EQUIVALENT CIRCUIT TECHNIQUE FOR ELECTROSTATIC MICROMOTORS

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Abstract

Industrial interest in micro electro-mechanical systems is rapidly increasing. Due to new processes for manufacturing microactuators and micromotors as e.g. X-ray LIGA, Deep UV-Lithography and plasma etching the number of possible structures has grown as well as the precision with which they can be manufactured. As the production of such devices is expensive there is an increasing demand for analysing tools that can simulate the operational behaviour of various designs. A technique to calculate the average torque and torque ripple for electrostatic micromotors has been developed. This technique is based on the use of an equivalent circuit model. It is described how to generate the parameters of this equivalent circuit model using finite elements. The same type of equivalent circuit can be used disregarding if the parameters are calculated from 2D or 3D field solutions.

The Equivalent Circuit Model

The idea of generating an equivalent circuit model for an electrostatic micromotor is, as for any electric motor, to provide the possibility to treat the motor as a circuit component, to integrate the motor in the electric supply circuit, to allow simulations of the dynamic behaviour especially in control systems and to allow prediction of the mechanical properties for a given excitation, e.g. electric torque and rotating speed.

The data needed to generate an equivalent circuit model is classically obtained from measurements. However, performing measurements on electrostatic micromotors is very difficult due to the extremely low values of both electrical and mechanical properties. Typical values are capacitances of a few to a few hundred fF (10^{-15}) and torques of a few to a few hundred pNm. Therefore, electrical and mechanical properties must be calculated using e.g. the finite element technique. Since finite element computation is more time consuming than computation using lumped parameters, it is in this case beneficial to calculate intrinsic properties. These are finally used to generate the equivalent circuit model.

The equivalent circuit model described here is specially designed to enable calculation of the electric torque as a function of the rotor position for variable capacitance micro motors. The natural choice for the intrinsic properties are hence capacitances. The equivalent circuit model for a motor with 6 stator electrodes is shown in fig. 1.

Due to the close link between the physical capacitive coupling between different parts in the motor and the lumped parameters of the equivalent circuit model, this equivalent circuit model provides a good representation of the motor geometry. Each parameter, i.e. each capacitive component is a function of the rotor position α. This makes it possible, using the energy stored in a capacitance and the torque formulation of virtual work, to express the torque as function of the rotor position and thus to calculate the average torque and the torque ripple.

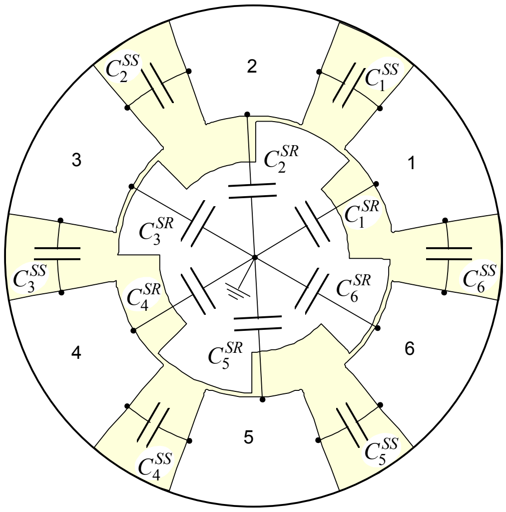

Fig. 1. The equivalent circuit model for a motor with 6 stator electrodes.

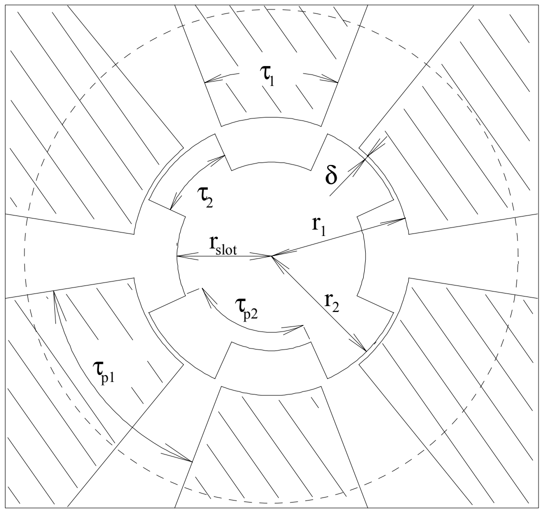

Fig. 2. Geometric parameters describing the geometry of the radial flux variable capacitance motor.

