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DESIGN AND OPTIMISATION OF ELECTROMAGNETIC DEVICES

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ABSTRACT

The development and design of electromagnetic devices reflects a complex process. Originating from an initial idea, the construction runs through different phases. This procedure is terminated when a final concept is selected and considered to be optimised, subject to various aims. As the whole, the task of the design engineer is to find solutions for technical problems. On the way to the latter physical and technical product, certain aspects have to be considered. Technological and material depending questions, the cost effectiveness and ecological constraints have to be taken into consideration. A cut-set of the mentioned boundary conditions controls the feasibility of the final design. With emphasis to electromagnetic devices, fig. 1 shows a simplified scheme of interdependencies of different constraints. This simple pattern makes clear, that the design process is strongly depending on the experience of the engineer and it reflects an optimisation procedure with partly contradictory aims. Therefore, the necessity of a systematic design with engineering tools is obvious.

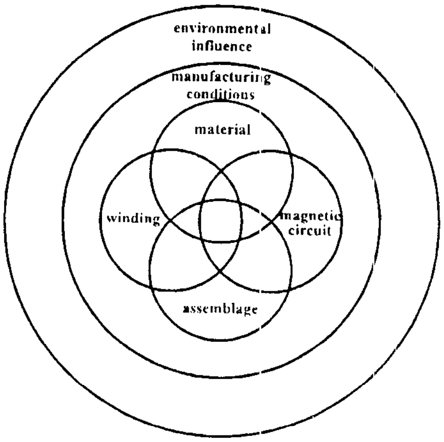


Figure 1: Interdependencies in the design of electromagnetic devices.

In the design of electromagnetic devices nowadays numerical methods become more and more common. With increasing computer speed complex problems can be solved with a high economical efficiency. To accelerate development expenditure field computation of complicated geometry's with various types of material can be performed avoiding expensive prototyping. As a consequence numerical optimisation algorithms are combined with field calculation methods. This paper introduces a heuristic optimisation technique of general application range. The optimisation algorithm used is a combination of evolution strategy and simulated annealing.

DEVELOPMENT AND DESIGN STRATEGIES

To put the numerical optimisation procedure in its proper place, the development process is commented briefly. The main part in the structured development of novel technical products is the analysis followed by a detailed synthesis. Analysing means to obtain information about partial functions of the desired overall function investigating single elements and their mutual interactions. In this way global connections between various principles of the partial functions are found.

In fig. 2 a structured development process is illustrated. There, a technical product has to be designed, able to fulfil three partial functions. Those individual functions to be connected to the overall task of the product are a linear motion, a continuos and a discontinuous rotation. After the analysis phase in the synthesis step different principles are selected. The selection process is governed by qualitative rules. In this way the partial functions are judged due to their feasibility with respect to given constraints. The constraints are derived from an object world. Properties of the materials and electronic circuits connected to the electromagnetic device with their restrictions concerning to the function of the principle, are found in the object world. The second influencing parameters are located in a world of rules. Those rules are related to the modelling of the materials and electric circuit theory .

The process of synthesis leads from qualitative decisions to quantitative statements. The whole process is accompanied by the consultation of experts and expert knowledge as illustrated in fig. 2. A detailed investigation and ranking, that means the precise calculation of the operating conditions, leads by a comparison to the final technical product. In this iteration loop between valuation and performing detailed predictions of qualified concepts, the numerical optimisation combined with field computation methods is found as an important and powerful engineering tool.

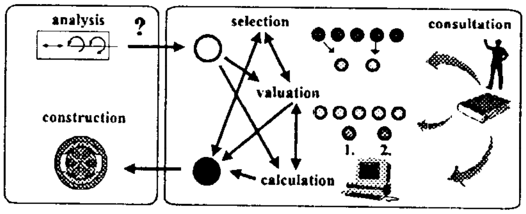


Fig. 2: Knowledge based and structured design.

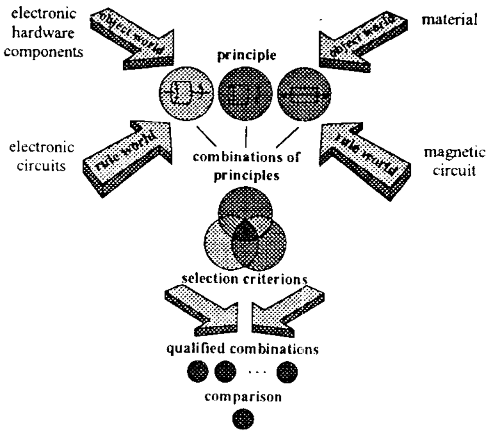


Fig. 3: Process of synthesis.

