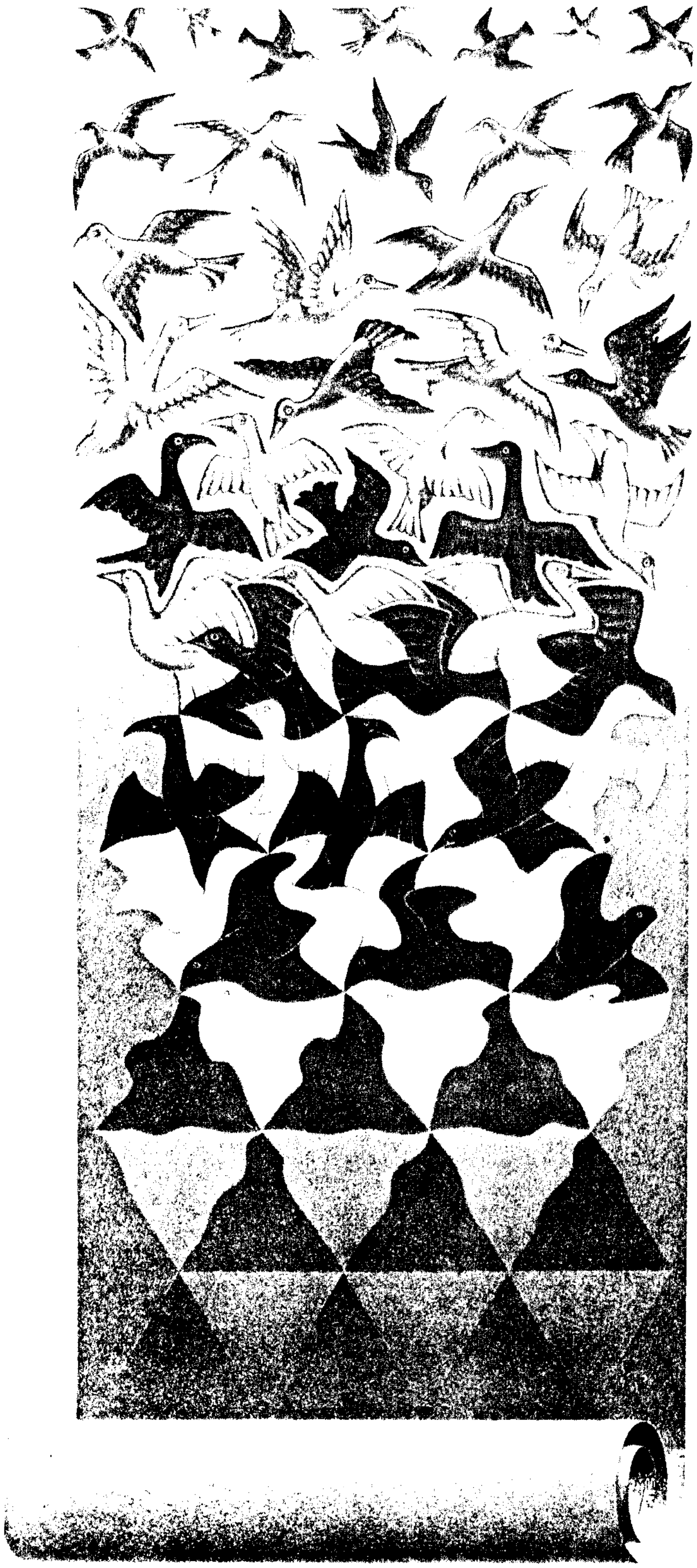
ADAPTIVE COMPUTING IN ENGINEERING DESIGN AND CONTROL - '96



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STOCHASTIC OPTIMISATION OF MATHEMATICAL MODELS FOR ELECTRIC AND MAGNETIC FIELDS

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ABSTRACT

The combination of numerical field computation methods with stochastic search methods permits computer simulations to be extended to the design stage. In modern systems, the trend is towards small packages, high efficiency and low cost. Aiming on this, in particular the optimisation of electromagnetic devices recommends field simulations applying different types of material to avoid expensive prototyping. In a relatively new research field, the micro mechatronic, new energy conversation principles have to be examined and must be geometrically optimised to obtain the desired properties. Due to overall dimensions in the range of some microns measurements are not possible. Therefore, reliable numerical methods are highly recommended.

