SIMULATION OF TONER PARTICLE MOTION UNDER DYNAMIC FIELD CONDITIONS IN ELECTROSTATIC PRINTING APPLICATIONS

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Abstract

The development of printing processes based upon electrostatic projection of charged toner particles onto paper, has to be supported by appropriate numerical simulation tools. This paper presents the numerical trajectory computation for charged toner particles subject to both direct and alternating electric fields, which are applied using control electrodes. The total and instantaneous electric field is the weighted superposition of two single electrostatic solutions from the finite element method: one for the dc and one for the ac field component. The presence of other charged particles is modelled by a homogeneous space charge. Air viscosity and collisions with the print-head geometry are considered. Electric field smoothing using a post-solving technique is necessary to obtain realistic particle trajectories.

ELECTROSTATIC PRINTING

A typical print-head geometry and the appropriate nomenclature are shown in Figure 1. A rotating cylinder, acting as *applicator electrode* (voltage V4), introduces new toner into the print-head. Here the toner particles are subject to an electric field created by applying specific voltages to a set of electrodes. The *back electrode* (voltage V1) is positioned behind the (moving) paper surface: its voltage creates a constant force (dc field) that attracts the charged toner as soon as it has passed through the nozzle opening. Two *control electrodes* (voltages V2 and V3) are mounted on a thin dielectric board, the first one facing the applicator and the second one facing the back electrode. The second control electrode's dc



Figure 1: Typical print-head geometry and nomenclature

voltage is set to either keep the toner away from the nozzle opening or to pull the toner through the nozzle. This way the nozzle can be switched in *open* or *closed* mode using the control electrode, as has been discussed in [1]. The toner particles are taken off the applicator and are captured for a short time by an alternating electric field between the applicator and the first control electrode. As soon as the second control electrode changes potential, toner is admitted through the nozzle and is projected onto the paper.

The main parameters in the design are those determining the electric field in the print-head:

- the geometrical shape and size of the print-head,
- the spatial positioning of the electrodes,

• the magnitude of the electrode voltages, considering both dc and ac potentials.

Design constraints are mainly determined by the desired printing quality and speed, but also by paper thickness and toner characteristics. The electrode voltages are limited by the dielectric properties of the print-head. The total and instantaneous electric field must however be sufficiently powerful to control the toner flow. The amount of toner present between the applicator and the dielectric board will be referred to as the *toner cloud*. This cloud represents a *space charge* contributing significantly to the electric field in the print-head. Unfortunately for simulation purposes, the toner cloud charge density ρ_c does not reach a steady-state value, since the nozzle switching pattern will change continuously due to printing different levels of colour intensity. The free design parameters can be optimised using the focusing or *defocusing action* of the nozzle as the main criterion. To evaluate the nozzle's defocusing performance, it is necessary to introduce a *particle trajectory* calculation.

TRAJECTORY CALCULATION

The calculation of the trajectory of a charged particle with a given initial position and velocity can be based on analytical formulae [2] or on finite element analysis. Figures 2 and 3 show 2D and 3D axisymmetric finite element models of the print-head. Preliminary to the trajectory calculation, the electrostatic field component \mathbf{E}_{DC} of the electric field in the print-head is calculated

for a given set of electrode potentials, using finite element software. This field \mathbf{E}_{DC} is determined by all dc voltages applied to the electrodes and by the toner cloud space charge. To include an alternating electric field component \mathbf{E}_{AC} , a second field solution is calculated with the potential of the electrodes carrying an ac voltage set to the corresponding rms-value. Typical dc fields are shown in Figures 4 and 5 for the nozzle *open* and the nozzle *closed* mode.



Figure 2: axisymmetric 2D finite element print-head model (initial mesh)



Figure 3: 3D finite element print-head model





$$\mathbf{F}(\mathbf{s},t) = q \,\mathbf{E}(\mathbf{s},t) - 3\pi \mathbf{v} D \,\mathbf{v}(t) \,. \tag{1}$$