ELECTROMAGNETIC FIELD COMPUTATION

Depending on the class of field problem different computation methods can be used. To select the most efficient method, again a decision process is necessary. The various possibilities are ranging from simplified analytical approaches over semi numerical up to numerical techniques. The drawbacks and strengths of the single technique can be read out of fig. 4. Here, the most common computation techniques for electromagnetic field problems are listed. The main difference between the listed methods is the possibility to approximate the geometry of the electromagnetic circuit. The required accuracy of the solution depends on the discretisation of the geometry of the device.

Looking to the point matching method (PM), it is obvious that the use of this method is limited to geometrically simple shapes. Non-linear ferromagnetic parts of the magnetic circuit can only be considered by constant factors. The magnetic equivalent circuit (MEC) represents flux paths by magnetic reluctance's. The field distribution must be known in principle before the magnetic circuit can be modelled. The main advantage of this lumped parameter approach is its computational speed. The boundary element method (BEM) approximates complex geometry's very precisely. Only the boundary of the interesting area has to be discretised. The treatment of non-linearity's is troublesome. With the finite difference method (FDM) local mesh refinements are not possible and this technique has its shortcoming in complex geometry's. The field computation method with the most general application range is the finite element method (FEM). Due to the discretisation of the field region by non overlapping triangles, complex geometry's can be approximated. Non-linearity's and local mesh refinements are possible. The high computational costs of this method may be a drawback.

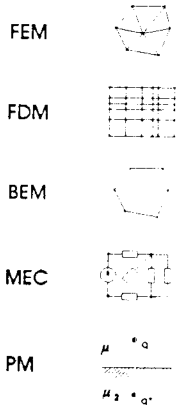


Fig. 4: Field computation methods.

In principle, field computation is performed in three major steps: Pre-processing, processing and post-processing. Figure 5 shows a typical pattern for the FEM approach. The first step consists of the definition of the geometry of the electromagnetic device. Material properties, electrical current densities and boundary conditions are defined. All the activities have to be carried out by the design engineer. Therefore, the pre-processing is time consuming. An estimated time expenditure for all three steps is given in fig. 5. The processing, i.e. the solution of a very large system of equations is done automatically in the second step. Only parameters to control the solution process have to be defined by the design engineer. In the last part of the FEM procedure, the interesting field quantities are computed from the solution out of the processing. If the geometrical data's can be parametrised the pre- and post-processing can be automated too. This represents an important prerequisite for the possibility of the combination of field computation and numerical optimisation.

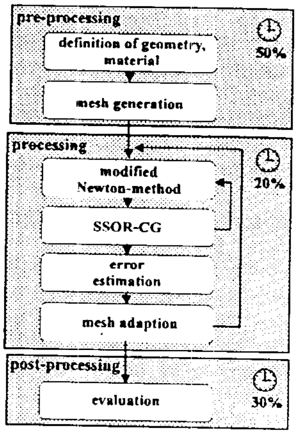


Fig. 5: FEM procedure.

To demonstrate the importance of electromagnetic field computation, i.e. the simulation of the physical behaviour of a technical system, the connection between classical theory, experiments and simulations is described. Figure 6 shows the influence of computer simulation to the theory and the experiment clearly. The numerical simulation is oriented to the existing theory and the verification of experimental results. Therefore, simulation represents the link between theory and experiment. If the mathematical model approximates the physical system with satisfying accuracy, cost intensive prototyping and measurements can be reduced to a minimum. This leads to development cycles of high economical efficiency. The supplementary combination with numerical optimisation strategies promises a self controlled automatic optimal design (AOD).

NUMERICAL OPTIMISATION

In general optimisation means to find the best solution for a given problem formulation under the consideration of prescribed constraints. In mathematical terms:

Define a point $\mathbf{x}_0 = (x_1, x_2, .. x_n)^T$ *with the independent variables* $x_1 x_2, .. x_n$ *in such a way, that by the variation of the objective variables, inside the admissible space, the value of a quality function* $Z(\mathbf{x}_0)$ *reaches a minimum or maximum. The point* $\mathbf{x}_0$ *is described as the optimum.*

For the optimisation the exact definition of the space of variables, the restricting constraints and a quality or objective function are necessary. The correct and functional expression of the objective function is of special importance. The latter product of the optimisation is a reflection of this function. The quality function must guaranty the existence of an extremum. This can be reached by the right choice of the restrictions. The objective variables should be normalised to get a well conditioned problem formulation, i.e. small variations of the variables result in a small change of quality. In the case multiple aims have to be followed, a weighted linear combination of the single targets is useful.

