COMPUTATION OF THE ELECTRIC AND MAGNETIC FIELD BELOW HIGH-VOLTAGE LINES

Kay Hameyer, Dr.-Ing. and Ronnie Belmans, Prof. Dr. Ir.,
Katholieke Universiteit Leuven, E. E. Dept., Div. ESAT/ELEN,
Kardinaal Mercierlaan 94, B-3001 Leuven/Heverlee, Belgium.

Abstract: Overhead transmission lines generate in their vicinity electric and magnetic fields. The source of the magnetic fields are the currents inside the phase conductors. The electric field is caused by the high potential of the phase conductors.

Nowadays an increasing sensibility to ecological problems can be stated. An injurious influence to the health of human beings caused by the direct effect of technical low frequency electromagnetic fields (50/60 Hz) is scientifically not proven yet. Since ca. 25 years research efforts to find a correlation mechanism between the field quantities and their effect to the human beings are going on but without significant success. In this situation, the electric and magnetic field quantities of high-voltage lines have to be examined in order to avoid EMC problems with the environment close to the power transmission line while planning high voltage lines.

The calculation of the electric and magnetic fields below overhead transmission lines requires a numerical method considering the three dimensional geometry of this particular technical object. Caused by the slag a two dimensional approach is not sufficient. In this paper two semi numerical methods will demonstrate the computation of the electric and magnetic field quantities below high voltage-lines.

1. INTRODUCTION.

Due to the geometry of electrical energy transmission lines, a wide expansion of the fields below the phase conductors is obvious. To avoid EMC problems with the environment, questions dealing with the field distribution are of great interest. To evaluate the influence of the energy line, it is not sufficient to calculate the coupling impedances or capacitances of the line. It is necessary to analyse the generated fields itself. The simulation of the high voltage transmission line while planning has the opportunity to know about possible risks and disturbing influences. With a trend towards higher values of the transmission voltage the necessity of this simulation is recommended.

The IRPA[2] and CENELEC standards supply maximum values for the duration of the stay of human beings exposed to electromagnetic fields of frequencies below 10 kHz. There, for the general public the permanently allowed effective value of field strength with 50/60 Hz is 10 kV/m for the electric- and 0.1 mT for the magnetic field. For short exposition times increasing limits are admissible.

Due to the slag of the phase conductors the field problem turn's out to be three dimensional. The problem specifies a small diameter conducting conductors above a large flat conducting ground plane. The phase conductors are at a time depending specified electrical potential and are carrying a time depending current as well. The numerical analysis using the finite element method is very costly and computer time consuming. Because of the very large wavelength the electric- and magnetic field problems may be considered to be quasi static. Therefore, the solutions can be determined by static field techniques. Here, the three dimensional field computation is performed by mirroring line charges at the before mentioned ground plane. The magnetic field quantities are obtained using the BIOT-SAVART law.

2. THE MODELLING OF THE TRANSMISSION LINE.

The geometry of the single transmission phase conductors are approximated by a polygon. Figure 1 illustrates this procedure. Due to symmetry between two high voltage poles one half of this arrangement is drawn only. The value of s indicates the slag and l is the distance between the two high voltage poles, the span field length. To consider the slag the quadratic approximation from eq. (1) is used to calculate the co-ordinates of the polygon.

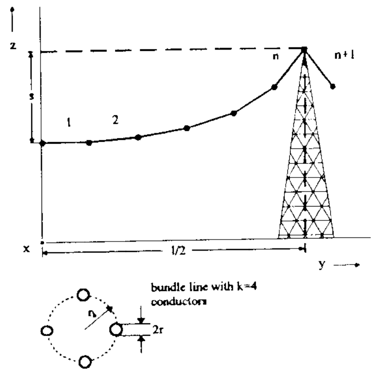


Fig. 1: Geometric modelling of a phase conductor in consideration of a slag s and of bundle lines.