In this paper a fast semi-numerical field computation method to compute the electromagnetic field is introduced. Field computations and optimisations using a servo motor demonstrate the reliability of the algorithms used. A second optimisation example shows results of finite element simulations of an electrostatic micro motor.

The optimisation method used is a combination of the evolution strategy with the simulated annealing approach, introduced in Hameyer (2). Here, a brief introduction and summary of the method is given only. The paper focuses on the application of the developed optimiser to practical engineering problems.

NUMERICAL OPTIMISATION

Mathematical optimisation requires a quality or objective function. All design- and optimisation-aims must be concentrated in this single function. Due to complicated dependencies of the objective variables, derivatives of the quality function may be troublesome. However, heuristic search methods are known to be reliable and stable tools to avoid such difficulties. On the other hand, this advantage is achieved by increased computational costs. Combining the stochastic search methods of evolution strategy with the algorithms of simulated annealing leads to a robust and global optimisation technique as reported in Hameyer (2).

Optimisation in general means to find the best solution for a given problem with the consideration of several restricting conditions. In mathematical terms:

Define a point $\mathbf{x}_0 = (x_1, x_2, \ldots, x_n)^T$ *with the independent variables* $x_1, x_2, \ldots, x_n$ *in such a way, that by the variation of the objective variables, inside the admissible space, the value of a quality function* $Z(\mathbf{x}_0)$ *reaches a minimum or maximum. The point* $\mathbf{x}_0$ *is described as the optimum.*

An exact and useful formulation of the objective function is strongly recommended because the latter product of the optimisation process is reflected by this function. Furthermore, the quality function must guarantee the existence of an optimum. With a proper choice of restrictions or constraints this demand can be fulfilled.

The objective variables should be normalised to obtain a well conditioned optimisation problem. If changes in the objective variables initiate changes of the same range in the quality function, the problem is well conditioned. With the existence of multiple partial optimisation aims, a weighted sum of the single goals can be used to form the overall objective function.

$$Z = \sum_{i=1}^{n} a_i Z_i \qquad (1)$$

Optimisation algorithms usually are set up, that temporarily steps lead to the optimum. This iteration is performed by specific rules to vary the steplength and the direction of the optimisation. The various methods only differ in the choice of generating steplength, search direction and stopping criterion.

The evolution strategy copies the principles of the biological evolution into the mathematical process of optimisation. It works with a fixed population of μ parent vectors each containing all n objective variables. In a second step the variables of the initial parents are mutated by adding a randomly generated steplength $\delta_{e,i}^{(k)}$ and an uniformly distributed search direction $p_i^{(k)}$.

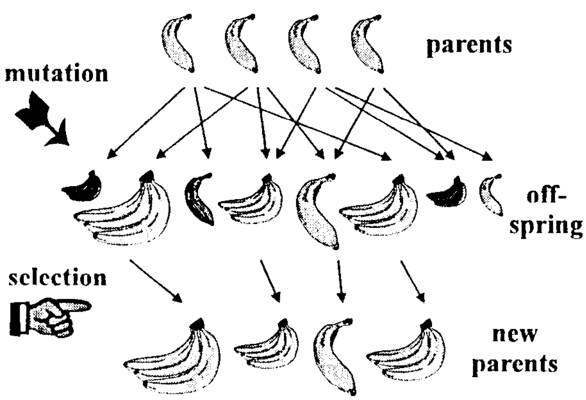


Fig. 1: Simple schematic of the evolution strategy.

$$x_{C,i}^{(k)} = x_{P,i}^{(k)} + \delta_{C,i}^{(k)}\, p_i^{(k)} \quad \text{with } i = 1(1)n \qquad . \quad (2)$$

Via a hereditary factor ρ, multiple parents are contributing to the generation of one of the λ child vectors. The last step of this iteration represents the selection of the best vectors to form the next parent generation. The *plus* strategy selects out of the children and former parent vectors and the *comma* strategy selects out of the generated child vectors only. In the optimisation process the repetition of mutation and selection of the objective variables leads from a temporary to an improved solution. Figure 1 illustrates the single steps of mutation and selection. There, a strategy (μ/ρ, λ) with (4/2, 8) is given. Eight offspring are produced with four parent vectors and two parents are contributing to one child vector.