$$Z = \sum_{i=1}^{n} a_i Z_i \qquad (1)$$

Here, a_i are the weighting factors and Z is the overall quality. A prerequisite for a successful optimisation is a clear definition of all partial target and the variability of the objective variables, i.e. the single targets must not contradict too strongly. In general a mathematical optimisation problem can be formulated as follows:

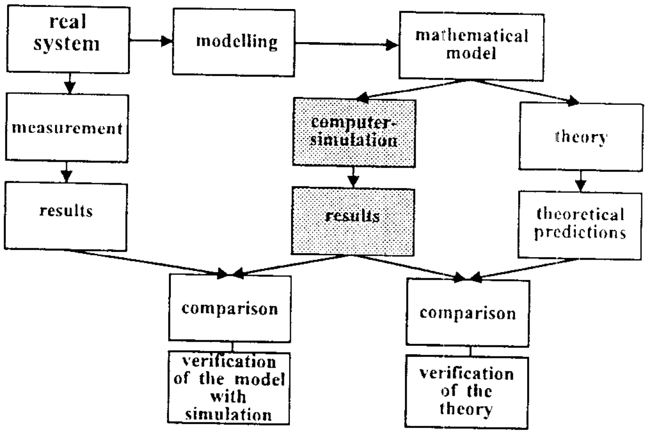
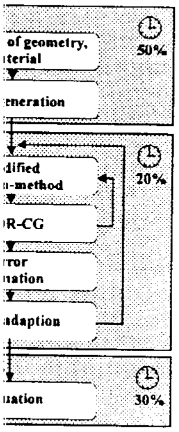


Fig. 6: Connection of theory and experiment with simulation.



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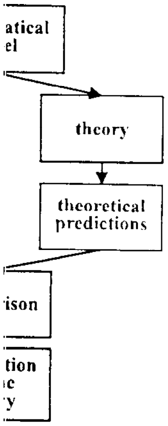


ı simulation.

$$Z(\mathbf{x}) = (x_1, x_2, \ldots, x_n) \rightarrow \text{min.} \tag{2}$$

with the independent variables $\mathbf{x} = \{x_i : i = 1(1)n\}$ in the space $\mathbf{x} \in IR^n$ and the $j=1(1)m$ constraints

$$C_j(\mathbf{x}) = C_j(x_1, x_2, \ldots, x_n) \begin{Bmatrix} = \\ < \end{Bmatrix} 0 \tag{3}$$

Optimisation algorithms are generally constructed in the way that the desired optimum is reached step by step. This happens through determined rules. The generation of the step width that varies the objective variables, the search direction on the way to the optimum and the stopping criterion distinguish between the various numerical optimisation algorithms. Detailed investigations and descriptions of different techniques can be found in [1]. Due to complicated dependencies of the free variables, the direct computation of derivatives of the quality function is troublesome or sometimes not possible. Stochastic methods do work without the use of derivatives. They are easy to implement and the treatment of constraints is simple. These are the reasons why stochastic methods are a reliable tool in the field of numerical optimisation. In the following only two stochastic search techniques and their methodology are described.

Evolution Strategy

The evolution strategy copies the principles of the biological evolution into the mathematical process of optimisation. The term mutation describes the variability of the objective variables and the term selection CHARLES DARWIN's (1809-1882) postulate *survival of the fittest*. The driving force of this strategy is the repetition of the mutation of the variables and selection in successive steps. In fig. 7 a simple pattern of the evolution strategy is drawn.

Some important features and possibilities of the algorithm can be taken from this drawing. If each fruit represents a variable, the procedure starts with a given number of μ parents. To generate the λ offspring's, mutation of the objective variables is done by random additions to the parent variables. The arrows indicate that ρ parent variables contribute to the generation of a single child. With the hereditary factor ρ in average $1/\rho$ of the properties of the parents are transferred to an offspring. If the step width is too small, this slows down and the algorithm gets trapped in a local optimum. The other case of a too large stepwidth results in pure fluctuation. To get an efficient algorithm, the step width must be self adaptive. Therefore, an intermediate value of steplength of the parents is transferred to the offspring's. The search direction is generated in the way, that the probability of every direction in the solution space is uniformly distributed. With the before declared control parameters it can be distinguished between several evolution strategies differing in the type of selection. The most powerful strategy is the at last listed *comma* variant.

- $(\mu + \lambda)$-*plus*-strategy:

  The population of the next generation is selected from μ parents *and* λ children.