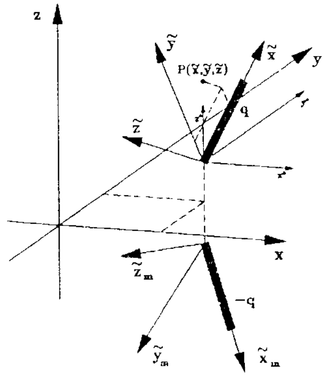


Fig. 2: Co-ordinate transformation.

$$f(y) = s \cdot \left\{ 1 - \frac{4}{l^2} y^2 \right\} \tag{1}$$

To apply the method to the model of the single phase conductors the rules of the static electric and magnetic fields a co-ordinate transformation is necessary. Every part of the polygon has to be transformed from the original system into a local co-ordinate system $(\tilde{x}, \tilde{y}, \tilde{z})$. This transformation is performed in two steps. A part of the polygon at any position in the original co-ordinate system (x, y, z) is shown in fig. 2. The first step of the transformation consists of a parallel shift of the origin into the starting point of the polygon. In a second step a rotation of this temporary co-ordinate system (x°, y°, z°) around the x°-axis is carried out in the way that the polygon lies in the x°-y° plane. The last rotation in this step is around the z°-axis so that the polygon lies in the x°-axis. In the new co-ordinate system $(\tilde{x}, \tilde{y}, \tilde{z})$ both of the desired field types can be computed. After the calculation the field quantities have to be re-transformed into the original co-ordinate system. Figure 2 illustrates the single steps and shows the mirrored co-ordinate system necessary for the electro static field calculation as well. Bundle lines constituting a single phase conductor are taken into consideration with an equivalent radius r_e

$$r_e = \sqrt[k]{k \cdot r \cdot r_b^{k-1}} \tag{2}$$

The definition of the variables used in eq. (2) can be taken from fig. 1. It is reported in Utmischi[4] that calculations with the equivalent radius and a model with 4 single conductors differ only around 1,4% in their solution. Therefore, the use of r_e is an admissible compromise with respect to the computational costs.

3. ELECTRIC FIELD.

In the mathematical model for the electric field each element of the polygon represents a constant line-charge. With the assumption of ideal conductivity of the ground plane below the transmission line, the rules of the electro static can be applied to the model. In the local co-ordinate system the potential φ of the line charge in the point $P(\tilde{x},\tilde{y},\tilde{z})$ is given with

$$\varphi(\tilde{x},\tilde{y},\tilde{z}) = \frac{q}{4\pi l\varepsilon}\ln\left[\frac{l-\tilde{x}+\sqrt{\tilde{y}^2+\tilde{z}^2+(l-\tilde{x})^2}}{-x+\sqrt{\tilde{x}^2+\tilde{y}^2+\tilde{z}^2}}\right] \qquad (3)$$

It is assumed, that the ground potential below the voltage line is set to φ = 0. To evaluate the field quantities with respect to this boundary condition, the line-charge has to be mirrored at the plane x-y. The superposition of line-charge q and mirror-charge $-q$, indicated in fig. 2, gives the potential φ in the point $P(x,y,z)$ inside the global co-ordinate system.

$$\varphi(\tilde{x},\tilde{y},\tilde{z},\tilde{x}_m,\tilde{y}_m,\tilde{z}_m) = \frac{q}{4\pi l\varepsilon}\ln\left\{\frac{\left[l-\tilde{x}+\sqrt{(l-\tilde{x})^2+\tilde{y}^2+\tilde{z}^2}\right]\cdot\left[-\tilde{x}_m+\sqrt{\tilde{x}_m^2+\tilde{y}_m^2+\tilde{z}_m^2}\right]}{\left[-\tilde{x}+\sqrt{\tilde{x}^2+\tilde{y}^2+\tilde{z}^2}\right]\cdot\left[l-\tilde{x}_m+\sqrt{(l-\tilde{x}_m)^2+\tilde{y}_m^2+\tilde{z}_m^2}\right]}\right\} \qquad (4)$$

With the known complex potentials $\underline{\varphi}_i$ of the i conductors and eq. (2), to compute the coefficient matrix **A**, a linear set of equations can be formulated by

$$\mathbf{A}\cdot\underline{\mathbf{q}} = \underline{\varphi} \qquad (5)$$

Now this equation system with two right hand sides, the real- and the imaginary component of the potential, is solved. The solution determines the charge $\underline{q}_i$ of each element of the transmission conductors. With this value the components of the electro-static field-strength in the point $P(x,y,z)$ can be computed with.

