31. EFFICIENT SIMULATION OF ELECTROMAGNETIC FIELDS USING MAGNETIC EQUIVALENT CIRCUITS FOR NUMERICAL OPTIMISATION

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Abstract

The increase of output power of an electromagnetic device by the raise of flux density is limited by the non-linear behaviour of the iron parts of the electromagnetic circuit. An improved power output can be achieved by optimising the non-linear magnetic circuit. A combination of evolution strategy and simulated annealing leads to a robust numerical optimisation method. Evaluation of the objective function is done by a non-linear flux tube model called magnetic equivalent circuit. Results from field calculations and optimisations are presented for the design of a electronically commutated servo motor.

INTRODUCTION

The design process of electromagnetic devices reflects an optimisation procedure. The construction and step by step optimisation of technical systems in practice is a *trial and error* - process. This design procedure may lead to suboptimal solutions, because its success and effort strongly depends on the experience of the design engineer. Hence, it is desirable to simulate the physical behaviour of the system by numerical methods. In order to get an automated optimal design, numerical optimisation is necessary to achieve a well defined optimum. A combination of two random based search methods, evolution strategy and simulated annealing is discussed in the paper.
Due to a high number of quality function evaluations, stochastic search methods demand for huge computational efforts [1]. Here, a fast and reliable numerical method is introduced to overcome this problem. Using the formal equivalence of the electric flow field, the magnetic field components of an electromagnetic device can be obtained using an magnetic equivalent circuit (MEC). With the rules of the circuit theory, this model of the electromagnetic circuit is solved. When compared to the finite element method (FEM), this approach offers the ability of low computational costs with an acceptable accuracy of the results. The inherent non-linearities caused by the characteristic of ferromagnetic parts are considered as well as non-linearities due to the relative displacement between moving parts. Results obtained by simulations using the FEM and MEC are compared with measurements.

NUMERICAL OPTIMISATION

Mathematical optimisation requires a quality or objective function. All design- and optimisation-aims must be concentrated in this single function. Due to complicated dependencies of the objective variables, derivatives of the quality function may be troublesome. However, heuristic search methods are known to be reliable and stable tools avoiding such difficulties. On the other hand, this advantage is achieved by increased computational costs. Combining the stochastic search methods of evolution strategy with the algorithms of simulated annealing leads to a robust and global optimisation technique [1]. Optimisation in general means to find the best solution for a given problem while accounting for several restricting conditions. In mathematical terms:

Define a point $\mathbf{x}_0 = (x_1, x_2, \ldots, x_n)^T$ *with the independent variables* $x_1, x_2, \ldots, x_n$ *in such a way, that by the variation of the objective variables, inside the admissible space, the value of a quality function* $Z(\mathbf{x}_0)$ *reaches a minimum or maximum. The point* $\mathbf{x}_0$ *is described as the optimum.*

Therefore, optimisation requires the concentration of all design aims into a single quality function. An exact and useful formulation of the objective function is strongly recommended, as the result of the optimisation process is reflected by this function. Furthermore, the quality function must guarantee the existence of an optimum. With a proper choice of restrictions or constraints, this demand can be fulfilled. The objective variables should be normalised to obtain a well conditioned optimisation problem. If changes in the objective variables initiate changes of the same range in the quality function, the problem is well conditioned. With the existence of multiple partial optimisation aims, a weighted sum of the single goals can be used to form the overall objective function.