The first term in (1) represents the *Coulomb* force, with *q* the particle charge and $\mathbf{E}(\mathbf{s},t)$ the electric field at position $\mathbf{s}=[x \ y \ z]^{T}$ and time *t*. The second term represents the *Stokes* force due to the viscosity of air v, with *D* the particle diameter and $\mathbf{v}(t)$ the particle velocity. The electric field $\mathbf{E}(\mathbf{s},t)$ at the particle's actual position $\mathbf{s}(t)$ is obtained as a weighted superposition of \mathbf{E}_{DC} and \mathbf{E}_{AC} :

$$\mathbf{E}(\mathbf{s}, \mathbf{t}) = \mathbf{E}_{\mathrm{DC}}(\mathbf{s}) + \sqrt{2} \, \mathbf{E}_{\mathrm{AC}}(\mathbf{s}) \, \sin(2\pi \mathbf{f} \mathbf{t} + \boldsymbol{\varphi}) \,. \tag{2}$$

where f and φ are the ac frequency and phase. First, looking over the mesh, the finite element containing position $\mathbf{s}(t)$ is retrieved. Second, the field values $\mathbf{E}_{DC}(\mathbf{s})$ and $\mathbf{E}_{AC}(\mathbf{s})$ are obtained from the local shape function at $\mathbf{s}(t)$ and the direct and alternating field solution respectively. Both components add to the total instantaneous electric field $\mathbf{E}(\mathbf{s},t)$ using (2) and the force $\mathbf{F}(\mathbf{s},t)$ on the particle is calculated using (1). Now the acceleration $\mathbf{a}(t)$, velocity $\mathbf{v}(t)$ and position $\mathbf{s}(t)$ are updated to calculate the next step of the particle trajectory, timestepping at a rate Δt :

$$\mathbf{a}(t + \Delta t) = \mathbf{F}(\mathbf{s}, t) / m$$

$$\mathbf{v}(t + \Delta t) = \mathbf{v}(t) + \mathbf{a}(t + \Delta t) \cdot \Delta t$$

$$\mathbf{s}(t + \Delta t) = \mathbf{s}(t) + \mathbf{v}(t + \Delta t) \cdot \Delta t$$

$$t = t + \Delta t$$
(3)

(4)

FIELD SMOOTHING USING LOCAL POST-SOLVING

The method mentioned above for calculating the particle trajectory requires an accurate value of \mathbf{E}_{DC} and \mathbf{E}_{AC} throughout the finite element model since trajectory calculation is very sensitive to errors in the electric field solution. A high degree of mesh adaptation is a first requirement. To prove the need for additional field smoothing, a particle trajectory was calculated using an thoroughly and adaptively refined 2D mesh only. The following typical particle and print-head parameter values were used:

- particle diameter: 10 micron
- particle charge: -4.10^{-15} Coulomb
- particle mass: 3.10^{-13} kg
- applicator voltage and both control electrode voltages: 0 V (ground)
- back electrode voltage: 1000 V

Figure 6 shows two particle trajectories calculated with and without additional field smoothing for two initial positions. As can be seen from the clearly different calculated paths, a fine mesh is insufficient and additional field interpolation is necessary to obtain a correct particle trajectory. A method for 2D and 3D *local post-solving* was introduced in [3] and [4] respectively. This 2D post-process solves *Laplace's equation* in a circular region free of space charge, using polar co-ordinates (r, Φ) to obtain the electric potential $u(r, \Phi)$ inside the circle as an analytical expression:

$$u(r,\Phi) = \frac{\alpha_0}{2} + \sum_{n=1}^{\infty} r^n \left\{ \alpha_n \cos(n\Phi) + \beta_n \sin(n\Phi) \right\}$$
(5)



Figure 6: Particle trajectories calculated with (a) and without (b) additional field smoothing, for two initial positions $\mathbf{s}_{0,1}$ and $\mathbf{s}_{0,2}$.

The coefficients α_n and β_n are determined by the potential values $u_i(R, \Phi_i) = u(R, i \cdot \frac{2\pi}{N}), i = l(1)N$ on the circular boundary:

$$\alpha_{n} = \frac{2}{N \cdot R^{n}} \sum_{i=1}^{N} u_{i} \cos(n\Phi_{i})$$

$$\beta_{n} = \frac{2}{N \cdot R^{n}} \sum_{i=1}^{N} u_{i} \sin(n\Phi_{i})$$
(6)

These values $u_i(R, \Phi_i)$ are retrieved from the finite element solution using the local shape function. The analytical expression (5) for the potential can be *derived* to obtain an expression for the electric field **E** and *evaluated* at the centre of the circle (*r*=0).