The comma variant is the most powerful evolution strategy. It permits steps backward to offer the chance to escape from local extremum on the way to a global optimum. Optimising hard problems however, often has the result that the evolution strategy does not find the global optimum. Therefore, a combination with simulated annealing is applied. The variation of the objective variables is done by the evolution strategy and the selection of the parent vectors of the new generation is performed by using the acceptance criterion of simulated annealing as introduced in (2, 3, 5, 6, 8).

Simulated annealing is well known as a global optimiser. Annealing describes the physical process of heating up a solid to a maximal temperature at which all molecules are freely moving and the process of slowly cooling down until a state of minimum free energy. This process describes a natural optimisation minimising the free energy. The probability of the energy E_i in a state i at temperature T is described by the BOLTZMANN distribution. Therefore, the probability of the change of energy can be expressed by:

$$\xi < prob\{\Delta E_i\} = \exp\left(-\frac{\Delta E_i}{k_B T}\right) \qquad . \quad (3)$$

Annealing is a natural optimisation process. Applying this to other systems is done by substituting the energy term by a quality term for the system. This is the basic idea behind simulated annealing and leads to a robust optimisation technique described in (2, 3, 5, 6).

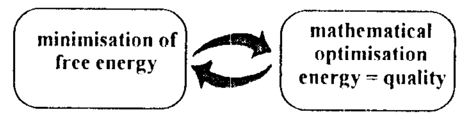


Fig. 2: Basic idea of simulated annealing.

OPTIMISATION OF A SERVO MOTOR

To simulate the electromagnetic field an equivalent magnetic circuit is used. The rules of the static electrical flow field are in formal accordance with those of the quasi static electromagnetic field. Analogue to electrical circuits an equivalent magnetic circuit can be given. This magnetic circuit consists of magnetic resistors, defined by flux tubes, and flux and/or mmf sources. The solution of this field problem is the analysis of a non-linear network. Figure 3 shows a fluxtube with its describing parameters.

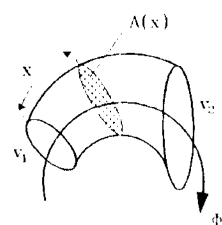


Fig. 3: Fluxtube.

A(x) is the area perpendicular to the direction of the flux Φ at the position x where v_1 and v_2 are the potentials of both ends of the fluxtube. The difference $v_1 - v_2$ corresponds to the magnetic voltage drop of the fluxpath. The magnetic resistor R_m for the equivalent magnetic circuit can be calculated with

$$R_m = \frac{1}{\Lambda_m} = \int_0^l \frac{dx}{\mu(x)A(x)} \qquad . \quad (4)$$

Since in iron parts of the electromagnetic device the permeability μ(x) is a function of the flux density, the field problem turns out to be non-linear. Permanent magnet material with its demagnetisation characteristic is included in the equivalent magnetic circuit illustrated in fig. 4.

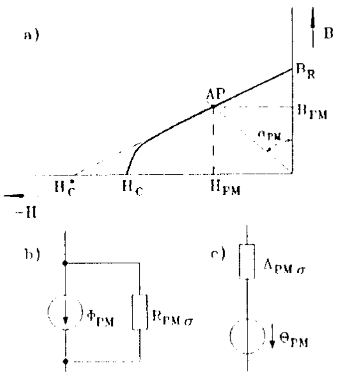


Fig. 4: a) Demagnetisation characteristic of PM material, b) as flux and c) as voltage source

An evaluation of the AMPERE's law leads to the mmf sources which model the windings in an electromagnetic device.

To verify the accuracy of the magnetic equivalent circuit method, comparisons with the very accurate finite element method and with measurements have been performed. Figure 5 shows a flux plot obtained by the finite element method of the servo motor prototype and its complete magnetic equivalent circuit.

A two pole diametrical magnetised permanent magnet rotor ring is centred in the stator bore. The two phase armature winding is arranged in closed stator slots. Electronic logic controls the angular position of the actuator over the range of 180°. Computations with different winding currents have been carried out to verify the accuracy of the magnetic equivalent circuit model at the different levels of saturation inside the iron parts. Figure 6 shows very good agreement between the methods. For the computed and measured torque versus position a good agreement can be stated as well.