$$Z = \sum_{i=1}^{n} a_i Z_i \tag{1}$$

A prerequisite for a successful optimisation is a clear definition of all partial targets and the variability of the objective variables, i.e. the single targets must not contradict too strongly.
Optimisation algorithms usually are set up, that consecutive steps lead to the optimum. This iteration is performed by specific rules to vary the steplength and the direction of the optimisation. The various methods only differ in the choice of generating steplength, search direction and stop criterion.
Due to complicated dependencies of the free variables, the direct computation of derivatives of the quality function is troublesome or sometimes not possible. Stochastic methods do work without the use of derivatives. They are easy to implement and the treatment of constraints is simple. If the objective function is complicated and includes local optima or saddle points, deterministic methods in general do not converge to the global optimum. These are the several reasons why stochastic methods are a reliable tool in the field of the numerical optimisation of electro magnetic fields. In the following only two stochastic search techniques and their methodology are described. In this contribution a combination of evolution strategy and simulated annealing is used. The evolution strategy is well known to be a stable local optimiser, but has only poor global convergence properties. Simulated annealing is a global optimiser with slow local convergence speed. In the combined optimisation algorithm the random variations of the design variables x_i are done using the principles of evolution strategy and the selection of the new parameter vectors is done according to the rules of simulated annealing. The overall process control is illustrated in fig. 1. This process introduces an Automated Optimal Design procedure. An initial set of design parameters, obtained from a basic design, accompanied by pre-set constraints is given to the process as input data. Using the MEC to evaluate the quality function of the electromagnetic device, the geometrical constraints are controlled in the pre-process. Question marks in fig. 1 indicate in which step the constraints are checked. If they are not violated, the field simulation is performed and in the post-process of the MEC the physical and technical constraints, in general generated forces and/or minimum/maximum field quantities, are checked. The resulting qualities for the single data set are transferred to the optimisation algorithm to be judged according to the strategy used. If a stopping criterion is fulfilled, the optimisation is finished. In the other case, the design parameter sets are varied by the optimisation algorithm and the design loop is closed by supplying the new parameters to the field computation.
This type of process offers the possibility of an easy change of numerical strategies used. The two branches, field simulation to evaluate the quality function and optimisation method, are independent. Therefore, different techniques may be introduced. Due to the stochastic optimisation method, the

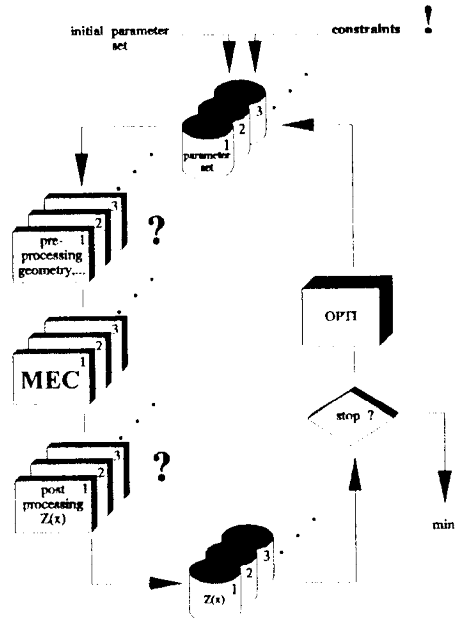

Fig. 1. Automated Optimal Design process control.

generated values of the design parameters are equally distributed. Indicated by dots in fig. 1, this offers the opportunity to use parallel computers in the process to reduce the computation time.

Evolution Strategy

The evolution strategy copies the principles of the biological evolution into the mathematical process of optimisation. The term mutation describes the variability of the objective variables and the term selection is the equivalent of CHARLES DARWIN's (1809-1882) postulate *survival of the fittest*. The algorithm works with a fixed population of μ parent vectors each containing all n objective variables. In a second step the variables of the initial parents are mutated by adding a randomly generated steplength $\delta_{C,i}^{(k)}$ and an uniformly distributed search direction $p_i^{(k)}$.

$$x_{C,i}^{(k)} = x_{P,i}^{(k)} + \delta_{C,i}^{(k)}\, p_i^{(k)} \quad \text{with} \quad i = 1(1)n \qquad (2)$$

The last step of this iteration represents the selection of the best vectors to form the next parent generation. The *plus* strategy selects out of the children and former parent vectors and the *comma* strategy selects out of the generated child vectors only. In the optimisation process the repetition of mutation and selection of the objective variables leads from a temporary to an improved solution. The comma variant is the most powerful evolution strategy [1, 6]. It permits steps backward to offer the chance to escape from local extremum on the way to a global optimum. Optimising hard problems however, often has the result that the evolution strategy does not find the global optimum. Therefore, a combination with simulated annealing is applied. The variation of the objective variables is done by the evolution strategy and the selection of the parent vectors of the new generation is performed by using the acceptance criterion of simulated annealing as introduced in [1-5].

