88. ERROR ESTIMATION AND ADAPTIVE MESHING USING A DUAL APPROACH FOR THE STUDY OF A MICROWAVE CAVITY

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Abstract

A dual approach is used to compute the harmonic electric and magnetic fields for the problem of a 3D loaded microwave cavity (ACES TEAM Workshop Problem 19). The structure is modelled with edge finite elements, which are well adapted to the approximation of the fields of interest. The local error estimation, based on magnetic and electric constitutive laws, is built up in order to guide the mesh optimisation. The latter is performed using a h-technique. A special attention is paid to the specific difficulties due to the microwave resonance of the cavity.

INTRODUCTION

The duality of the microwave electric and magnetic field formulations is very attractive to perform mesh optimisation. The mesh optimisation is based on the a posteriori estimation of an error field that is a combination of the lack of fulfilment of the magnetic and electric constitutive laws. This error field constitutes the starting point of the mesh refinement process, it helps to distribute the characteristic mesh length for the adapted mesh (h-technique). The method is applied to the analysis of a three-dimensional resonant loaded cavity supplied with a TE_{10} mode coming from a rectangular waveguide (ACES TEAM Workshop Problem 19). The sharp edges of the iris and the dielectric rod in the middle of the cavity are particular places where the mesh quality is crucial. The rod is made of a dissipative dielectric modelled by a complex permittivity.

ELECTROMAGNETIC FORMULATIONS

The problem of the electromagnetic field distribution inside a resonant metallic cavity can be considered as an inner problem because no other exit way than the waveguide is possible. All the walls are made of sufficiently conductive metals in such a way that they act as quasi perfect reflectors. The problem of microwave distribution by the finite element method, using respectively the electric and the magnetic field formulation is expressed as

$$\int_D j\omega\varepsilon\,\mathbf{e}.\mathbf{e}' + \int_D (j\omega\mu)^{-1}\,\mathrm{curl}\,\mathbf{e}.\mathrm{curl}\,\mathbf{e}' = \int_{S_{ne}} \mathbf{n}\times\mathbf{h}^d.\mathbf{e}' \quad , \quad \text{in D} , \tag{1}$$

$$\int_D j\omega\mu\,\mathbf{h}.\mathbf{h}' + \int_D (j\omega\varepsilon)^{-1}\,\mathrm{curl}\,\mathbf{h}.\mathrm{curl}\,\mathbf{h}' = -\int_{S_{nh}} \mathbf{n}\times\mathbf{e}^d.\mathbf{h}' \quad , \quad \text{in D} , \tag{2}$$

* This author is a Senior Research Assistant with the Belgian National Fund for Scientific Research.

This text presents research results of the Belgian programme on Interuniversity Poles of Attraction initiated by the Belgian State, Prime Minister's Office, Science Policy Programming. The scientific responsibility is assumed by its authors.

where **e** and **h** are the electric and magnetic field vectors of phasors, **e'** and **h'** are test functions, μ is the magnetic permeability and ε the complex permittivity ($\varepsilon = \varepsilon' - j\,\varepsilon''$), D is the studied domain, S_{ne} and S_{nh} are the Neumann boundaries of the associated formulation, ω the angular frequency, $\mathbf{n}\times\mathbf{h}^d$ and $\mathbf{n}\times\mathbf{e}^d$ the tangential components of the a priori known magnetic and electric field on the Neumann boundaries. These formulations result from the minimisation of a functional also based on the lack of fulfilment of both magnetic and dielectric constitutive laws [1,4].

The meaning of the complex permittivity used in the harmonic formalism is the following: its real part represents the actual permittivity of the material, while its imaginary part simulates dissipation effect in dynamic polarisation phenomenon. In fact, it is linked to the loss factor (tan δ) by the relation

$$\varepsilon'' = \varepsilon_0 \varepsilon_r \tan(\delta) \ . \tag{3}$$

APPROXIMATION SPACE

e and **h** are vector fields whose tangential component must satisfy some continuity conditions. That is the reason why edge finite element spaces are the most appropriate approximation spaces to interpolate them [1, 3, 5]. Moreover, edge elements avoid the presence of spurious modes which can appear when other kinds of finite elements are used.