The computation time to solve the non-linear magnetic equivalent circuit with 210 elements is of the order of a few seconds. About 8 minutes are necessary to solve the finite element model with ≈10.000 elements. Therefore, a relatively short time to optimise the actuator can be expected.

The geometric dimensions of the actuator represent the objective variables to be tuned during the optimisation process. Different quality functions are used to demonstrate the solutions obtained by the different optimisation aims. Figure 7 shows the parameters used. The constraints are a result of the actuator dimensions. The slot height must be greater than zero, the tooth width b_z must be smaller than one slot pitch and for fabrication purposes the outer radius R_a must be smaller or equal to R_j.

For the evolution strategy a comma variant (μ/ρ, λ) with (5/5, 20) is chosen. The starting stepwidth is set to $5x10^{-2}$ and the initial temperature for the simulated annealing loop is set to $1x10^{-3}$. The initial parameters are the dimensions of the prototype actuator. After the optimisation all results were verified by solutions obtained by the finite element method.

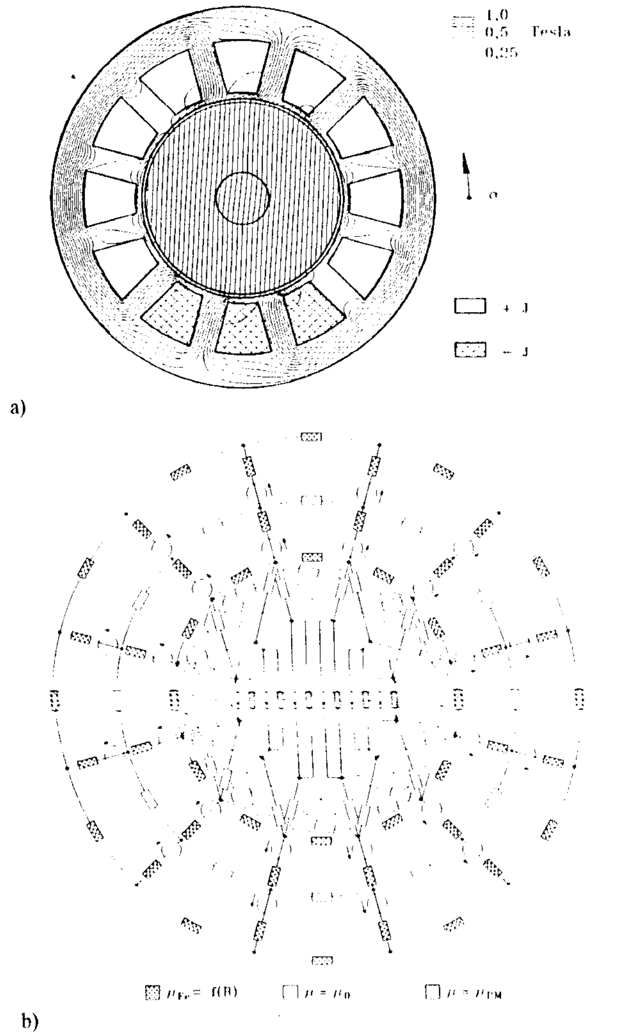


Fig. 5: a) Flux plot of the motor computed by FEM and b) the complete magnetic equivalent circuit.

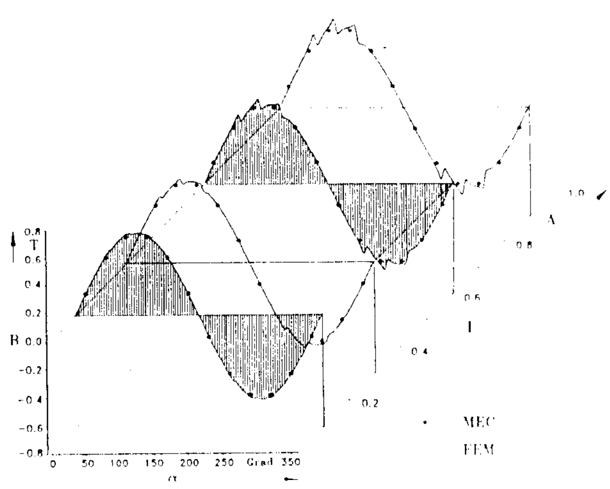


Fig. 6: Air gap flux density with different winding currents.