Simulated Annealing

Simulated annealing is well known as a global optimiser. Annealing describes the physical process of heating up a solid to a maximum temperature at which all molecules are freely moving, followed by the process of slowly cooling down until a state of minimum free energy. This process describes a natural optimisation, minimising the free energy. The probability of the energy E_i in a state i at temperature T is described by the BOLTZMANN distribution. Therefore, the probability of the change of energy can be expressed by:

$$\xi < prob\{\Delta E_i\} = \exp(-\frac{\Delta E_i}{k_B T}) \qquad (3)$$

Transferring this natural optimisation process into a general optimisation scheme, means to substitute the term energy by the term quality. Simulated annealing introduces by the Boltzmann distribution the control parameter temperature into the search process. Eqn. (3) is used as an acceptance criterion for the modified design parameter vector from the previous step of iteration. If the variation of objective variables results in a better quality, the configuration is accepted. On the other hand, if the change in quality is larger than zero, the configuration is treated as follows. A uniformly distributed random number ξ out of the interval [0,1] is generated. If the METROPOLIS criterion (3) is true [5], the new configuration is accepted. In the other case, a new mutation on the free variables is performed. The acceptance criterion aims to avoid the system getting stuck in a local minimum. Barriers of height $\sim k_B T$, where k_B is the Boltzmann constant and T the temperature, can be surmounted on the way to a better solution. During the optimisation the artificial temperature T is reduced by a simple schedule $T^{(k)} = T^{(0)}\alpha^{k}$, where k denotes the step of iteration and the reduction factor is $0 < \alpha <$.

FIELD COMPUTATION

To simulate the electromagnetic field, an equivalent magnetic circuit is used. The rules of the static electrical flow field are in formal accordance with those of the quasi static electromagnetic field. Analogue to electrical circuits, an equivalent magnetic circuit is derived. This magnetic circuit consists of magnetic resistors, defined by flux tubes, and flux and/or mmf sources. The solution of this field problem is the analysis of a non-linear network. Figure 3 shows a fluxtube with its describing parameters. A(x) is the area perpendicular to the direction of the flux Φ at the position x where v_1 and v_2 are the potentials at both ends of the flux tube. The difference v_1 - v_2 corresponds to the magnetic voltage drop of the fluxpath. The magnetic resistor R_m of the equivalent magnetic circuit can be calculated by

$$R_m = \frac{1}{\Lambda_m} = \int_0^l \frac{dx}{\mu(x)A(x)} \quad . \tag{4}$$

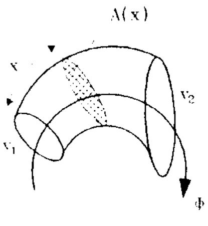

Fig. 3. Fluxtube.

Since in the iron parts of the electromagnetic device, the permeability $\mu(x)$ is a function of the flux density, the field problem turns out to be non-linear. Permanent magnet material with its demagnetisation characteristic is included in the equivalent magnetic circuit as well. Figure 4 shows the complete magnetic equivalent circuit of the servo motor prototype. The motor consists of a two pole diametrically magnetised permanent magnet rotor ring that is

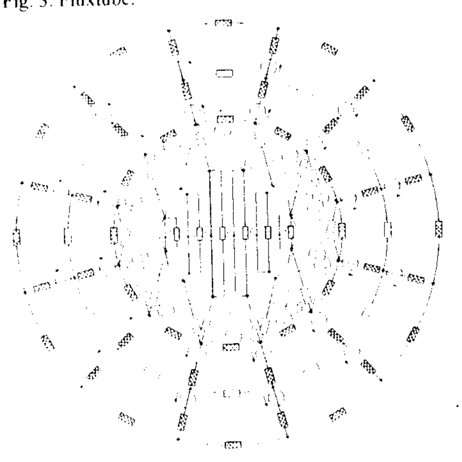
Fig. 4. Complete magnetic equivalent circuit.

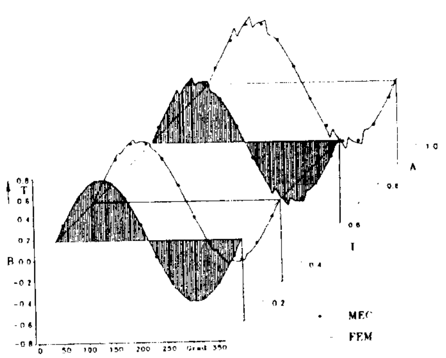

Fig. 5. Flux density distribution with different winding currents.

centred in the stator bore. The two phase armature winding is arranged in closed stator slots. An evaluation of AMPERE's law leads to the mmf sources representing the windings in an electromagnetic device. To verify the accuracy of the magnetic equivalent circuit method, comparisons with the very accurate finite element method and with measurements have been performed. Computations with different winding currents have been carried out to verify the accuracy of the magnetic equivalent circuit model at the different levels of saturation inside the iron parts. Figure 5 shows very good agreement between both methods. For the computed and measured torque versus position a good agreement can be stated as well [2]. The computation time to solve the non-linear magnetic equivalent circuit with 210 elements is of the order of a few seconds. About 8 minutes are necessary to solve the finite element model with ≈10.000 triangular elements of first order. When compared to the FEM a relatively short time to optimise the motor can be expected using the MEC.

