FLOATING POTENTIALS IN VARIOUS ELECTROMAGNETIC PROBLEMS USING THE FINITE ELEMENT METHOD

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Abstract
An efficient method for the treatment of floating potentials, associated with the finite element method and applied to various electromagnetic problems, i.e. electrostatics, electrokinetics, magnetostatics and magneto-thermal coupling, is presented. A key feature of the method is that it leads to naturally define global flux quantities associated with floating potentials. Advantages are numerous, e.g. local and global quantities are coherently coupled within the finite element model and the matrix of the system remains symmetrical.

INTRODUCTION

Many electromagnetic problem formulations make use of scalar potentials, of which the gradient is a physical vector field (e.g. $\mathbf{e} = -\,\mathrm{grad}\,\mathrm{v}$ where $\mathbf{e}$ is the electric field and v is the electric scalar potential, in electrostatics). These potentials define fields of local quantities in the studied domain and can be approximated with the finite element method.

Certain boundary conditions on parts of the boundary of the studied domain can imply the definition of floating values for scalar potentials [1, 2, 6, 7]. A floating value is an unknown constant on a region and comes from a homogeneous boundary condition for the tangential component of the associated physical vector field (e.g. $\mathbf{n} \times \mathbf{e} = -\,\mathbf{n} \times \mathrm{grad}\,\mathrm{v} = 0$ on surface Γ implies that v is a constant on Γ). Also, in some cases, it can be advantageous to extract some regions from the studied domain when they exhibit particular properties, usually large values of physical properties, to prevent numerical difficulties during computation. Therefore, only the boundaries of these regions have to be taken into account in the finite element model. This results in defining associated boundary conditions leading again to floating potentials.

A natural and general method can be applied to efficiently take floating potentials into account in the frame of the finite element method. It only makes use of the information contained in the weak finite element formulation of the main problem, without any intermediate computational procedure, to express in the correct weak sense the global flux quantities which are always necessary for a complete definition of floating potentials [6, 7]. The method is described and applied to electrostatics, electrokinetics, magnetostatics and magneto-thermal coupling, to naturally define electric charges and floating potentials, currents and voltages, magnetic fluxes and magnetomotive forces, and heat fluxes and floating temperatures, respectively. Illustrative examples are given to point out the main characteristics of the method.

FLOATING POTENTIALS

Discrete scalar potentials and their formulations

A Green formula is generally involved in the establishment of weak formulations of partial differential equations using scalar potentials or scalar fields. It is the *grad–div* formula applied to a domain Ω of boundary Γ, i.e.

$$(\,\mathbf{u}\,,\,\mathrm{grad}\,\mathrm{v}\,)_\Omega + (\,\mathrm{div}\,\mathbf{u}\,,\,\mathrm{v}\,)_\Omega = <\,\mathbf{n}\cdot\mathbf{u}\,,\,\mathrm{v}\,>_\Gamma\,, \qquad (1)$$

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where $\mathbf{u} \in \mathbf{H}^1(\Omega)$ and $v \in H^1(\Omega)$; $(\cdot\,,\cdot)_\Omega$ and $<\cdot\,,\cdot>_\Gamma$ respectively denote a volume integral in Ω and a surface integral on Γ of products of scalar or vector fields [7]; normal $\mathbf{n}$ is exterior to Ω. A suitable treatment of the surface integral term in (1) can be made to naturally define global quantities of flux types in a weak sense, as it will appear in the following. Those weak global quantities will be associated with the strongly defined ones, being floating potentials.

A discrete characterization is developed for a scalar field $v \in H^1(\Omega)$ in *grad–div* formula (1). Such a field is generally discretized in a nodal finite element space, defined on a mesh of Ω and denoted $S^0(\Omega)$ [3, 4] — associated finite elements can be of various geometries and degrees, in 2D and 3D —, i.e.

$$v = \sum_{n\in N} v_n\, s_n \;, \;\; v \in S^0(\Omega) \;, \tag{2}$$

where N is the set of nodes of Ω, s_n is the nodal basis function associated with node n and v_n is the value of v at node n. Functions s_n, $\forall\, n \in N$, form a basis for the nodal finite element space without constraint, e.g. boundary conditions or fixed global quantities. In case constraints exist, functions s_n in (2) are no longer linearly independent, i.e. relations exist between some of their coefficients. The direct expression of these constraints reveals the basis functions to consider, i.e. which can serve as test functions in the finite element method.