ERROR ESTIMATION AND MESH REFINEMENT

The discretization error in the dual approach is based on the lack of fulfilment of the constitutive relations [4,7]. The absolute error over an element E is defined by

$$e_E^2 = \int_E \frac{1}{\varepsilon}|\mathbf{d} - \varepsilon\mathbf{e}|^2\, dE + \int_E \frac{1}{\mu}|\mathbf{b} - \mu\mathbf{h}|^2\, dE \tag{4}$$

and the relative error over this element is defined by

$$\varepsilon_E^2 = \frac{e_E^2}{d^2} \ , \tag{5}$$

with

$$d^2 = \int_\Omega \frac{1}{\varepsilon}|\mathbf{d} + \varepsilon\mathbf{e}|^2\, d\Omega + \int_\Omega \frac{1}{\mu}|\mathbf{b} + \mu\mathbf{h}|^2\, d\Omega \ . \tag{6}$$

The global relative error ε over the domain Ω is defined as a function of the elementary errors, i.e.

$$\varepsilon^2 = \sum \varepsilon_E^2 \ . \tag{7}$$

This error is always positive and is equal to zero only when both constitutive relations are exactly satisfied in the whole domain. The goal of mesh adaptation is to obtain a prescribed error $\varepsilon^* = \varepsilon^0$ for an optimised mesh M^* while using a minimum number of elements to discretize the problem. Let M be a relatively coarse initial mesh. If h_E is the size of the element E in M and h_E^* is the size of the elements of M^* on the area defined by E, the classical convergence rate theory of finite element schemes leads to

$$\frac{\varepsilon_E^{*2}}{\varepsilon_E^2} = O\left(\frac{h_E^*}{h_E}\right)^{2p} = O(r_E)^{2p} \ , \tag{8}$$

where p is a constant depending on the polynomial order of the interpolating functions (p=1 for first order edge elements). The condition $\varepsilon^* = \varepsilon^0$ then becomes

$$\sum \varepsilon_E^2 r_E^{2p} = \varepsilon^{02} \ , \tag{9}$$

where the unknowns are the reduction factors r_E. It is the case of the h-refinement method. The number of elements N^* of the optimised mesh can be approximated by

$$N^* = \sum_E \frac{1}{r_E^3} \tag{10}$$

for three-dimensional discretization. The optimisation process consists therefore in minimising N^* while keeping $\varepsilon^* = \varepsilon^0$. This can be achieved by using Lagrangian multipliers and a golden section algorithm.

PRACTICAL PROBLEM

The under consideration cavity (ACES TEAM Workshop Problem 19 [6, 8]) consists in a cylindrical metallic box (diameter 9 cm) loaded with a dissipative centred rod (diameter 9 mm) and coupled, through an inductive rectangular iris, with a rectangular waveguide carrying a TE_{10} mode. The rod is made of a dielectric material of complex relative permittivity $\varepsilon' - j\,\varepsilon''$ equal to 4 - 1 j. The supply frequency is chosen near resonance, i.e. 2.412 GHz.

Actually, the walls are made of nonperfect conductors, which should necessitate the use of an impedance boundary condition [2]. However, a very good approximation is to consider perfect reflectors, where the electric tangential field is equal to zero. The use of homogeneous Dirichlet or Neumann conditions on symmetry planes, depending on the geometry and the kind of microwave excitation mode, is very useful. This allows to reduce the size of the system by a factor four in the case of the studied problem.

The source of electromagnetic waves is introduced by the imposition of the TE_{10} tangential electric field distribution at the entrance of the rectangular waveguide, where all evanescent modes due to the iris proximity are supposed to be negligible. Nevertheless, the imposition of the **e** profile is satisfied weakly in the case of the magnetic field formulation. Since areas concerned with zero tangential electric field are more extended (certain kinds of symmetry conditions and perfect conductors), the **e** problem includes less degrees of freedom and less weakly satisfied boundary conditions. That is why the **e** formulation leads to a better conditioned system of equations and to more accurate results for a given mesh.

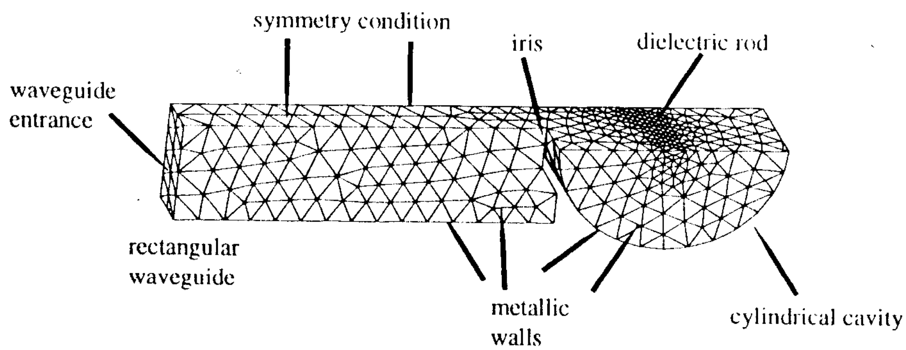

Fig. 1. A quarter of the studied structure with the initial mesh and the boundary conditions.

