Technique calculating Equivalent Circuit Models for electrostatic Micromotors using 2D or 3D finite elements

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Abstract-- **The continuous improvement in finite element software for electromagnetics and electrostatics together with the continuous increase of the speed/cost ratio for computers have lead to extended possibilities of solving field problems in order to extract complex global quantities of electrical devices. However, analysing electric rotating machines for global quantities as average torque or a complex characteristics as torque as a function of the excitation wave form, remains tedious to do using only finite elements. The main reason is the amount of field solutions that such quantities and characteristics require. A more efficient approach of calculating global properties and characteristics directly with finite elements, is to first generate an equivalent circuit model of the electrical machine, and then derive all desired properties from this equivalent circuit.**

This paper presents a technique to compute equivalent circuit models (ECMs), for electrostatic micromotors of the type of variable capacitance motor. The technique applies to any pole configuration i.e. combinations of number of stator poles and number of rotor poles. The same technique can be used with both, 2D and 3D finite elements.

I. Defining the Equivalent Circuit Model

The ECM used for variable capacitance micromotors is in its simplest representation a circuit of capacitances. Fig. 1 shows an ECM for a motor having 3 stator and 4 rotor poles, a 3/4 pole motor.

The capacitances of the ECM are periodic functions of the rotor position α. As can be seen in Fig. 1, the ECM consists of two principally different capacitance functions. The first type is the capacitance function representing capacitances between each stator electrode and the rotor, $C_k^{SR}(\alpha)$. The second type represents capacitance between each pair of consecutive stator electrodes $C_k^{SS}(\alpha)$. For $C_k^{SR}(\alpha)$ k represents the electrode number. For $C_k^{SS}(\alpha)$ k is the number of the first electrode in the pair. These functions are periodic with an angle equal to is the rotor pole pitch τ_{p2} Functions with different index k are all identical except for a phase shift equal to a multiple of the stator pole pitch τ_{p1}. Numbering the stator electrodes in the same direction as defined by the positive rotor rotation, the capacitance functions for different

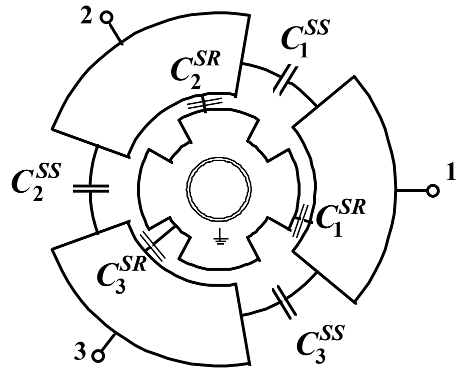

Fig. 1: Outline and equivalent circuit model of a variable capacitance motor with 3 stator poles.

k can be written as:

$$\left.\begin{aligned} C_k^{SR}(\alpha) &= C_1^{SR}(\alpha-(k-1)\tau_{p1}) \\ C_k^{SS}(\alpha) &= C_1^{SS}(\alpha-(k-1)\tau_{p1}) \end{aligned}\right\} \quad \left\{k = 1,2,3,\ldots n_{p1}\right\}. \tag{1}$$

If the phase angle $(k-1)\tau_{p1}$ becomes larger than the period of the capacitance function, being equal to τ_{p2}, it is substituted for the smallest positive angle difference between the functions. This smallest difference is extracted using the mod function:

$$\beta_k = (k-1)\tau_{p1} \bmod \tau_{p2}. \tag{2}$$

Using (2), (1) can be rewritten as

$$\left.\begin{aligned} C_k^{SR}(\alpha) &= C_1^{SR}(\alpha-\beta_k) \\ C_k^{SS}(\alpha) &= C_1^{SS}(\alpha-\beta_k) \end{aligned}\right\}. \tag{3}$$

II. Extracting the ECM Parameters

To extract the values of each capacitance function, a system of equations must be set up. The equation used is the formulation of the stored electric energy in the capacitances:

$$W_e(\alpha) = \tfrac{1}{2}\sum_{k=1}^{n_{p1}}\left[C_k^{SR}(\alpha)\cdot V_k^2 + C_k^{SS}(\alpha)\cdot\left(V_{k+1}-V_k\right)^2\right], \tag{4}$$

where V_k is the potential of the electrode k, n_{p1} = number of stator electrodes and $V_{(n_{p1}+1)} = V_1$. The rotor is assumed to be at reference potential. Using 2D or 3D finite elements, the stored energy of the motor is calculated for two different

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excitations and a number of rotor positions. With these energies the system of equations is set up. As the ECM contains two principally different capacitance functions, exactly two different excitations must be set and solved for each rotor position. Any two excitations can be used as long as they, for the same rotor position, energise a different set of capacitances. However, these two sets may have some capacitances in common.