- (μ, λ)-comma-strategy:

  The population of the next generation is selected from the μ best children. Parents survive only one single generation.

- $(\frac{\mu}{\rho} + \lambda)$-strategy:

  ρ parents contribute to the creation of a child. $1/\rho$ of the properties of one parent is transmitted to a child.

- $(\frac{\mu}{\rho}, \lambda)$-strategy:

  Comma variant of the last listed strategy.

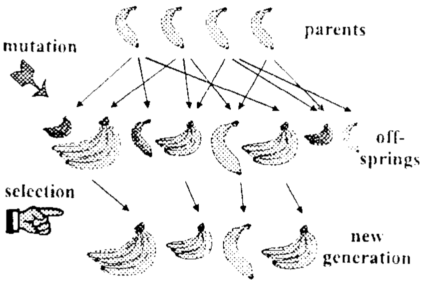


Fig. 7: Simple pattern of schematic of the evolution strategy.

Simulated Annealing

Annealing describes the physical process of heating up a solid to a maximal temperature at which all molecules are freely moving and the process of slowly cooling down until a state of minimum free energy. The decrease of temperature must be that slow that at every temperature the solid is in a thermal equilibrium. This process describes a natural optimisation with object to minimise the free energy. The probability of the energy E_i in a state i at temperature T is described by the BOLZMANN distribution. Therefore, the probability of the change of energy can be expressed by:

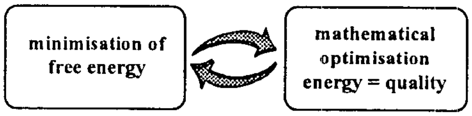


Fig. 8: Idea of simulated annealing.

$$\xi < prob\{\Delta E_i\} = \exp(-\frac{\Delta E_i}{k_B T}) \quad . \tag{4}$$

Substituting the term energy with the term quality, as indicated in fig. 8, the physical process and (4) can be used in the mathematical optimisation as an acceptance criterion. If the variation of objective variables results in a better quality, the configuration is accepted. On the other hand, if the change in quality is larger then zero, the configuration is treated as follows. A uniformly distributed random number ξ out of the interval [0,1] is generated. If the METROPOLIS criterion (4) is true [6], the new configuration is accepted. In the other case, a new mutation on the free variables is performed. To complete the algorithm, a temperature schedule is necessary. The most simple one is a linear schedule [4].

Combined Search algorithm

Test calculations discovered a weakness of the evolution strategy. Not in every case the global optimum is found. In contrast, simulated annealing is well known for its reliability to find a global extremum. Therefore, the evolution strategy is combined with simulated annealing. The algorithm is drawn in fig. 9.

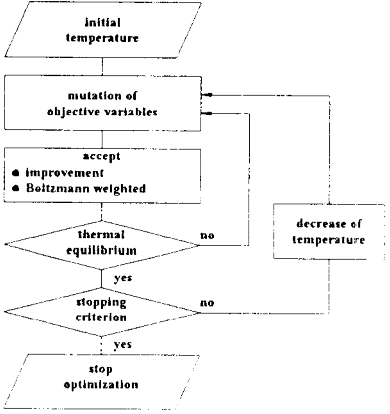


Fig. 9: Combined search algorithm.

SHAPE OPTIMISATION OF A SMALL DC-MOTOR

With the combined optimisation technique, the shape of a small dc-motor is optimised, aiming to minimise the overall material expenditure. Quality function evaluation is done with the FEM. Figure 10 illustrates the problem formulation. The free variables are the co-ordinates of the data sample of a polygon describing the rotor slot shape, the rotor- and the outer-radius of the motor. In this example 14 objective variables are tuned. The quality function is formulated in the way, that a minimum quantity of torque T_{min} is obtained due to a penalty term.

$$Z(x) = 10^{\left(\frac{cost(x) - cost_{max}}{cost_{max}}\right)} + penalty \tag{5}$$

$$penalty = \begin{cases} T < T_{min} & : \quad 10^{\left(\frac{T_{min} - T(x)}{T(x)}\right)} \\ T \geq T_{min} & : \quad 1 \end{cases} \tag{6}$$

The overall costs are estimated by the sum of copper-, lamination- and permanent magnet-expenditure. Constraints result from manufacturing conditions. The air gap is set to a fixed value, minimal tooth width and slot opening are given. The initial shape of the motor generates 75% of the desired torque. The change of geometry during the optimisation is shown in fig. 11. The quality expressed by the material costs decreased by 10%. In this example a (12/12, 60)-strategy has been used.