The problem of cavity resonance is relatively complex from the error estimation point of view. Indeed, the same mesh used with two different formulations leads to two different estimations of the resonance frequency. Consequently, the same mesh excited at a given frequency can give electromagnetic fields that are very different one from another inside the cavity, while they are quite similar inside the waveguide. For example, opposite signs of electric z component can be found when the supplied frequency is between the estimations of the resonance frequency given by the different formulations.

The more the quality factor is important, the greater the sensibility of the results will depend on the choice of a formulation.

As a consequence, the estimated field of electric error is expected to be more important inside the cavity, resulting to a sharper mesh refinement in this area. The dielectric rod will also present a greater electric error because of its greater permittivity (reduced wave length). As a consequence, the curvature of the radial electric field distribution is greater in the dielectric medium than in the air, which needs more elements, given the fixed edge element degree of shape functions. Because there is an important current along the vertical edge of the iris, a quasi singularity of the magnetic field happens at that point and consequently a greater error on the computed field.

Maps of elementary relative error given by the initial mesh are shown in Fig. 2.

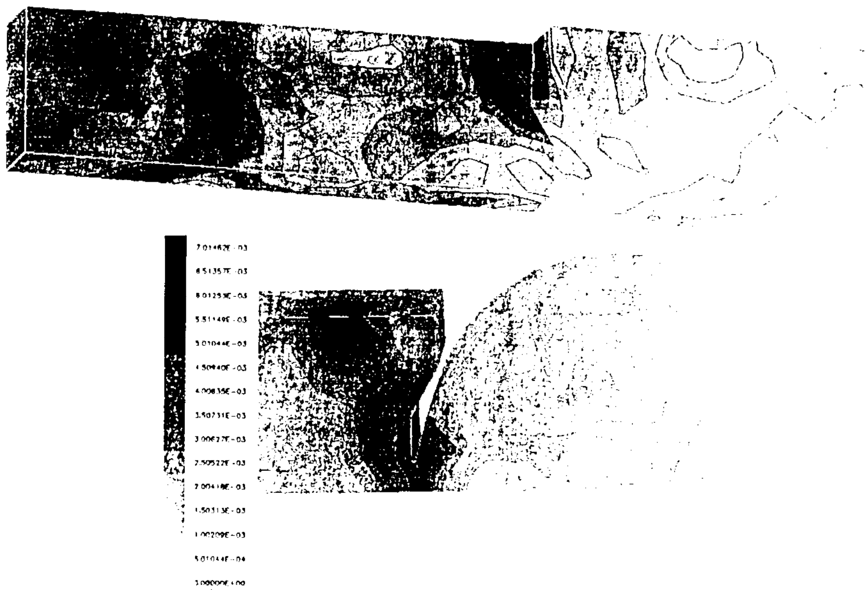

Fig. 2. Maps of error interpolated at nodes.

As expected, the greatest error is located along the iris edge, mostly near the side opposite to the symmetry plane. After two adaptive iterations, an adapted mesh (Fig. 3) is obtained by cutting the edges in order to respect the field of characteristic length. As it can be seen in Fig. 4, 5, 6 and 7, this last mesh increases the similarity between the results given by the dual formulations. These figures represent a cut of e_z or h_y obtained with both formulations and respectively with the initial and adapted mesh. The cut is taken parallel to the structure symmetry axis and close to the iris edge. Its left end is at the entrance of the waveguide while its right end is at the opposite side of the cavity.

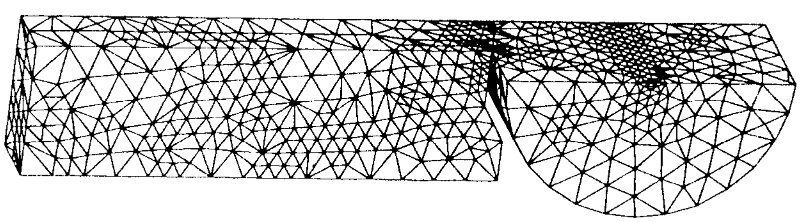

Fig. 3. Adapted mesh.