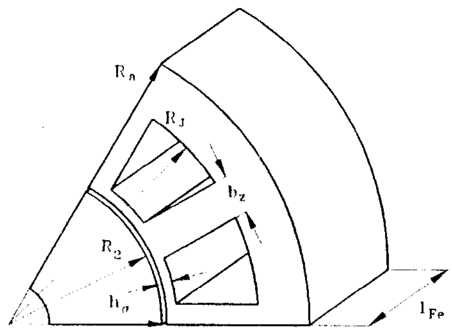


Fig. 7: Object variable used during optimisation.

Now, two different objective functions are supposed to demonstrate the usability of the design tools. The first quality function

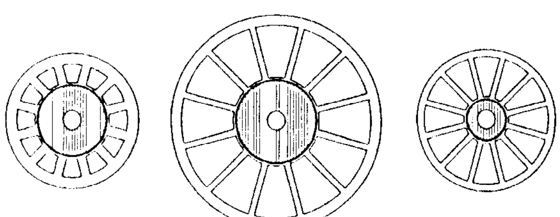
$$Z_1 = 10\exp\left(\frac{T_{min} / w_{max} - T / w}{T / w}\right) \rightarrow \text{min.} \qquad (5)$$

generates an actuator with optimised ratio of weight to generated torque. This is a weak formulation of the quality. If either the constraint of maximal weight or minimal torque is violated, the parameter set is not rejected it is merely penalised by an increase of Z_1. This is in contrast to a strict formulation methodology where the parameter set would have been rejected. The second quality function

$$Z_2 = 10\exp\left(\frac{C - C_{max}}{C_{max}}\right) + penalty \rightarrow \text{min.} \qquad (6)$$

$$\text{where } penalty = \begin{cases} T \leq T_{min} & : \; 10\exp((T_{min} - T)/T) \\ T \geq T_{min} & : \; 1 \end{cases}$$

generates a cost optimised device. It refers to the sum of the overall material costs $C = C_{Fe} + C_{Wdg} + 0.07\, C_{PM}$. The costs for the permanent magnet material are weighted by a factor of 0.07 because they are high compared to the winding and iron lamination costs. A penalty term is added to ensure that the cost optimised device generates exactly the given torque T_{min}.



a) b) c)

Fig. 8: a) Initial-, b) Z_1 and c) Z_2 optimised geometry.

The torque/weight ratio of the actuator optimised by Z_1 improved nearly by a factor of three compared to the prototype. The generated torque improved about six times. After optimisation by Z_2 the resulting device generates exactly the torque desired. Simultaneously the ratio T/w increased about 38%. Compared to the prototype device, the cost optimised actuator will cost only 26% of the prototype.

OPTIMISATION OF AN ELECTROSTATIC MICRO MOTOR

The same approach, the combination of a field computation method and the numerical optimisation algorithm, is used for a electrostatic micro motor. Here, the torque production is based on the variation of the capacitance between the rotor and stator electrodes as they move relative to each other. Designing such motors requires the use of the very expensive LIGA technology as reported in Mohr et al (9). Therefore prototyping to optimise the motor properties is out of question. Here, the field computation is performed by the well known finite element method using standard tetrahedron element types to obtain a three dimensional field solution. Due to the complicated geometry, as shown in fig. 9, the three dimensional field approach is strictly recommended.