Such a potential v can be involved in a scalar potential formulation of the generalized problem

$$\mathbf{r} = -\,\mathrm{grad}\, v \;, \quad \mathrm{div}\, \mathbf{s} = \eta \;, \quad \mathbf{s} = \alpha\, \mathbf{r} \;. \tag{3-4-5}$$

Note that (3) comes from an equation of the form curl $\mathbf{r} = 0$, and in case this original equation contains a source term $\mathbf{k}_s$, i.e. curl $\mathbf{r} = \mathbf{k}_s$, (3) becomes $\mathbf{r} = \mathbf{r}_s - \mathrm{grad}\, v$ where $\mathbf{r}_s$ is a source field satisfying curl $\mathbf{r}_s = \mathbf{k}_s$. Generalized fields $\mathbf{r}$, $\mathbf{s}$, v, η, $\mathbf{k}_s$ and characteristic α will be particularized to physical quantities involved in some physical problems in the next section.

The scalar potential weak formulation is obtained from the weak form of (4), together with (3) and (5), i.e.

$$(-\alpha\, \mathrm{grad}\, v, \mathrm{grad}\, v')_\Omega - <\mathbf{n}\cdot\mathbf{s}_s\,, v'>_{\Gamma_s} + (\eta, \mathrm{grad}\, v')_\Omega = 0 \;, \; \forall\; v' \in F_r^0(\Omega) \,, \tag{6}$$

where $\mathbf{n}\cdot\mathbf{s}_s$ is a constraint on the generalized flux density $\mathbf{s}$ associated with nonfixed potential boundaries Γ_s of domain Ω, e.g. on floating potential boundaries Γ_f, $f \in C_f$. $F^0(\Omega)$ is a function space of scalar fields defined in Ω, with essential boundary conditions when subscripted.

Floating scalar potential constraints

In order to explicitly define constraints of floating potential type, the nodes of Ω are classified in complementary subsets: N_v, which is the set of nodes inside Ω, and N^f, $\forall\, f \in C_f$, which are the sets of nodes of parts Γ_f (Fig. 1). Floating potentials being constant on each Γ_f, (2) can then be decomposed as

$$v = \sum_{n\in N_v} v_n\, s_n + \sum_{f\in C_f} v^f\, s^f \;, \;\; v \in S^0(\Omega) \;, \;\; \text{with} \;\; s^f = \sum_{n\in N^f} s_n \;, \;\; \forall\, f \in C_f \;, \tag{7-8}$$

where s_n, $\forall\, n \in C_n$, and s^f, $\forall\, f \in C_f$, are basis functions for the constrained potential.

Each function s^f is associated with the group of nodes — a global geometrical entity, while nodes $n \in N_v$ are elementary entities — of boundary Γ_f (Fig. 1). The support of s^f (i.e. its domain of nonzero values) is limited to a transition layer containing all the geometrical elements having nodes on Γ_f.

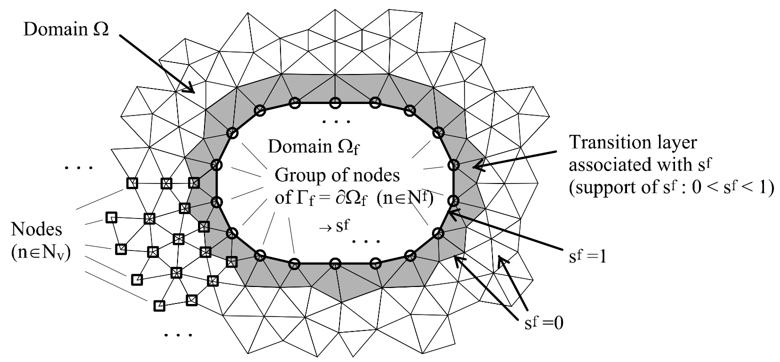


Fig. 1. Nodes and groups of nodes associated with the characterization of a scalar potential with floating values (7).

Discrete global fluxes

The discretization of the generalized weak formulation (6), by using test functions appearing in (7), gives regular symmetrical systems of equations. Test functions s_n, $\forall\, n \in N_v$, are classically treated, while test functions s^f, $\forall\, f \in C_f$, need attention.