The two different excitations are referred to as A and B. For excitation A (Fig. 2), a voltage V is applied to only one node in the ECM, corresponding to one stator electrode of the motor. All the other nodes are at the reference potential.

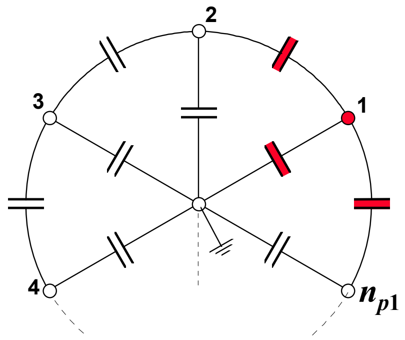

Fig. 2. Excitation type A. n_{p1} indicates the number of stator electrodes.

By this excitation, the three capacitances around the excited node are storing energy and the total stored energy for excitation A is given by:

$$W_1^A(\alpha) = \tfrac{1}{2}V^2\left[C_1^{SR}(\alpha) + C_1^{SS}(\alpha) + C_{n_{p1}}^{SS}(\alpha)\right]. \tag{5}$$

For excitation B (Fig. 3), a voltage V is applied to two consecutive nodes in the ECM. Using this excitation four capacitances around the excited nodes are storing energy. Since there is no voltage difference between the two excited nodes, the capacitance between these terminals does not store any energy.

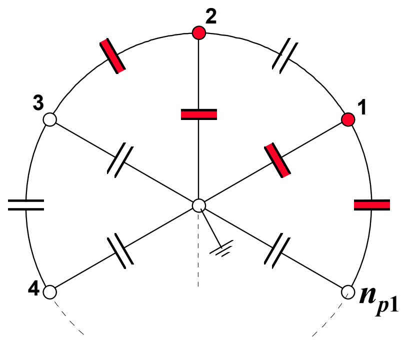

Fig. 3. Excitation type B. n_{p1} indicates the number of stator electrodes.

The overall stored energy for excitation B is:

$$W_1^B(\alpha) = \tfrac{1}{2}V^2\left[C_1^{SR}(\alpha) + C_2^{SR}(\alpha) + C_2^{SS}(\alpha) + C_{n_{p1}}^{SS}(\alpha)\right]. \tag{6}$$

III. Number of FE Solutions Required

The number of excitations A and B which can be set is, disregarding the pole configuration, equal to the number of stator poles of the motor. The system of equations set up in this way requires twice as many energy values as the numbers of stator poles, and this for each rotor position where the capacitance functions should be defined. To discretice one period of the capacitance functions by 9 rotor position would for a motor with 3 stator poles require 2*3*9=54 different energy values and thus as many finite element solutions. However, by substitutions and a careful choice of rotor positions this number can be reduced to 2*9=18, thus, two times the number of rotor positions.

Since the energy functions from (5) and (6) are periodic with the same period as the capacitance functions, using (3) also the energies with different index k may be substituted for energies with index 1 and a phase angle.

$$\left.\begin{aligned} W_k^A(\alpha) &= W_1^A(\alpha - \beta_k) \\ W_k^B(\alpha) &= W_1^B(\alpha - \beta_k) \end{aligned}\right\}. \tag{7}$$

Using (7), the system of equations for the motor from Fig. 1 can be written as:

$$\begin{bmatrix} W_1^A(\alpha-\beta_1) \\ W_1^A(\alpha-\beta_2) \\ W_1^A(\alpha-\beta_3) \\ W_1^B(\alpha-\beta_1) \\ W_1^B(\alpha-\beta_2) \\ W_1^B(\alpha-\beta_3) \end{bmatrix} = \tfrac{1}{2}V^2 \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} \bullet \begin{bmatrix} C_1^{SR}(\alpha) \\ C_1^{SS}(\alpha) \\ C_2^{SR}(\alpha) \\ C_2^{SS}(\alpha) \\ C_3^{SR}(\alpha) \\ C_3^{SS}(\alpha) \end{bmatrix}. \tag{8}$$