The main problem with such electrostatic micro motors is to find a design generating a sufficient torque to fulfil given requirements and motor properties. The generated torque must overcome friction losses to guarantee the function of the micro device. With the dimensions of a rotor diameter of 320 microns, a rotor thickness of 4 microns and an axial air gap of 3 microns the generated average torque referred to the applied voltage, is in the range of 0.06 pico Nm/V^2. The movement of the rotor is obtained by cyclic switching of the voltage, exciting the

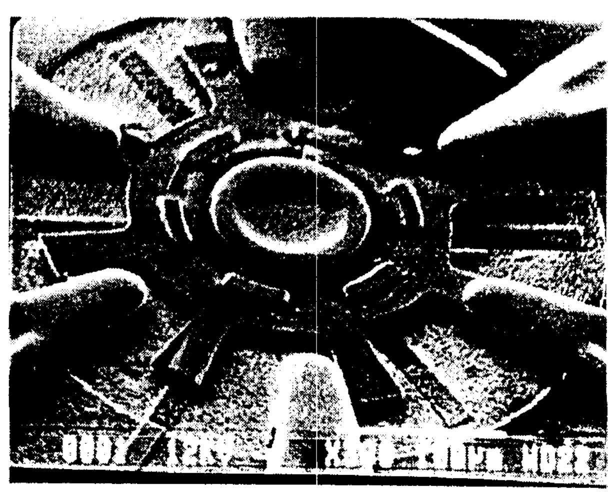


Fig. 9: Axial field type electrostatic micro motor with double stator system (Photo: ESIEE, Paris).

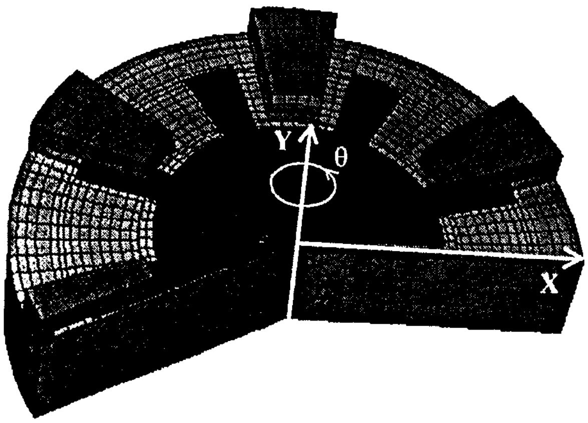


Fig. 10: Three-dimensional finite element model for an electrostatic micro motor.

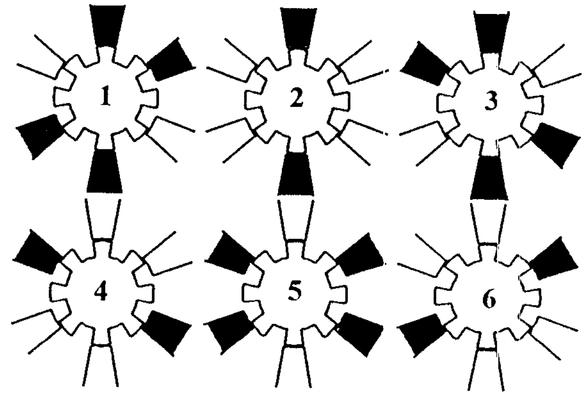


Fig. 12: Possible symmetric excitation sequence.

stator electrodes. Caused by this construction of the motor, the torque is inherently superposed by a torque ripple.

In fig. 10 the construction of the finite element model is illustrated. To generate the model, the two-dimensional radial cross section of the motor is rotated around the y-axis to build the complete extruded three-dimensional model of the motor. The stator is designed as a double system to allow the rotor move between the upper and lower part of the stator electrodes.

To consider the torque ripple and to minimise it while maximising the average torque of the micro motor, several rotor positions, applying different stator excitation sequences, are simulated and evaluated by the optimisation algorithm. The variation of the capacitances as a function of the rotor position is depending on the geometry of the electrodes. Hence, the pole arc of the stator electrodes and respectively the pole arc of the rotor electrodes are the design parameters. To tune those objective variables the optimisation is performed. Figure 11 shows a typically torque characteristic of an axial field micro motor consisting of 6 stator and 8 rotor poles. The corresponding excitation sequence can be taken from fig. 12. There, the voltage supplied electrodes are indicated in black ink.

To minimise radial contact forces generating increasing friction losses in the rotor bearings, only symmetrical excitation sequences are considered. By switching the voltage at the crossovers, marked in fig. 11, from one excitation to another the highest generated average torque is achieved. The full- and half-step techniques known from stepper motor applications are applied in this way. Full angular steps are performed by sequences 2, 4 and 6 and the half-steps by sequences 1, 3, and 5 respectively.