The surface integral term in (6) has, for test function s^f, equal to one on Γ_f [3], [4], a contribution equal to $<\mathbf{n}\cdot\mathbf{s}_s,1>_{\Gamma_f}$ and thus to the flux of $\mathbf{s}_s$ leaving Ω through surface $f\in C_f$, noted ψ^f. This contribution is related to a physical flux. Then, to the surface integral on Γ_f in (6) can be substituted the value of the global flux ψ^f, i.e.

$$< \mathbf{n}\cdot\mathbf{s}_s , v' >_{\Gamma_f} = < \mathbf{n}\cdot\mathbf{s}_s , 1 >_{\Gamma_f} = \Psi^f \ , \quad \text{for } v' = s^f \in S^0(\Omega),\ f \in C_f . \tag{9}$$

Consequently, the computation of the global flux can be performed in average by the volume integral in (6) in a transition layer (support of s^f ; Fig. 1), i.e.

$$\Psi^f = (-\alpha \operatorname{grad} v, \operatorname{grad} s^f)_\Omega + (\eta, \operatorname{grad} s^f)_\Omega \ , \quad f \in C_f . \tag{10}$$

This approach is in perfect accordance with the discretized weak formulation of the problem, i.e. with (6), and thus with an only weakly satisfied conservation of flux. The computation of the global flux based on the explicit surface integration of $\mathbf{n}\cdot\mathbf{s}$ (i.e., $-\alpha\ \mathbf{n}\cdot\operatorname{grad} v$) would be affected by the choice of the integration surface. There would be generally no reason for the so computed flux to be equal to the flux given by the volume integral in the transition layer, whatever the surface is.

Compared to another method considering floating regions as whole volume regions with sufficiently high values of the generalized physical characteristic α, the proposed approach has the advantage of directly giving flux quantities. This can be useful for the computation of lumped parameter models or when both potential and flux have to be considered, e.g. in case of physically coupled problems. Moreover, the number of unknowns is lower with the proposed method, while keeping a symmetrical matrix of the system of equations as the discrete space defined by (7) concerns both test and shape functions.

For a fixed potential region, (10) can also be used for an efficient computation of the global flux ψ^f at the post-processing stage (this has similarities with the method in [5]). For that, it is sufficient to define also basis functions of type s^f (8) for such regions.

APPLICATIONS TO ELECTROMAGNETIC PROBLEMS

Applications of floating potentials to electromagnetic problems are numerous. The proposed method is applied to electrostatics, electrokinetics, magnetostatics and magneto-thermal coupling, to naturally define floating potentials — of electric, magnetic or thermal types —, and their associated global flux quantities, being respectively electric charges, electric currents, magnetic fluxes and heat fluxes. All these problems involve similar properties and are derived from the generalized problem (3-4-5) as shown in Table 1. Illustrative examples are given below to point out the main characteristics of the method.

TABLE 1. APPLICATION OF THE GENERALIZED PROBLEM OF FLOATING POTENTIALS TO PARTICULAR PHYSICAL PROBLEMS

Generalized problem	Electrostatics	Electrokinetics	Magnetostatics	Magneto-thermal (thermal part)
Equations				
curl $\mathbf{r} = \mathbf{k}_s$	curl $\mathbf{e} = 0$	curl $\mathbf{e} = 0$	curl $\mathbf{h} = \mathbf{j}_s$	curl $\mathbf{p} = 0$
div $\mathbf{s} = \eta$	div $\mathbf{d} = \rho$	div $\mathbf{j} = 0$	div $\mathbf{b} = 0$	div $\mathbf{q} = Q$
$\mathbf{s} = \alpha\,\mathbf{r}$	$\mathbf{d} = \varepsilon\,\mathbf{e}$	$\mathbf{j} = \sigma\,\mathbf{e}$	$\mathbf{b} = \mu\,\mathbf{h}$	$\mathbf{q} = k\,\mathbf{p}$
Potential def.				
$\mathbf{r} = \mathbf{r}_s -$ grad v	$\mathbf{e} = -$ grad v	$\mathbf{e} = -$ grad v	$\mathbf{h} = \mathbf{h}_s -$ grad ϕ with curl $\mathbf{h}_s = \mathbf{j}_s$	$\mathbf{p} = -$ grad T
Fields				
field $\mathbf{r}$	$\mathbf{e}$ = electric field	$\mathbf{e}$ = electric field	$\mathbf{h}$ = magnetic field	$\mathbf{p}$=gradient of temperature
field $\mathbf{s}$	$\mathbf{d}$ = electric flux density	$\mathbf{j}$ = current density	$\mathbf{b}$ = magnetic flux density	$\mathbf{q}$ = heat flux density
field η	ρ = electric charge density	—	—	Q = thermal source density
field $\mathbf{k}_s$	—	—	$\mathbf{j}_s$ = source current density	—
scalar potential v	v = electric scalar potential	v = electric scalar potential	ϕ = magnetic scalar potent.	T = temperature
characteristic α	ε = electric permittivity	σ = electric conductivity	μ = magnetic permeability	k = thermal conductivity
Global flux (weak sense)	electric flux, i.e. electric charge	electric current	magnetic flux	heat flux
Global floating potential	floating electric potential	floating electric potential	floating magnetic potential	floating temperature

