voltage). This clearly indicates that the electromagnetically generated audible noise components get less pronounced at high speed. Furthermore, at high speed the airborne audible noise dominates. These supplementary components are the basis of the extra forces and audible noise components that are observed in practice [7-10].

3. INFLUENCE OF DAMPING

The eccentricity fields are damped by currents in the rotor bars, both in amplitude and in phase. The phase shift is not considered here, as it does not influence the force components to a major extend. Therefore, the imaginary part is neglected and the factor is [11]:

$$\alpha_{p\pm1} = 1 - \frac{s_{p\pm1}^2}{\beta_{p\pm1}^2 + s_{p\pm1}^2}\frac{k^2\xi_{p\pm1}^{*''2}}{1 + \sigma_{gRp\pm1}''} \tag{8}$$

with the slip

$$s_{p\pm1} = \begin{cases} s \mp \dfrac{1-s}{p} & \textit{static eccentricity} \\ s & \textit{dynamic eccentricity} \end{cases} \tag{9}$$

At synchronous speed (s = 0), the static eccentricity is damped while the dynamic eccentricity remains undamped. The amplitudes of the eccentricity fields are

$$B_{\varepsilon,p\pm1} = \frac{\varepsilon}{2}B_p\alpha_{p\pm1} \tag{10}$$

with number of polepairs, pulsations and phase angles:

$$\nu_{\varepsilon,p\pm1} = p \pm 1 \tag{11}$$

$$\omega_{\varepsilon,p\pm1} = \omega \pm \omega_{\varepsilon} \tag{12}$$

$$\varphi_{\varepsilon,p\pm1} = \varphi_m \pm \varphi_{\varepsilon} \tag{13}$$

When calculating the u.m.p., the damped amplitudes have to be used. For the radial forces other than eccentricity generated, the damping may be neglected, as it concerns mostly high pole fields yielding $\alpha_{\mu} \approx 1$. The amplitudes of the components are the product of the stator harmonics, damped by the rotor and accounting for the addition using the appropriate phase angles for the slot and slotting harmonics. When an eccentricity is present, the eccentricity harmonics have to be taken into consideration. The undamped eccentricity harmonics (amplitude $\frac{\varepsilon}{2} B_p$) induce at rated speed in the rotor bars current having the r.m.s.-value:

$$\underline{I}^{"}_{R,\varepsilon p\pm1} = -\frac{js_{\varepsilon,p\pm1}}{\left(\beta_{p\pm1} + js_{\varepsilon p\pm1}\right)} \frac{k\xi^{*}_{p\pm1}}{\left(1+\sigma^{"}_{g,R,p\pm1}\right)} \cdot \frac{\delta^{"} \frac{\varepsilon}{2} B_{pu}}{\mu_0 \sqrt{2}} \tag{14}$$

with pulsation

$$\omega_{\varepsilon,p\pm1} = \omega\left[1 - \frac{(p\pm1)(1-s)}{p}\right] \tag{15}$$

These currents generate rotor harmonics, having the amplitudes:

$$B_{\varepsilon,p\pm1} = \frac{\mu_0}{\delta^{"}} k\xi^{*}_{\lambda,p\pm} \underline{I}_{R\varepsilon,p\pm1}\sqrt{2} = \frac{s_{\varepsilon,p\pm1}}{\sqrt{\beta^2_{p\pm1} + s^2_{\varepsilon,p\pm1}}} k^2 \frac{\xi^{"}_{\lambda,p\pm1}\xi_{p\pm1}}{1+\sigma^{"}_{gR,p\pm1}} \left(\frac{\varepsilon}{2} Bpu\right) \tag{16}$$

with the number of polepairs

$$\lambda_{p\pm1} = g^{"} N_2 + (p\pm1) \tag{17}$$

and with the pulsation as seen by a stator observer:

$$\omega_{\varepsilon,p\pm1} = (\omega + \omega_{\varepsilon}) + \frac{g^{"} N_2 \omega (1-s)}{p} \tag{18}$$

When the teeth and the yoke are highly saturated, the amplitudes $B^{"}_{\varepsilon,p\pm1}$ may differ a lot from the results as shown here with an overall correction to the airgap $\delta^{"}$. Due to saturation, an extra reluctance can be introduced, being high where the flux density is high and vice versa [12]

$$\lambda_{sat}(\alpha,t) = \lambda_{average}\left[1 - \Delta\lambda_{saturation}\cos\left(2p\alpha - 2\omega_m t - 2\varphi_m\right)\right] \tag{19}$$

In the machine discussed further on, saturation is not critical.