By inverting the coefficient matrix the capacitance functions may be expressed explicit.

$$\begin{bmatrix} C_1^{SR}(\alpha) \\ C_1^{SS}(\alpha) \\ C_2^{SR}(\alpha) \\ C_2^{SS}(\alpha) \\ C_3^{SR}(\alpha) \\ C_3^{SS}(\alpha) \end{bmatrix} = \frac{1}{V^2} \begin{bmatrix} 0 & -1 & -1 & 1 & 0 & 1 \\ 1 & 1 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & -1 & 0 \\ -1 & -1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \bullet \begin{bmatrix} W_1^A(\alpha-\beta_1) \\ W_1^A(\alpha-\beta_2) \\ W_1^A(\alpha-\beta_3) \\ W_1^B(\alpha-\beta_1) \\ W_1^B(\alpha-\beta_2) \\ W_1^B(\alpha-\beta_3) \end{bmatrix} \tag{9}$$

The way to ensure that not more finite element solutions than twice the number of rotor positions are needed, is to choose the series of rotor positions α equidistantly and in that way that the angles β_k are a part of this series. For a 3/4 pole motor the stator pole pitch τ_{p1} is 120°mech. and the rotor pole pitch τ_{p2} is 90°mech. Using (2) giving $\beta_1 = 0$,

$\beta_2 = 30$ and $\beta_3 = 60\,°$mech., and discretising one electric period by nine rotor positions, a set of α and corresponding angles $(\alpha - \beta_k)$ are derived.

TABLE I

Rotor positions α and corresponding set of angles (α–β_k) for which the stored energy must be known in order to solve the ECM for nine rotor positions. Rotor positions equal to β_1, β_2 and β_3 , are underlined.

α	<u>0</u>	10	20	<u>30</u>	40	50	<u>60</u>	70	80
$(\alpha - \beta_1)$	0	10	20	30	40	50	60	70	80
$(\alpha - \beta_2)$	60	70	80	0	10	20	30	40	50
$(\alpha - \beta_3)$	30	40	50	60	70	80	0	10	20

IV. NUMERICAL RESULTS

The technique described has been applied during optimisation of electrostatic variable capacitance motors [5]. The capacitance functions were derived for each analysed motor and are given here for two specific motors. One radial flux motor designed for a high average torque and one axial flux motor designed for a low torque ripple.

The radial flux motor has six stator electrodes and four rotor teeth. The corresponding ECM is shown in Fig. 4. The geometric parameters are indicated in Fig. 5. For a specific motor is r_1 = 300 µm, r_{slot} = 210 µm, δ = 10 µm, τ_1 = 42°, τ_2 = 40.5°, τ_{p1} = 60°, τ_{p2} = 90° and the axial length 100 µm.

The capacitance functions $C_k^{SR}(\alpha)$ and $C_k^{SS}(\alpha)$ of this motor is shown in Fig. 6 and 7 and are derived using 18 equidistant rotor positions. Choosing the rotor positions equidistant is required in order to make the functions continuous using discrete Fourier transform. The continuous functions of Fig. 6 and 7 are filtered to contain 7 of the maximum 9 harmonics which can be reconstructed from 18 samples. The rotor position of rotor in Fig. 4. is 25° mech. and positive rotation is counter clockwise.

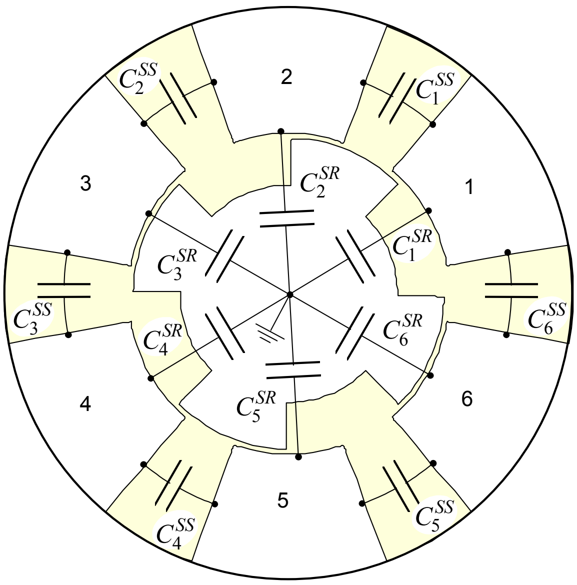

Fig. 4: The ECM for a motor with 6 stator electrodes.