$$W_e(\alpha) = \tfrac{1}{2} C(\alpha) \cdot V^2 \qquad (1)$$

$$T(\alpha) = \frac{\partial W_e(\alpha)}{\partial \alpha} \qquad (2)$$

The equivalent circuit model in fig. 1 consists of 12 capacitances, or rather, capacitance functions. However, it contains only two principally different capacitance functions. The first type is the function representing the capacitance between each stator electrode and the rotor, $C_k^{SR}(\alpha)$. The second type is the function representing the capacitance between each pair of consecutive stator electrodes $C_k^{SS}(\alpha)$. In $C_k^{SR}(\alpha)$ k represents the electrode number. In $C_k^{SS}(\alpha)$ k is the number of the first electrode in the pair. Figs. 3 and 4 show two of these capacitance functions. They are derived from the 3D finite element solutions for a motor with the following geometric parameters from fig. 2.: $r_1 = 300\ \mu m$, $r_{slot} = 210\ \mu m$, $\delta = 10\ \mu m$, $\tau_1 = 42°$, $\tau_2 = 40.5°$, $\tau_{p1} = 60°$, $\tau_{p2} = 90°$ and an axial length of 100 μm.

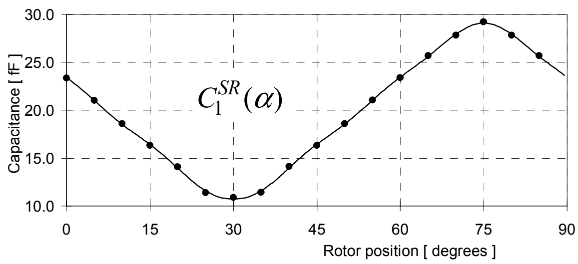

Fig. 3. The parameter $C_1^{SR}(\alpha)$ representing the capacitance function between electrode 1 and the rotor.

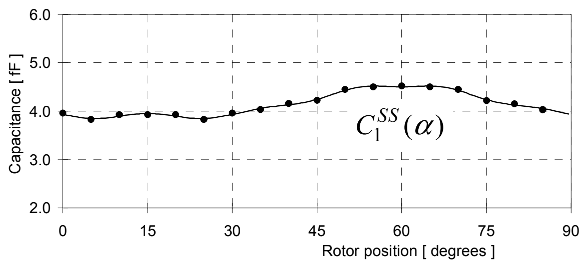

Fig. 4. The parameter $C_1^{SS}(\alpha)$ representing the capacitance function between electrode 1 and 2.

Deriving the Parameters of the equivalent circuit model

Using (1) and computing the stored energy for different rotor positions a system of equations can be set up. In order to solve this system of equations successfully, there is one condition that has to be fulfilled. As the equivalent circuit model consists of two principally different capacitance functions, two different excitations must be set and solved for each rotor position. This is in order to accomplish two linear independent left-hand sides of (1), which in turn is required in order to solve for two unknowns. Any two excitations can be used as long as they, for the same rotor position, energise a different set of capacitances. However, these two sets may have some capacitances in common.

The two excitations chosen are denoted A and B. For excitation A, a voltage V is applied to one stator electrode of the motor only. All other electrodes are at the reference potential. Therefore, the three capacitances surrounding the excited electrode are storing energy and the total stored energy for excitation A is given by (3). For excitation B, a voltage V is applied to two consecutive electrodes and four capacitances surrounding the excited electrodes are storing energy. The capacitance between the excited electrodes does not store any energy since there is no voltage difference between them. The total stored energy for excitation B is given by (4).

$$W_k^A(\alpha) = \tfrac{1}{2}V^2\left[C_k^{SR}(\alpha) + C_k^{SS}(\alpha) + C_{k-1}^{SS}(\alpha)\right] \qquad \left(k = 1,2,\ldots n_{p1}\right) \tag{3}$$

$$W_k^B(\alpha) = \tfrac{1}{2}V^2\left[C_k^{SR}(\alpha) + C_{k+1}^{SR}(\alpha) + C_k^{SS}(\alpha) + C_{k-1}^{SS}(\alpha)\right] \quad \left(k = 1,2,\ldots n_{p1}\right) \tag{4}$$