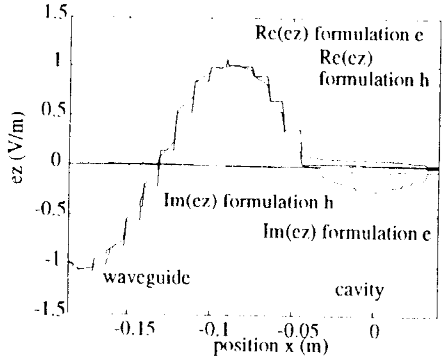

Fig. 4. Electric field distribution for the initial mesh.

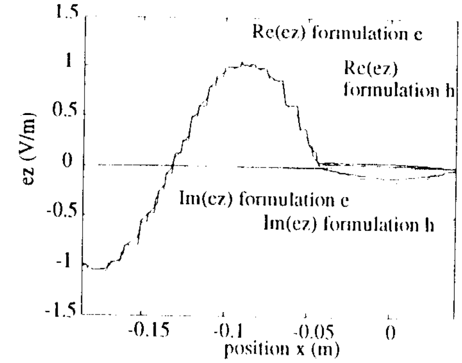

Fig. 5. Electric field distribution for the adapted mesh.

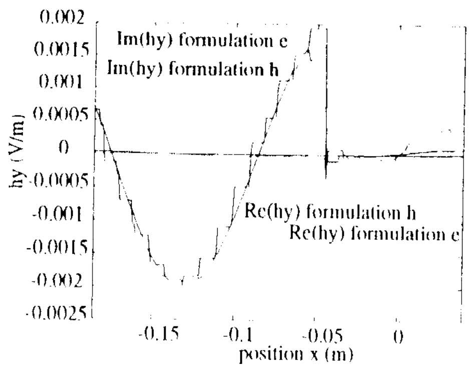

Fig. 6. Magnetic field distribution for the initial mesh.

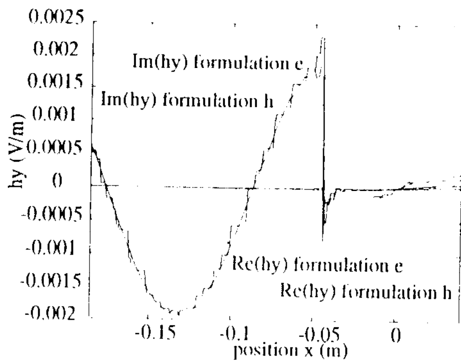

Fig. 7. Magnetic field distribution for the adapted mesh.

The figures relative to the initial mesh clearly show the phase error in the waveguide and the magnitude error in the cavity. These errors are significantly reduced when the mesh is adapted.

Finally, Fig. 8 and 9 show the magnetic field distribution at phases 0° and 90°.

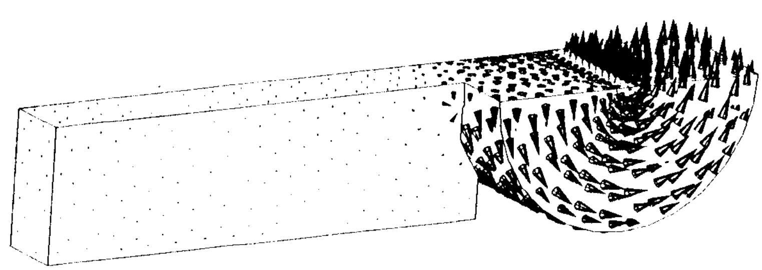

Fig. 8. Real part of the magnetic field in the structure

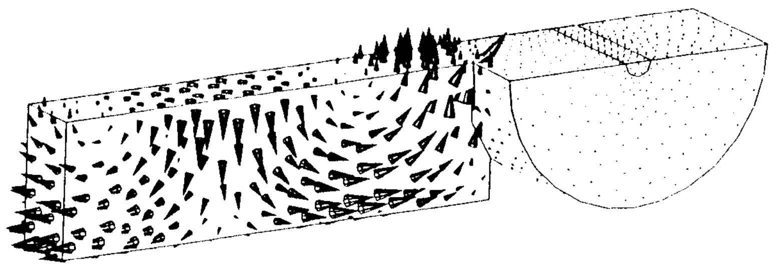

Fig. 9. Imaginary part of the magnetic field in the structure.

CONCLUSIONS

The problem of a three-dimensional loaded resonant cavity is solved by using dual formulations and edge finite elements. An error estimator based on the lack of fulfilment of the constitutive relations is built up in order to define by the h-refinement method an adapted mesh with an a posteriori prescribed error and with a minimised number of elements. This procedure leads to a good convergence of the results given by both formulations. Better estimation of resonance frequency and electromagnetic fields in microwave devices can be performed thanks to the presented method.

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