Electrostatics

The boundary of a perfectly electric conducting region is, under static or quasi-static conditions, i.e. in electrostatics, an equipotential surface for the electric scalar potential. In case this equipotential value is unknown, it is of floating type and can be well defined by the knowledge of the electric charge contained in the region [1, 2, 6, 7]. The global flux ψ^f given by (9) is here the opposite of the total electric charge Q^f of Ω_f ($\partial\Omega_f = \Gamma_f$). In particular, the method enables an efficient computation of capacitances thanks to a coherent definition of both electric potential and charge [6].

An example of application concerns insulators leading high-voltage conductors through grounded walls, floors and metal tanks, which are called "bushings" [8] (Fig. 2). They consist of an insulator, mainly porcelain filled with oil, around the high-voltage conductor. This configuration suffers from local high electric field strengths on the triple junction area. This difficulty is overcome by the conductor bushing principle. A number of concentric conducting cylinders form a series connection of capacitors [9] and redistribute the electric field towards the top of the bushing. Two bushings, one without conducting cylinders (left in Fig. 3a), one with conducting cylinders (right in Fig. 3a) [10] are modelled applying axisymmetry. The vertical conductor is applied with a voltage of 75 kV. The bushing is fixed on a transformer tank on ground potential.

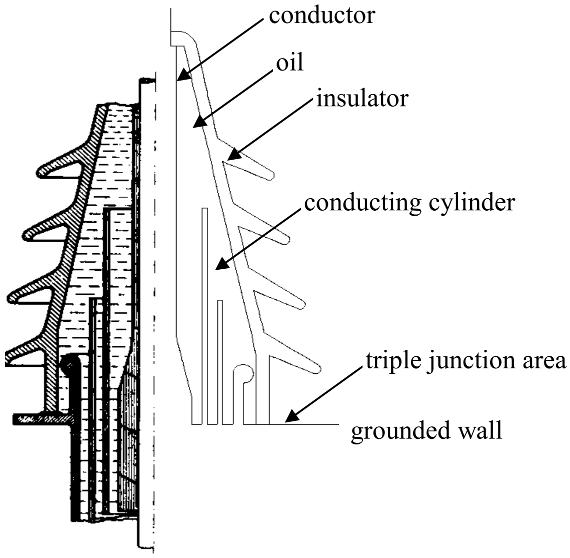
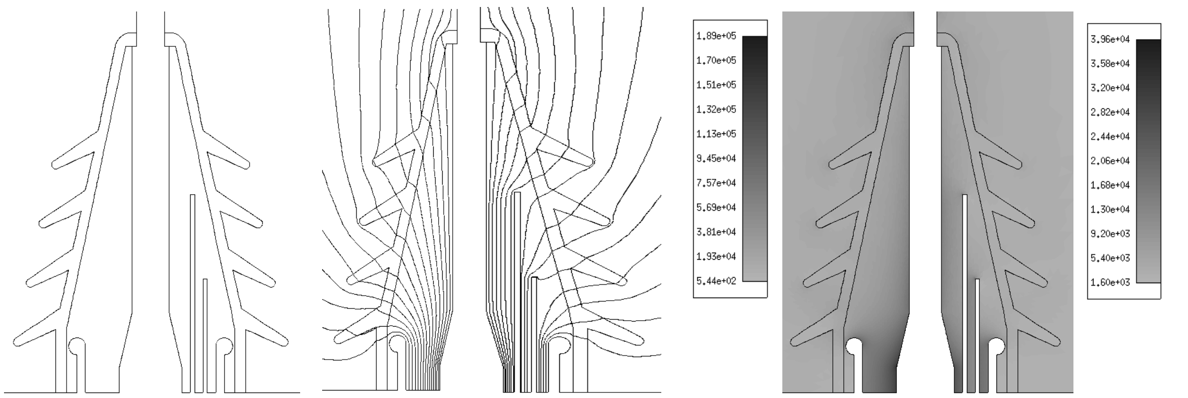


Fig. 2. Condenser bushing [10].