In (3) and (4) n_{p1} is the number of stator electrodes. For index $k = 1$, $C_{k-1}^{SS}(\alpha) = C_{n_{p1}}^{SS}(\alpha)$. The number of possible different excitations A and B that can be defined is always equal to the number of stator electrodes. For the motor of fig. 1, a system of equations is set up using first the 6 possible excitations A and then the 6 possible excitations B. This system of equations is written as a matrix equation:

$$\left[W(\alpha)\right] = \tfrac{V^2}{2}\left[\mathrm{K}\right]\cdot\left[C(\alpha)\right] \tag{5}$$

Thus, in this example (fig. 1), $\left[W(\alpha)\right]$ is a 12 element column vector containing the energy functions, $\left[\mathrm{K}\right]$ is a 12x12 coefficient matrix containing only ones or zeros and $\left[C(\alpha)\right]$ is a 12 element column vector containing the capacitance functions. The 6 first rows of $\left[W(\alpha)\right]$ contain the A series and the last 6 rows the B series. Both series are ordered by the electrode number. By assembling $\left[W(\alpha)\right]$ in this way, $\left[\mathrm{K}\right]$ will have an inverse and the capacitance functions can be expressed explicit. The same holds for any number of stator electrodes.

$$\left[C(\alpha)\right] = \tfrac{2}{V^2}\left[\mathrm{K}\right]^{-1}\cdot\left[W(\alpha)\right] \tag{6}$$

In fig. 1 the equivalent circuit model has 12 capacitance functions. Each excitation resulting in one energy value, means 12 excitations per rotor-position. If one electric period is discretised by 18 rotor-positions, the total number of energy values or excitations required, is 12*18 = 216. However, by a careful choice of the rotor positions, energy values between the different sets of equations can be substituted. The number of energy values required can always be reduced to twice the number of rotor positions.

Reducing the Required Number of Finite Element Calculations

The parameters of the equivalent circuit model, $C_k^{SS}(\alpha)$ and $C_k^{SR}(\alpha)$, are functions of the rotor position and hence are periodic with a period equal to the rotor pole pitch τ_{p2}. Therefore, the energy functions (1) are periodic as well. This implies further that functions with different electrode number *k*, are all identical except for a phase angle β. This angle is equal to a multiple of the stator pole pitch τ_{p1}. Defining a positive rotor rotation and numbering the stator electrodes in the same direction, functions with different *k* can be substituted as

$$W_k^A(\alpha) = W_1^A(\alpha - \beta_k) \qquad \left(k = 1,2,\ldots n_{p1}\right) \qquad (7a)$$

$$W_k^B(\alpha) = W_1^B(\alpha - \beta_k) \qquad \left(k = 1,2,\ldots n_{p1}\right) \qquad (7b)$$

where $\beta_k = (k-1)\tau_{p1}$.

For a given pole configuration, i.e. a combination of number of electrodes in the stator and teeth in the rotor, the set of phase angles β is fixed. However, the set of rotor positions α may be chosen. The way to ensure that not more finite element calculations than twice the number of rotor positions are needed, is to choose the values of α equidistant and making the values of β a subset of them. The smallest set of rotor positions for which the parameters of the equivalent circuit model can be derived is a set equal to the set of phase angles.

The capacitance functions of figs. 3 and 4 have a period of τ_{p2} = 90°. They are sampled by 18 rotor positions resulting in a rotation step of 5°. With τ_{p1} = 60° the phase angles for this motor are 0°, 30° and 60°. Using a discrete Fourier transform, the discrete functions are transferred into continuous ones and are represented by their Fourier series. This makes it possible to calculate the torque from (2) very accurately by means of an analytic differentiation.

Conclusions

Deriving an equivalent circuit model of a motor has undoubtedly many benefits. The equivalent circuit model for a variable capacitance micromotor has been one of the most important tools when performing optimisation of these motors [1,2]. Using the equivalent circuit model, the total computational time is reduced significantly. Furthermore, the equivalent circuit model presented here can easily be complemented with other parameters independent of the rotor position as e.g. resistance, friction, rotor inertia, etc. The equivalent circuit model complemented by such parameters offers the possibility for the dynamic analysis of micromotors [3].

Acknowledgements

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