A Topological Method used for Field-Circuit Coupling

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Abstract - **A topological method is applied to couple a magnetic field description with external electric circuits including both stranded and solid conductors simultaneously. No assumptions concerning the kind of circuit elements and connections are made. The matrix coupling is performed symmetrically and involves a minimum amount of additional equations. The method is demonstrated on two eddy current examples.**

INTRODUCTION

Field-circuit coupling is performed by adding extra equations, describing the external circuit, to the system matrix. From the magnetic point of view, it is obvious to model solid conductors and stranded conductors by unknown voltages and currents respectively [1]. In order to consider those restrictions, published implementations mainly consider one type of conductor only [2,3] and do not symmetrize the problem [1,4]; if considered, on the matrix level only [5]. Instead a unique way to describe an arbitrary connection of magnetically coupled conductors, impedances and sources is developed.

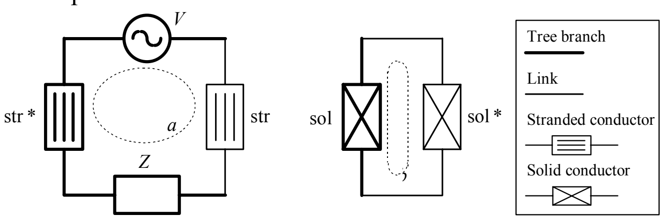

Fig. 1: Electric circuit with a) stranded and b) solid conductors

SIGNAL FLOW GRAPH

A *loop* is a circuit path that has the same begin and end node (Fig. 1). A *cutset* is a set of branches which removal splits the circuit in two parts. A *tree* is a set of branches that connects all nodes and has no loops. The set of the remaining branches is the *cotree*. A member of the cotree is a *link*. A *fundamental loop* is a loop formed by one link and a set of tree branches. A *fundamental cutset* is a cutset formed by one tree branch and a set of links [6].

It is assumed that the voltages sources contain no loops and that the current sources do not form a cutset. Branches are assembled in the tree in a given preference: voltage sources, solid conductors, impedances and stranded conductors. The preferred order for links is: current sources, stranded conductors, admittances and solid conductors.

A system of equations describing the circuit can be represented by a Signal Flow Graph (SFG). Each unknown is represented by a *node* of the graph. Each node equals the sum of the nodes of the incoming graph branches multiplied by the branch weights of the graph. A node with only outgoing branches is a *source node*. A node with only incoming branches is a *sink node*. A node with some incoming branches is a *dependent node*. A SFG is arranged by applying the Kirchhoff Current Law (KCL) for each fundamental cutset and the Kirchhoff Voltage Law (KVL) for each fundamental loop [6]. The unknowns of the system are the link currents and the tree branch voltages. The SFG of a stranded and a solid conductor is shown in Fig. 2. The SFG of the circuit of Fig. 1 is represented in Fig. 3.

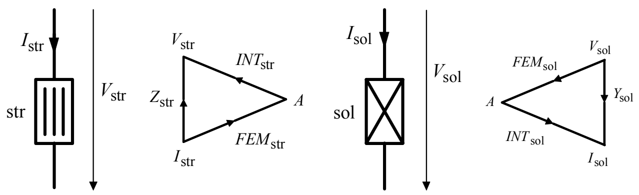

Fig. 2: SFG of a a) stranded and b) solid conductor

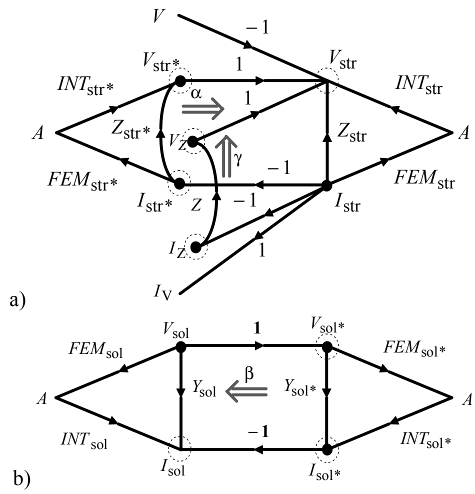

Fig. 3: Primitive Signal Flow Graph

Combining magnetically coupled branches causes two difficulties.

- stranded conductor tree branches are described by an unknown voltage (Fig. 3a),
- solid conductor links are described by an unknown current (Fig. 3b).

This can be surmounted in three steps.