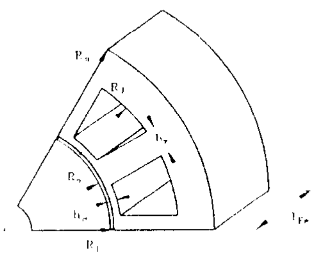

Fig. 7. Object variable used during optimisation.

OPTIMISATION EXAMPLE

The geometrical dimensions of the motor represent the objective variables to be tuned during the optimisation process (Fig. 7). Several quality functions are used to demonstrate the solutions obtained by the different optimisation targets. The constraints are a result of the motor dimensions. The slot height must be larger than zero, the tooth width b_z must be smaller than one slot pitch and for manufacturing purposes the outer radius R_a must be smaller or equal to R_j. For the evolution strategy a comma variant with (5/5, 20) is chosen. The starting stepwidth is set to $5x10^{-2}$ and the initial temperature for the simulated annealing loop is set to $1x10^{-3}$. The initial parameters are the dimensions of the prototype. After the optimisation all results are verified by solutions obtained by the FEM. Now, two different objective functions are supposed to demonstrate the flexibility of the design tools. The first quality function

$$Z_1 = 10\exp\left(\frac{T_{\min}/w_{\max} - T/w}{T/w}\right) \rightarrow \min. \tag{5}$$

generates a motor with optimised ratio of weight to generated torque. This is a weak formulation of the quality. If either the constraint of maximal weight or minimal torque is violated, the parameter set is not rejected. It is merely penalised by an increase of Z_1. This is in contrast to a strict formulation methodology where the parameter set would have been rejected. The second quality function

$$Z_2 = 10\exp\left(\frac{C - C_{\max}}{C_{\max}}\right) + penalty \rightarrow \min.$$

$$\text{where } penalty = \begin{cases} T < T_{\min} & : \ 10\exp((T_{\min} - T)/T) \\ T \geq T_{\min} & : \quad 1 \end{cases} \tag{6}$$

generates a cost optimised device. It refers to the sum of the overall material costs $C=C_{Fe}+C_{Wdg}+0.07\,C_{PM}$. The costs of the permanent magnet material are weighted by a factor of 0.07 because they are very high when compared to the winding and iron lamination costs. A penalty term is added to ensure that the cost optimised device generates exactly the

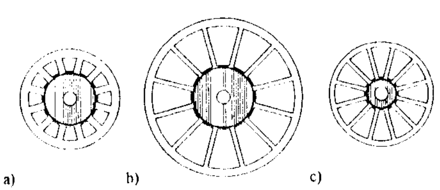

Fig. 8. a) Initial-, b) Z_1 and c) Z_2 optimised geometry.

given torque T_{min}. The torque/weight ratio of the motor optimised by Z_1 improved nearly by a factor of three compared to the prototype. The generated torque improved about six times. After optimisation by Z_2 the resulting device generates exactly the torque desired. Simultaneously the ratio T/w increased about 38%. Compared to the prototype device, the cost optimised motor only costs 26% of the prototype. Fig. 8 shows the initial and optimised shapes.

CONCLUSIONS

This paper describes the application of a combination of a numerical field computation method with a stochastic optimisation method to optimise the electromagnetic field. A stable and robust optimisation technique using a combination of evolution strategy and simulated annealing is demonstrated. Two quality functions are applied to optimise an electromagnetic servomotor and demonstrate the suitability and the general application range of the methods used.

The field computation for the electromagnetic field is performed by a magnetic equivalent circuit. The decision to use this approach is a compromise between accuracy and computational effort. By using this method to evaluate the quality function on a PC 486-33MHz, the overall computation time for the optimisation lies in the range of a couple of hours. In comparison, the stochastic optimisation using the very accurate finite element method to evaluate the quality function can take days or even weeks [1,2].

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