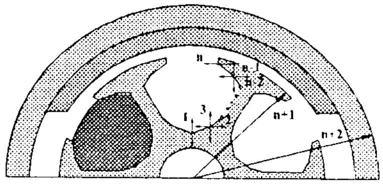


Fig. 10: Motorgeometry with variable definition.

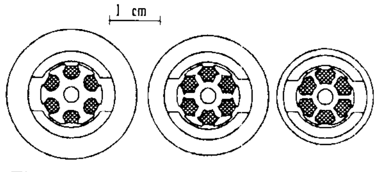
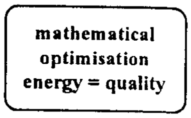


Fig. 11: Initial, temporary and optimised shape.

MINIMISATION OF THE ELECTRIC FIELD OF A HIGH-VOLTAGE LINE

To demonstrate the general application range of the optimisation algorithm, in this example the objective function evaluation is performed by the point matching method, in order to minimise the average value of the electrostatic field generated by high-voltage lines. Due to their influence on the field distribution, the objective variables are the co-ordinates of the ground lines. With respect to the slack of the phase cables a three dimensional field computation



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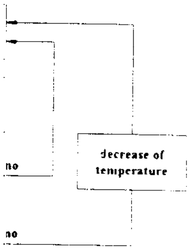

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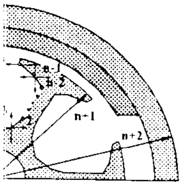

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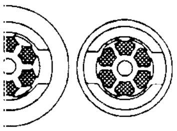

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is required. The single transmission line is approximated by a polygon as shown in fig. 12. Each element of the polygon represents a constant line-charge. It is assumed, that the ground potential below the high-voltage line is zero. To evaluate the field quantities with respect to this boundary condition, the potential of the line-charge has to be mirrored at the ground plane xy. Superposition of the original and the arbitrary mirror charge gives the resulting potential. In fig. 13 the initial and optimised pole configuration are drawn. From fig. 14 the computed field distribution, one meter above the ground, can be taken. During optimisation the average value of electric field-strength decreased about 8,6% in total. For the computation a (4/4,12)-strategy is used.

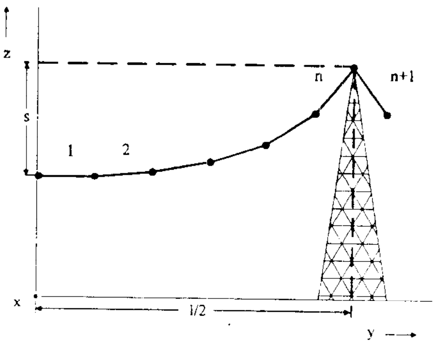

Fig. 12: Modelling of a conductor by a polygon.

CONCLUSION

The paper explains, where the numerical optimisation finds its proper place inside a development process. A combined stochastic search method is introduced. This technique together with field computations leads to a powerful tool for the design of electromagnetic devices. Two examples out of the area of electromagnetic fields demonstrate the general application range of the methods.

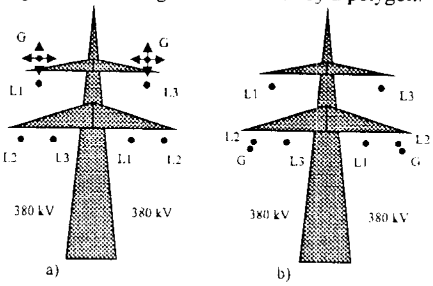

Fig. 13: a) Initial and b) final pole configuration.

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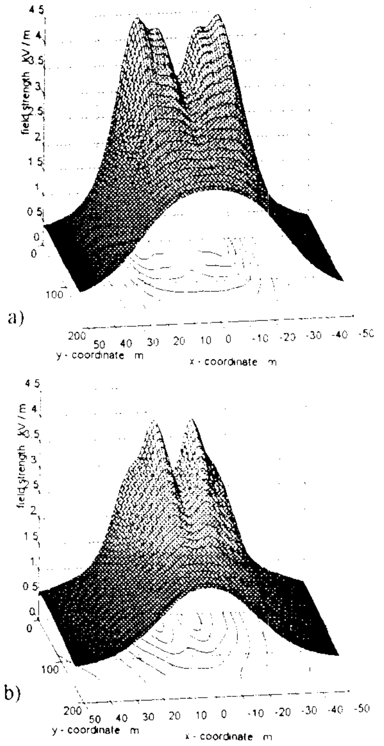

Fig. 14: a) Initial and b) optimised field distribution.