4. STATOR FLUX DENSITY

In a three phase machine with q slots per pole and per phase, the flux density is composed of a number of waves (eccentricity and saturation neglected):

$$b(\alpha,t) = \sum_{\nu} B_{\nu} \cos\left(\nu\alpha - \omega t - \varphi\nu\right) \tag{20}$$

with

$$\nu = 6gp + 1 \quad with \;\; g = 0 \tag{21}$$

In general the slot and slotting harmonics are sufficient:

$$B_n^{'} = \left(K_c^{'} - 1\right)\xi_n^{'} B_{pu} \qquad \text{(slotting)} \tag{22}$$

$$B_{Wn}^{'} = \frac{p}{\nu}\frac{\xi_n^{'}}{\xi_p^{'}} B_{pu} \frac{I^{'}}{I_m}\alpha_n \qquad \text{(slot)} \tag{23}$$

with

$$n = p\left(6g^{'}q^{'} + 1\right) \qquad g^{'} = \pm 1, \pm 2 \tag{24}$$

$$\alpha_n = 1 - \frac{k^2 \xi_n^{*"2}}{1 + \sigma_{gRn}^{"}} \tag{25}$$

The resulting amplitudes of the undamped stator fields are ($\sin\varphi^{'} = \sqrt{1 - \cos^2\varphi^{'}}, \cos\varphi^{'}$ power factor):

$$B_{rnu}^{'} = \sqrt{B_n^{'2} + B_{wn}^{'2} + 2B_n^{'}B_{wn}^{'}\sin\varphi^{'}} \tag{26}$$

5. FORCE CALCULATION

The Maxwell stress from the combination of eccentricity and slotting is:

$$\sigma_{\varepsilon p \pm 1} = \frac{B_{\varepsilon p \pm 1} B_{rna}^{'}}{\mu_0} \tag{27}$$

They have the number of polepairs:

$$r_{\varepsilon, p\pm 1} = \lambda_{p\pm 1} \pm n = g^{"}N_2 + \left(p \pm 1\right) + n = \left[\left(g^{"}N_2 + p\right) \pm n\right] \pm 1 = r \pm 1 \tag{28}$$

They differ from the normal one by ± 1. An as such silent machine can become loud due to the fact that the frequency of the forces generated by the eccentricity are close to the natural frequencies. The frequencies of the radial forces are

$$f_{r\pm 1} = f_{\sup ply}\left\{\frac{g^{"}N_2}{p}(1-s) \pm \frac{\omega_\varepsilon}{\omega} + \begin{matrix}2\\0\end{matrix}\right\} \tag{29}$$

The ± is dealing with the p ± 1 choice, while $\begin{matrix}2\\0\end{matrix}$ deals with the sum a difference in the combination of trigonometric functions. For the static eccentricity, $\omega_\varepsilon = 0$, yielding as frequencies

$$f_{r\pm 1} = f_{\sup ply}\left\{\frac{g^{"}N_2}{p}(1-s) + \begin{matrix}2\\0\end{matrix}\right\} \tag{30}$$

6. PRACTICAL EXAMPLE

The following data are available of a practical motor that showed serious audible noise problems. Number of polepairs p=3. Number of stator slots N_1=54. Number of rotor slots N_2=68. Number of slots per pole and per phase in the stator q=3. Coil pitch in the stator w/τ_p=8/9. At rated speed, the slip is small compared to 1 (s≈1), leading to

$$f_{r\pm 1} = f_{\sup ply}\left\{\frac{g"68}{3} + {}^{2}_{0}\right\}$$

The results of this analysis are shown in the following table for a supply frequency of 40 Hz.

g	1	-1	2	-2	3	-3	4	-4
2	987	827	1893	1733	2800	2640	3707	3547
0	907	907	1813	1813	2720	2720	3627	3627

When these results are compared to the values as found in the measured curves, the evidence is obvious. A main audible noise frequency was experimentally present at 3625 Hz with two side bands at ±80 Hz. This is exactly predicted by the theory (both main frequency and side bands). Also the 2800 Hz value is correctly predicted. Also the lower audible noise values around 1800 Hz probably coincide with the theoretical values, but are not possible to assess in great detail on the measured curves.

7. CONCLUSIONS

It is demonstrated how a static eccentricity can contribute to the generation of audible noise. The influence of the supply frequency and the number of rotor bars is analysed. When the frequency of the generate forces coincides with a bending natural frequency of the stator assembly, problems may be expected. As the static eccentricity is not checked in the manufacturing process, the possibility is clearly present. It should be stressed that choosing a different number of rotor bars will not help, as the problem will occur at a different supply frequency. Only a reduction of the cause, i.e. the eccentricity will cure the problem. As the problem is due to the fundamental supply frequency, higher inverter switching frequencies will not influence the phenomenon.