$$\mathbf{E} = -grad\varphi = -\left(\frac{\partial\varphi}{\partial x}\mathbf{a}_x + \frac{\partial\varphi}{\partial y}\mathbf{a}_y + \frac{\partial\varphi}{\partial z}\mathbf{a}_z\right) \qquad (6)$$

The interesting equivalent value of E_e (DIN VDE 0848/T. 1) is the geometrical sum of the single components in x-, y- and z-direction in the global co-ordinate system.

$$E_e = \sqrt{E_x^2 + E_y^2 + E_z^2} \qquad (7)$$

4. MAGNETIC FIELD.

The magnetic field problem is considered to be linear. Therefore, the field quantities can be computed and superposed using the BIOT' SAVART law. Each element of the polygon from fig. 1 carries now a current $i(t)$. The generated flux density of this part of the conductor is given by

$$|d\mathbf{B}| = \frac{\mu_0}{4\pi r^2}\cdot i(t)\cdot dl\cdot\sin\alpha \qquad (8)$$

integrating eq. (8), the flux density is calculated with

$$|\mathbf{B}_j| = \frac{\mu_0}{4\pi r}\cdot i(t)\cdot\left(\frac{l-\tilde{x}}{\sqrt{(l-\tilde{x})^2+r^2}} + \frac{\tilde{x}}{\sqrt{\tilde{x}^2+r^2}}\right) \qquad (9)$$

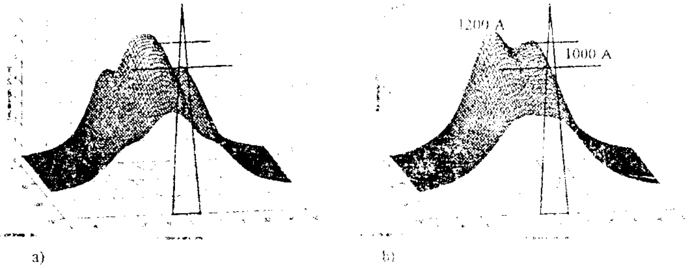
If n is the number of current leading conductors, the superposition of the partial flux densities results in the overall flux density.

$$\mathbf{B} = \sum_{j}^{n}\mathbf{B}_j \qquad (10)$$

5. RESULTS AND CONCLUSIONS.

Figure 3 shows the plotted results of a field computation of a high voltage transmission line with two 400 kV systems. The plotted values are calculated in a heights of 1 m above the ground plane. As expected, the field strengths of both, electric and magnetic field rapidly decrease perpendicular to the direction of the high voltage line. The highest values are calculated at the point of largest slag of the span field.

The computational costs depend on the number of polygon elements approximating the slag of the phase conductors. In Hameyer[1] it is shown that an approximation with 14 elements per span field length an acceptable accuracy is achieved. The paper discussed an efficient method for the three dimensional computation of the electric and magnetic field quantities generated by high voltage lines.



a) b)

Fig. 3.: a) Electro static- and b) magnetic field distribution of two 400 kV voltage systems 1 m above ground

6. ACKNOWLEDGEMENTS.

The authors express their gratitude to the Belgian Ministry of Scientific Research for granting the IUAP No. 51 on Magnetic Fields and the Council of the Belgian National Fond of Scientific Research.

7. REFERENCES.

[1] Hameyer, K., Hanitsch, R. and Belmans, R.: "Optimisation of the electro static field below high-voltage lines", *Proc. 6th International IGTE Symposium on Numerical Field Calculation in Electrical Engineering*, September 24-28, 1994, Graz, Austria, pp. 264-269.

[2] IRPA: "Interim guidelines on limits of exposure to 50/60 Hz electric and magnetic fields", *Health Physics*, Vol. 58, No. 1, January 1990, pp. 113-122.

[3] Schwetz, P.: "Ein Verfahren zur Berechnung von dreidimensionalen Magnetfeldern im Nahbereich von Leitungen der elektrischen Energieversorgung", *Elektrizitätswirtschaft*, 85 (1986) 21, 827-831.

[4] Utmischi, D.: *"Das elektrische Feld unter Hochspannungsleitungen"*, Ph.D. Thesis Universität München, 1976.