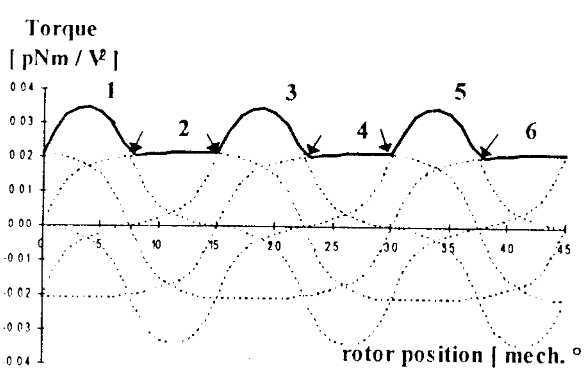


Fig. 11: Torque vs. rotor position for 6 different excitation sequences of a 6/8 pole micro motor.

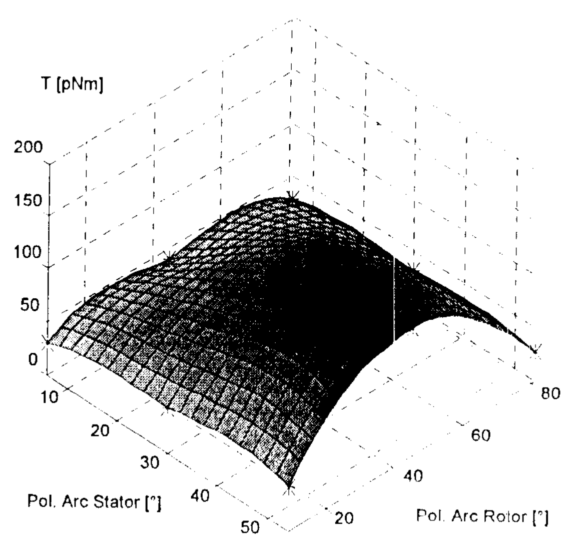


Fig. 13: Average torque as function of stator and rotor pole arc.

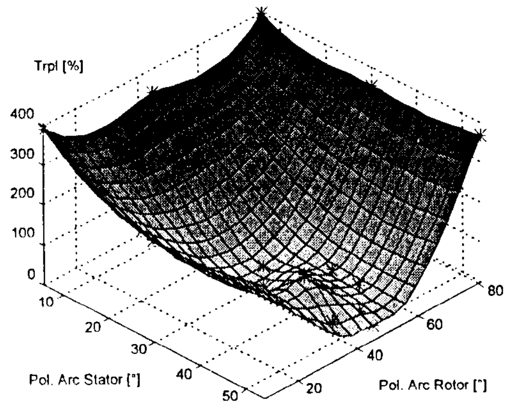


Fig. 14: Torque ripple as function of stator and rotor pole arc.

Using reliable automated mesh generation tools for the finite element model, as described in Johansson (1), a fully automated optimisation procedure is realized. Using those methods several motor configurations having different numbers of stator and rotor electrodes where investigated.

Computed results for the average torque obtained by the mentioned methods are drawn in fig. 13. Exciting an axial field 6/4 pole configuration by 50 Volt, the maximum average torque reaches a value of 150 pico Nm. The optimiser tuned the design parameters for the stator pole arc to 44.4 mech. deg. and to 37.8 mech. deg. for the rotor pole arc to obtain the maximum average torque. At this point inside the solution space the torque ripple passes through minimal values (fig. 14).

CONCLUSIONS

This paper describes the application of a combination of a numerical field computation methods with a stochastic optimisation method to optimise the electromagnetic and electric field. A stable and robust optimisation technique using a combination of evolution strategy and simulated annealing is demonstrated. Two optimisation examples, an electromagnetic servo motor and an electrostatic micro motor, demonstrate the suitability and the general application range of the methods used.

Field computation for the electromagnetic field is performed by a magnetic equivalent circuit. The decision to use this approach is a compromise between accuracy and computational costs. By using this method to evaluate the quality function, the overall computational time for optimisation lies in the range of a couple hours. In comparison optimisation using the very accurate finite element method applied to the optimisation of a micro motor can take days or even weeks (1, 2).

ACKNOWLEDGEMENTS

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