This leads to an accurate estimate of the field E_c in the centre of the circle:

$$E_{x}\Big|_{r=0} = -\frac{\partial u}{\partial x}\Big|_{r=0} = -\alpha_{1} = -\frac{2}{N \cdot R} \sum_{i=1}^{N} u_{i} \cos \Phi_{i}$$

$$E_{y}\Big|_{r=0} = -\frac{\partial u}{\partial y}\Big|_{r=0} = -\beta_{1} = -\frac{2}{N \cdot R} \sum_{i=1}^{N} u_{i} \sin \Phi_{i}$$

$$(7)$$

This post-solving technique can be used to obtain field values \mathbf{E} everywhere inside the circle whenever the circular area is free of space charge. However, when the circular area has a constant non-zero space charge (e.g. due to the toner cloud), the same formulae can be used but only for the field \mathbf{E}_{c} in the centre. Due to symmetry, a constant space charge in the circular region does not contribute to the field in the centre of the circle.

SIMULATIONS

To evaluate the overall performance of a specific print-head set-up, two analyses have to be performed:

1. a characteristic analysis of a small set of *representative* particle trajectories,

2. a statistical analysis of a large set of randomly chosen particle trajectories.

This way it is possible to evaluate the specific characteristics of the electric field for the set-up chosen, as well as the defocusing action of the print-head for various random initial conditions of particle position and velocity.

The presence of other charged toner particles between applicator and control electrode, the toner cloud, is modelled by a homogeneous space charge in this area. The magnitude of this space charge can be estimated roughly using experimental data of toner supply. The space charge is taken into account in the electrostatic problem definition and contributes to E_{DC} directly. The toner cloud space charge has a

considerable effect on the particle trajectory. Collisions of toner particles with the print-head geometry are detected and modelled as ideal elastic collisions. The relative importance of several design parameters can now be investigated based on the resulting toner particle trajectories, performing both a characteristic and a statistical analysis.

Influence of back electrode and control electrode voltage

The back electrode's electrostatic field is due to attract the toner particles as soon as they have passed through the nozzle opening. The control electrode facing the back electrode (voltage V2) can be set to repel the toner from the nozzle opening, or to attract the toner representative it. Figure 7 shows particle to trajectories for several values of the second control electrode potential V2 relative to the back electrode's potential V1 (no ac field). Negative values of V2/V1 refer to a 'nozzle closed' mode. This analysis can lead to an appropriate setting of the nozzle *closed* voltage.



Influence of the ac field

The goal of the ac field is to keep the toner particles moving between the applicator and the closed nozzle until the nozzle opens. Due to the great number of

toner particles in the cloud, the ac excitation maintains a continuous colliding between the particles and the geometry, rather than a coherent movement of the entire cloud. The ac field frequency is set to 2 kHz. Figure 8 shows, for an *open* nozzle, several particle trajectories for different levels of ac potential. In some cases, too high an ac field will be able to push toner through a *closed* nozzle, overruling the dc field. This analysis can be used to balance dc and ac field components.





Figure 9: Particle trajectory end positions and time in histogram format

Statistical analysis

Figure 7: Particle trajectories for several values of V2/V1

For randomly chosen initial conditions like position, velocity and ac field phase, a great number of particle trajectories is calculated and the end position and time is stored. The end positions can be visualised using a histogram to represent the dot the toner leaves on the paper. The end position and total travelling time for 1000 particle trajectories with randomly chosen initial conditions (the ac phase was fixed to 0°) was calculated. The histogram in Figure 9 shows the end positions and travel times for those particle that reached the paper surface. The majority of these particles reaches the paper after 700 ms. The dot formed by the toner on the paper is fairly uniform. This analysis can be used to determine the effect of several parameters on the dot formation, e.g. the ac field's phase, the nozzle *open* voltage, etc.

CONCLUSIONS

To simulate, analyse and optimise the performance of an electrostatic printing device, it is necessary to calculate the trajectory of toner particles under combined dc and ac field conditions. The finite element analysis requires an adaptively refined mesh as well as additional field smoothing (post-solving). The total and instantaneous field inside the print-head is obtained as a weighted sum of two single electrostatic finite element solutions. A characteristic analysis can be performed based on representative trajectories. A statistical analysis can be performed based on a large number of trajectories with random initial conditions. These analyses allow for optimising the print-head geometry, voltage levels or any other free design parameter.

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