Floating potential boundary conditions are applied to model the equipotential surfaces of the conducting cylinders (Fig. 3a). The equipotential plots and the field strength plots of the electric fields are shown in Figs. 3b and 3c. The conducting cylinders push the electric field towards the top of the bushing. As a result, the electric field strength diminishes at the grounded tank whereas the field strength raises at the top of the bushing.



a) b) c)

Fig. 3. a) Geometry; b) equipotential lines and c) electric field strengths of non-condenser and condenser bushings.

Electrokinetics

An electrokinetic formulation using a scalar potential can benefit from floating potentials to define global quantities such as electric voltages and currents. Either voltages or currents can be prescribed, leading to the computation of the nonprescribed quantities and then to the determination of electric resistances. Fig. 4 shows a metallic plate of which the right surface is fixed at zero potential while the left one is associated either with a prescribed current and a floating potential, or a prescribed potential and an unknown current. Fig. 5 shows a ground in which a current enters and leaves through two surfaces and of which the resistance can be computed as well.

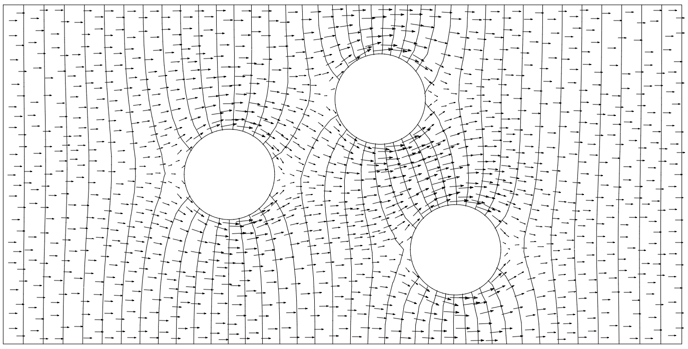


Fig. 4. Potential lines and electric current density in a metallic plate with circular cavities (the opposite sides are equipotential surfaces).

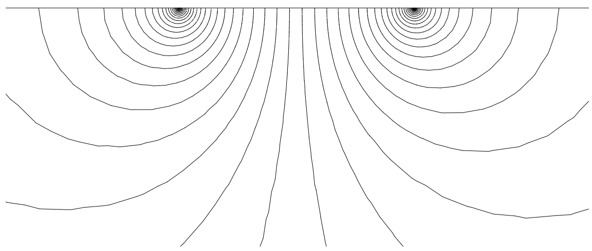


Fig. 5. Potential lines in the ground due to a current entering and leaving through two small surfaces.

Magnetostatics

A magnetic scalar potential can be of floating type on the boundary of a perfectly magnetically conducting region ($\mu \rightarrow \infty$) as well as on surfaces crossed by magnetic flux paths [7]. Here, the total flux ψ^f given by (9) is the magnetic flux. For the first case, this flux is directly taken to be equal to zero through the closed surface Γ_f of the perfectly magnetic region (Fig. 6). The theoretical limit $\mu \rightarrow \infty$ is practically obtained with a good accuracy when the permeability of the material is higher than 300 times the one of the exterior region. Then, the flux can be considered as a degree of freedom and its direct natural coupling with magnetomotive force and local magnetic field can be obtained. This enables the modeling of magnetostatic circuits with definition of associate lumped parameters, i.e. reluctances (Fig. 7) [7].

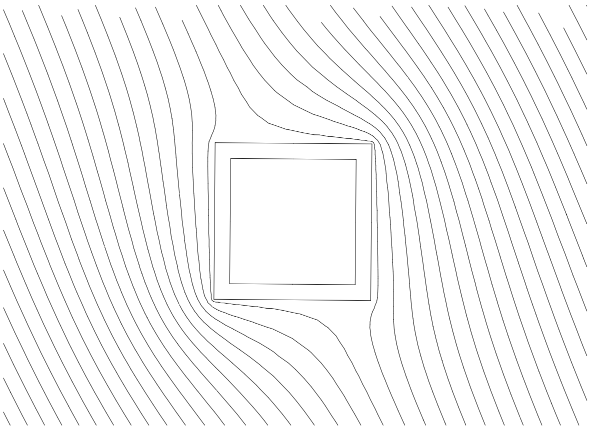


Fig. 6. Magnetic potential lines around a perfectly magnetic screen in an initially uniform oblique source field.