1. *Partial cutset transformation*

   A cutset associated with a stranded conductor tree branch only exists of current sources and stranded conductors. A partial cutset transformation

$$\boldsymbol{I}_{str*} = -\boldsymbol{D}_{str*,i} \cdot \boldsymbol{I} - \boldsymbol{D}_{str*,str} \cdot \boldsymbol{I}_{str} \tag{1}$$

contracts the graph in direction α (Fig. 3a). Now, the stranded conductor is described by a linear combination of known and unknown currents.

2. *Partial loop transformation*
   A loop associated with a solid conductor link only exists of voltage sources and solid conductors. A partial loop transformation
   $$\boldsymbol{V}_{\mathrm{sol}*} = -\boldsymbol{B}_{\mathrm{sol}*,\mathrm{v}}.\boldsymbol{V} - \boldsymbol{B}_{\mathrm{sol}*,\mathrm{sol}}.\boldsymbol{V}_{\mathrm{sol}} \qquad (2)$$
   contracts the graph in direction β (Fig. 3b).
3. *Symmetrizing the system*
   Finally, a contraction in direction γ (Fig. 3a), leads to a Compact Signal Flow Graph (CSFG) (Fig. 4).

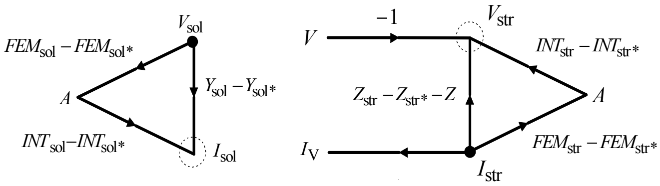

Fig. 4: Compact Signal Flow Graph

From this graph the coupling matrices are extracted easily. The unknowns are the unknown source nodes of the CSFG (Fig. 4). The additional equations are represented by the sink nodes of the CSFG (Fig. 4). The coupling terms are kept symmetric. Compared to other methods like tableau analysis, Modified Nodal Analysis (MNA) and Compact Modified Nodal Analysis (CMNA), a reduction of additional circuit equations is obtained.

NUMERICAL EXAMPLES

In the first example, eddy currents are induced in a conducting plane, passing between two symmetric inductors (Fig. 5). The external circuit is shown in Fig. 1. In the second example an induction machine at load is analyzed (Fig. 6). Stator windings and rotor bars are connected as shown in Fig. 7. The additional number of circuit equations of the different methods is collected in Table I.

TABLE I: Number of circuit equations

Circuit Analysis	Tableau	MNA	CMNA	SFG
Inductor	13	5	4	2
Induction motor	166	46	43	31

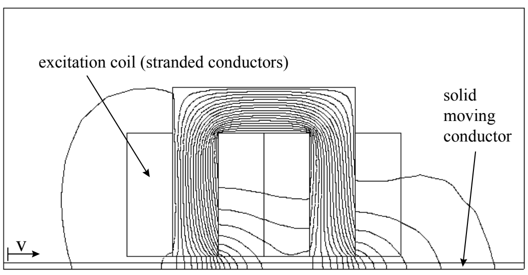

Fig. 5: Equipotential plot of an inductor and a conducting plane moving at 10 m/s to the right

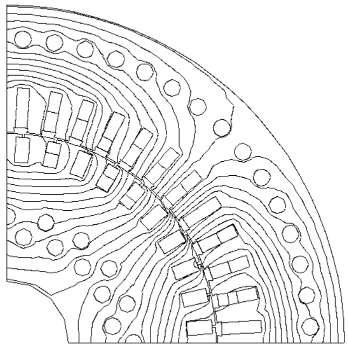
Fig. 6: Equipotential plot of a time-harmonic 50 Hz solution of a loaded induction motor

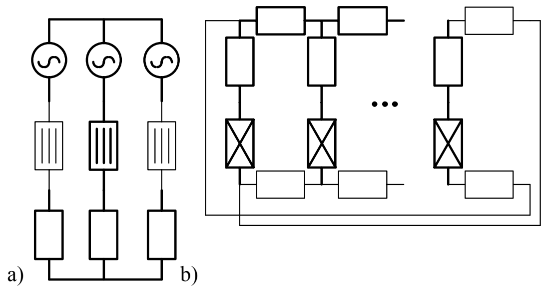

Fig. 7: Electric circuit of a) the stator and b) of the rotor of an induction motor

CONCLUSIONS

External circuits can be modelled by a Signal Flow Graph. The method adds a minimum set of additional equations to the system matrix. Both stranded and solid conductors, arbitrarily connected in the network, are described. The matrix retains symmetry.

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