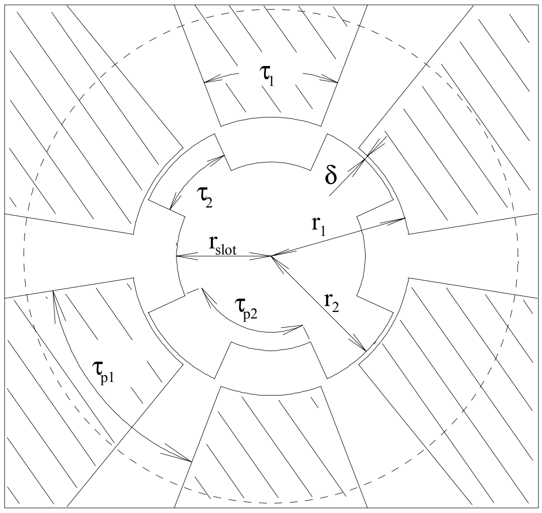

Fig. 5: The geometric parameters describing the motor geometry.

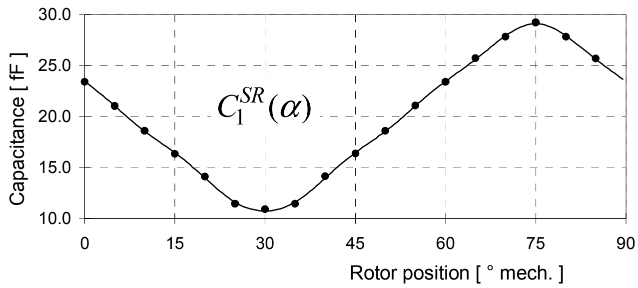

Fig. 6: Capacitance function stator to rotor of a radial flux.

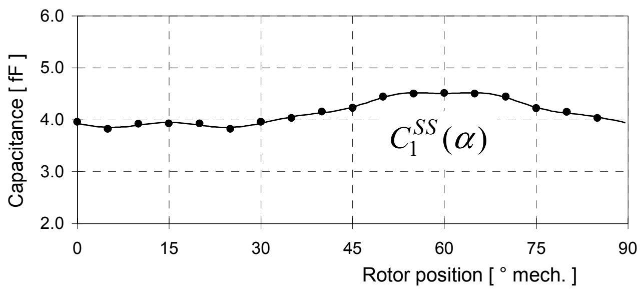

Fig. 7: Capacitance function between consecutive stator electrodes of a radial flux motor.

For the axial flux motor in Fig. 8 the stator electrodes overlap the rotor teeth on two sides. The stator electrodes are excited by different potentials and the rotor is set to reference potential. This causes the electric field to be directed along the motor axis, thus in axial direction.

For this motor the same ECM as shown in Fig. 4 applies. The only differences are the shapes of the capacitance functions and their periodicity. This axial flux motor has six stator electrodes and eight rotor teeth and one electric period is thus 45°mech. instead of 90°mech. for the radial flux motor.

The capacitance functions $C_k^{SR}(\alpha)$ and $C_k^{SS}(\alpha)$ of this motor are shown in Fig. 9 and 10 and are as in the previous example derived using 18 equidistant rotor positions. The geometric parameters of the axial flux motor are: r_1 = 160 µm, r_2 = 100 µm, r_{slot} = 90 µm, δ = 3 µm, τ_1 = 18°, τ_2 = 13.5°, τ_{p1} = 60°, τ_{p2} = 45° and the rotor thickness is 4 µm. The rotor position of the rotor in Fig. 8 is 0° mech.