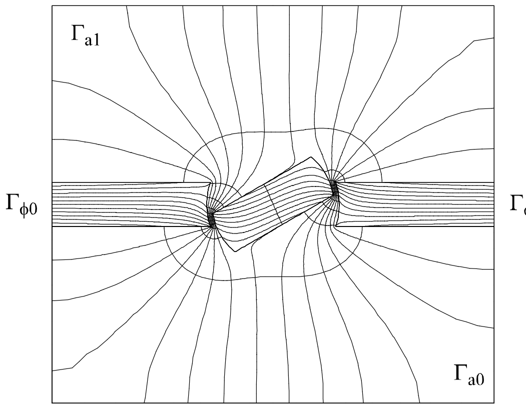


Fig. 7. Magnetic circuit element with injected magnetic flux through $\Gamma_{\phi 1}$ (potential lines and field lines).

Another example is a 2D magnetic vector potential of floating type on a boundary where no flux is passing. A region can be omitted out of the model if it is embedded in a region with a relatively high reluctance and when the boundary carries a floating potential. If the cooling channels in the stator and rotor of an asynchronous machine have small influence on the magnetic behavior of the device, the reluctance of the cooling air is far lower than the reluctance of the surrounding iron. The size of the discretization is reduced by omitting these regions. Floating potential boundary constraints make these boundaries impenetrable for a magnetic flux (Fig. 8).

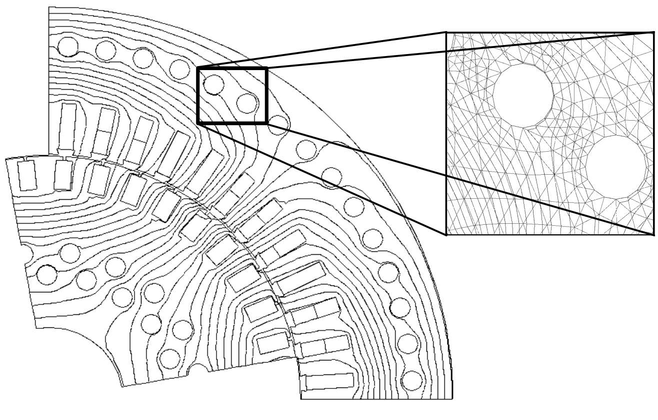


Fig. 8. Flux line plot and mesh detail of an asynchronous machine.

Magneto-thermal coupling

A thermal formulation can also use a floating temperature on boundaries of highly thermal conductive materials, naturally associated with heat flux as thermal source (computed e.g. from an magnetodynamic problem). Such a heat flux is directly the flux ψ^f given by (9).

Ohmic, dielectric and iron losses are responsible for the heat generation inside an electromagnetic device. The conductivity, permeability and remanence of materials depend on the temperature. Therefore, the magnetic and thermal fields are physically coupled. The numerical coupling of both phenomena gives often raise to excessive solution times [11]. With respect to coupled field problems it is important to reduce the thermal and magnetic mesh sizes. This can be done by omitting thermally highly conducting material out of the thermal problem and applying floating temperatures on those boundaries. The magnetic problem is reduced as described above. The floating potential approach has been successfully applied to a coupled thermal-electromagnetic calculation of a three-phase power cable [12].

CONCLUSIONS

A method has been proposed which finds its place into a general frame of definitions of floating potentials, these being global quantities related to fluxes. It has been applied to formulations of various physical problems, such as electrostatics, electrokinetics, magnetostatics and magneto-thermal coupling, to naturally define the implied global quantities in both weak and strong senses.

The generality of the method, which is set at the formulation level, enables its application to all kinds of geometrical models (2D or 3D), with linear or nonlinear material characteristics. The method is independent of the properties of the finite elements used (geometry and degree of basis functions). All the advantages of the method appear when local and global quantities have to be coupled, either within a finite element problem or through external lumped circuits, expressed by circuit equations, which directly opens the method to the coupling of physical problems.

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