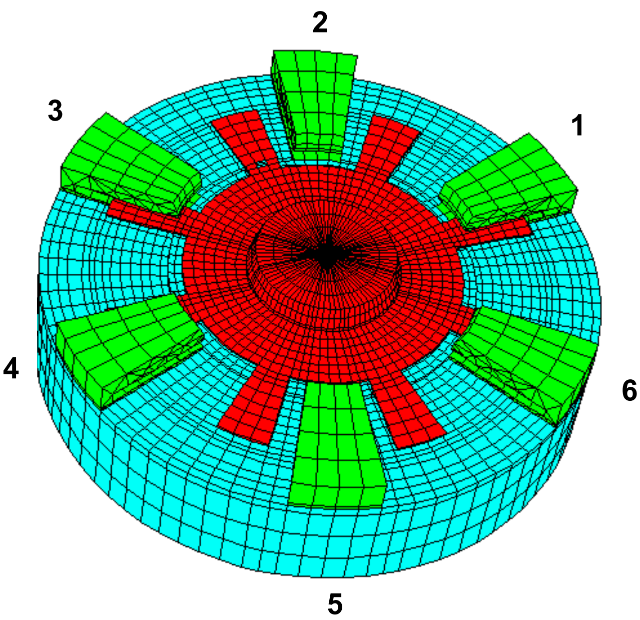

Fig. 8: The 3D mesh of the axial flux motor design to give a low torque ripple.

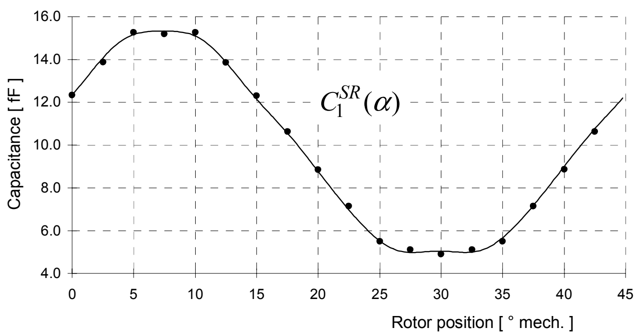

Fig. 9: Capacitance function stator to rotor of a axial flux motor designed to give a low torque ripple.

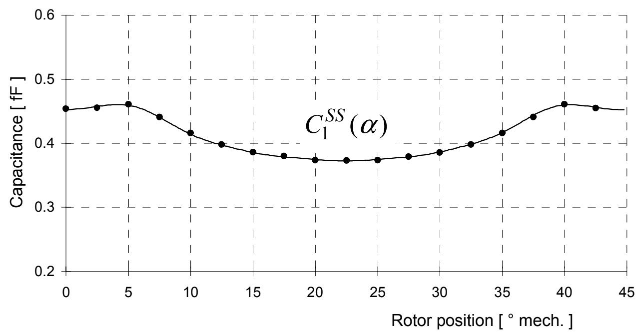

Fig. 10: Capacitance function between consecutive stator electrodes of a axial flux motor designed to give a low torque ripple.

V. Conclusions

Calculating the ECM of a motor has undoubtedly many benefits. The ECM for a variable capacitance micromotor has been one of the most important tools when using the it for computing the average torque and the torque ripple for electrostatic micromotors. This was done in order to perform an optimisation of these motors [4] and [5]. The time consumption was in these cases dramatically reduced. Using an ECM does also save storage space. Finite element calculations, especially in 3D, are creating a substantial amount of data . The main part of this data is omitted once the parameters of the ECM are found. The ECM technique presented here can furthermore easily be complemented with other parameters independent from the rotor position as e.g. resistance, friction, rotor inertia, etc. The ECM complemented with such parameters offers the possibility for the dynamic analysis of micromotors [6].

References

[1] David K. Cheng, "*Field and Wave Electro-Magnetics*" Addison-Wesley Publishing Company, Menlo Park, Don Mills, Amsterdam, Sydney, Singapore, Tokyo, 1983.

[2] D.A. Lowther P.P. Silvester, "*Computer Aided Design in Magnetics*" Springer-Verlag, Berlin New York Heidelberg Tokyo, 1985

[3] W.Trimmer: "Micromechanical systems", *The Third Toyota Conference Program*, October 22-25, 1989, Toyota Motor Corporation, Nisshin International Education Building, pp.1.1-1.32.

[4] T.B. Johansson, M. Van Dessel R. Belmans W. Geysen, "Technique for finding the optimum geometry of electrostatic micromotors. " *IEEE Transactions on Industry Applications*, July / August 1994, Volume 30 Number 4, pp. 912 - 919.

[5] T.B. Johansson, K. Hameyer, R. Belmans, "3D Optimisation of Average Torque of Electrostatic Micromotors", *MST Journal on Microsystem Technologis, Sensors Actuators System Integration*, Volume1, Number 2, March 1995, pp 98-103

[6] E.Sarraute, Y.Lefèvre, M.Lajoie-Mazenc, "Modelisation and simulation of the dynamics of variable capacitance micro motors", *ICEM '94*, Paris, France, 5-8 Sept. 1994, pp. 305-310.